U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

October 2024 Supplementary Examinations

Programme: B.E.	**Semester: VI**
Branch: Institutional Elective	**Duration: 3 hrs.**
Course Code: 23MA6OESFE	**Max Marks: 100**
Course: Mathematical Statistics for Engineers	

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

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Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

<table>
<tr><th></th><th></th><th>UNIT - I</th><th>CO</th><th>PO</th><th>Marks</th></tr>
<tr><td>1</td><td>a)</td><td>Discuss the four levels of measurement scales. Give examples of each and explain in detail.</td><td>CO1</td><td>PO1</td><td>06</td></tr>
<tr><td></td><td>b)</td><td>Indian times to work of employees of a company were as follows:
<table>
<tr><td>Times in minutes</td><td>0 - 10</td><td>10 - 20</td><td>20 - 30</td><td>30 - 40</td><td>40 - 50</td><td>50 - 60</td><td>60 - 80</td></tr>
<tr><td>No of Employees</td><td>14</td><td>26</td><td>64</td><td>46</td><td>28</td><td>16</td><td>8</td></tr>
</table>
Obtain the mean, median and mode of travel time to work.</td><td>CO1</td><td>PO2</td><td>07</td></tr>
<tr><td></td><td>c)</td><td>A survey was conducted on the inmates of a prison and the following data were found:
<table>
<tr><td><i>Period of jail (in months)</i></td><td>6</td><td>10</td><td>12</td><td>20</td><td>24</td><td>36</td><td>40</td><td>48</td></tr>
<tr><td><i>No. of prisoners</i></td><td>7</td><td>11</td><td>14</td><td>36</td><td>16</td><td>9</td><td>5</td><td>1</td></tr>
</table>
Calculate the coefficient of quartile deviation and mean deviation from median.</td><td>CO1</td><td>PO2</td><td>07</td></tr>
<tr><td></td><td></td><td>UNIT - II</td><td></td><td></td><td></td></tr>
<tr><td>2</td><td>a)</td><td>Roll a six faced fair die. Suppose X denote the number appear on the top of a die.
(i) Find the probability that an even number appear on the top.
(ii) Find the probability that the number appear on the top is less than 3.
(iii) Compute mean and variance of X.</td><td>CO1</td><td>PO1</td><td>06</td></tr>
<tr><td></td><td>b)</td><td>A batch of parts contains 100 from a local supplier of tubing and 200 from a supplier of tubing in the next state. If four parts are selected randomly and without replacement. Find the probability that
(i) All are from the local supplier.
(ii) At least 2 parts in the sample are from the local supplier.
Also, Calculate the mean, variance and finite population correction factor.</td><td>CO1</td><td>PO1</td><td>07</td></tr>
<tr><td></td><td>c)</td><td>Derive an expression for the mean and variance of a student’s t-distribution.</td><td>CO1</td><td>PO1</td><td>07</td></tr>
<tr><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
</table>

	3	a)	The probability that a wafer contains a large particle of contamination is 0.01. If it is assumed that the wafers are independent, what is the probability that exactly 125 wafers need to be analyzed before a large particle is detected? Also, find the mean and standard deviation of the number of wafers before a large particle is detected.	*CO1*	*PO2*	**06**
		b)	The total service time of a multistep manufacturing operation has a gamma distribution with mean 18 minutes and standard deviation 6. (i) Determine the parameters α and β. (ii) Find the probability that the total service time taken is at least 20 minutes. (iii) Find the probability that the total service time taken is at most 10 minutes.	*CO1*	*PO2*	**07**
		c)	Prove that χ^2-distribution tends to a normal distribution as $n \to \infty$.	*CO1*	*PO1*	**07**
			UNIT - III			
	4	a)	A computer has a random number generator that generates random numbers uniformly distributed in [0, 1]. We run it 100 times and let S_n be the sum of the 100 numbers. Estimate $P(S_n \geq 52)$.	*CO1*	*PO2*	**06**
		b)	The average height of a raccoon is 10 inches. (i) Given an upper bound on the probability that a certain raccoon is at least 15 inches tall. (ii) The standard deviation this height distribution is 2 inches. Find a lower bound on the probability that a certain raccoon is between 5 and 15 inches tall.	*CO1*	*PO2*	**07**
		c)	A random variable X has the geometric distribution $G(p)$. Show that the moment-generating function of X is $M_X(t) = \frac{pe^t}{1-qe^t}$ and also find mean and variance of the geometric distribution using $M_X(t)$.	*CO1*	*PO2*	**07**
			UNIT - IV			
	5	a)	Suppose $X_1, X_2, \ldots\ldots, X_n$ are independent $Uniform[0,\theta]$ random variables. Let $\hat{\theta} = \frac{3(X_1 + X_2 + \ldots + X_n)}{n}$ be an estimator of θ. Find the bias, variance and risk of $\hat{\theta}$	*CO1*	*PO2*	**06**
		b)	Suppose it is known that a sample consisting of the values 27, 79, 87, 45, 79, 85, 34 comes from a population with the density function $f(x) = \begin{cases} \frac{x}{\theta} e^{-x/\theta} & x > 0 \\ 0 & otherwise \end{cases}$ Find the maximum likelihood estimate of θ.	*CO1*	*PO2*	**07**

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<table>
<tr><td></td><td></td><td>c)</td><td>Two machines are used to fill plastic bottles with dishwashing detergent. The standard deviations of fill volume are known to be $\sigma_1 = 0.1$ fluid ounces and $\sigma_2 = 0.15$ fluid ounces for the two machines, respectively. Two random samples of $n_1 = 12$ bottles from machine 1 and $n_2 = 10$ bottles from machine 2 are selected, and the sample mean fill volumes are $\overline{x}_1 = 30.87$ fluid ounces and $\overline{x}_2 = 30.68$ fluid ounces. Assume that the populations are normally distributed. Construct a 90% confidence interval on the mean difference in fill volume.</td><td><i>CO1</i></td><td><i>PO2</i></td><td>07</td></tr>
<tr><td></td><td></td><td></td><td>UNIT - V</td><td></td><td></td><td></td></tr>
<tr><td></td><td>6</td><td>a)</td><td>A marketing company places an advertisement for a new brand of deodorant on two different platforms: television and social media. The company wants to study the proportion of people who remembered seeing the advertisement two hours later. In a sample of 200 people who saw the advertisement on television, 74 remembered seeing it two hours later. In a sample of 300 people who saw the advertisement on social media, 129 remembered seeing it two hours later. At 5% level of significance, is there a significant change for the difference in the proportion of people from the two different platforms that remember seeing the advertisement two hours later.</td><td><i>CO1</i></td><td><i>PO2</i></td><td>06</td></tr>
<tr><td></td><td></td><td>b)</td><td>The following figures give the systolic blood pressure of 10 joggers before and after an 8-kilometer run:
<table>
<tr><td>Jogger</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td></tr>
<tr><td>Before</td><td>158</td><td>149</td><td>160</td><td>155</td><td>164</td><td>138</td><td>163</td><td>159</td><td>165</td><td>145</td></tr>
<tr><td>After</td><td>164</td><td>158</td><td>163</td><td>160</td><td>172</td><td>147</td><td>167</td><td>169</td><td>173</td><td>147</td></tr>
</table>
At 0.05 level of significance apply Wilcoxon-signed Rank test to test that the jogging 8 kilometers increases the median systolic blood pressure.</td><td><i>CO1</i></td><td><i>PO2</i></td><td>07</td></tr>
<tr><td></td><td></td><td>c)</td><td>Given the following contingency table for hair color and eye color. At 1% level of significance, test whether the two attributes are independent by applying Chi-square test.
<table>
<tr><td rowspan="5">Eye color</td><td colspan="4">Hair color</td></tr>
<tr><td></td><td>Fair</td><td>Brown</td><td>Black</td></tr>
<tr><td>Blue</td><td>15</td><td>5</td><td>20</td></tr>
<tr><td>Grey</td><td>20</td><td>10</td><td>20</td></tr>
<tr><td>Brown</td><td>25</td><td>15</td><td>20</td></tr>
</table></td><td><i>CO1</i></td><td><i>PO2</i></td><td>07</td></tr>
<tr><td></td><td></td><td></td><td>OR</td><td></td><td></td><td></td></tr>
<tr><td></td><td>7</td><td>a)</td><td>At a certain college it is estimated that at most 25% of the students ride bicycles to class. Does this seem to be a valid estimate if, in a random sample of 90 college students, 28 are found to ride. Use 5% level of significance.</td><td><i>CO1</i></td><td><i>PO2</i></td><td>06</td></tr>
<tr><td></td><td></td><td>b)</td><td>The following data relating to the prices of commodities in different months in five cities.
<table>
<tr><td>A</td><td>B</td><td>C</td><td>D</td><td>E</td></tr>
<tr><td>29</td><td>31</td><td>26</td><td>19</td><td>28</td></tr>
<tr><td>24</td><td>35</td><td>28</td><td>21</td><td>28</td></tr>
<tr><td>27</td><td>28</td><td>24</td><td>23</td><td>25</td></tr>
<tr><td>26</td><td>28</td><td>25</td><td>29</td><td>26</td></tr>
<tr><td>21</td><td>22</td><td>20</td><td>25</td><td>20</td></tr>
</table>
Apply one-way ANOVA to test whether the difference between mean prices of commodities in cities is significant or not. Use 5% level of significance.</td><td><i>CO1</i></td><td><i>PO2</i></td><td>07</td></tr>
</table>

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<table>
<tr>
<td></td>
<td></td>
<td>c)</td>
<td>A new approach to prenatal care is proposed for pregnant women living in a rural community. The new program involves in-home visits during the course of pregnancy in addition to the usual or regularly scheduled visits. A pilot randomized trial with 15 pregnant women is designed to evaluate whether women who participate in the program deliver healthier babies than women receiving usual care. The outcome is the APGAR score measured 5 minutes after birth. Recall that APGAR scores range from 0 to 10 with scores of 7 or higher considered normal (healthy), 4-6 low and 0-3 critically low. The data are shown below.
<table>
<tr><td>Usual Care</td><td>8</td><td>7</td><td>6</td><td>2</td><td>5</td><td>8</td><td>7</td><td>3</td></tr>
<tr><td>New Program</td><td>9</td><td>9</td><td>7</td><td>8</td><td>10</td><td>9</td><td>6</td><td></td></tr>
</table>
At 5% level of significance, is there statistical evidence of a difference in APGAR scores in women receiving the new and enhanced versus usual prenatal care by applying Mann-Whiney test?</td>
<td>CO1</td>
<td>PO2</td>
<td>07</td>
</tr>
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