B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June / July 2024 Semester End Main Examinations

Programme: B.E.
Branch: Institutional Elective
Course Code: 23MA6OESFE
Course: Mathematical Statistics for Engineers

Semester: VI
Duration: 3 hrs.
Max Marks: 100

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

				CO	PO	Marks
			UNIT - I			
1	a)		Discuss the four levels of measurement scales. Give examples of each and explain in detail.	CO1	PO1	**06**
	b)		An incomplete frequency distribution is given as follows:	CO2	PO1	**07**

Variables	Frequency	Variables	Frequency
10-20	12	50-60	Y
20-30	30	60-70	25
30-40	X	70-80	18
40-50	65	Total	229

Given that the median value is 46, determine the value of X and Y.

				CO	PO	Marks
	c)		In a school, the intelligent students are admitted to batch *A* of a class and the others are admitted to batch *B*. The students in the two batches are aged as follows. Compare their average age and variations.	CO2	PO2	**07**

$Age(years)$	10	11	12	13	14	15
$students(Batch\,A)$	14	20	11	2	2	1
$students(Batch\,B)$	2	12	11	13	4	2

				CO	PO	Marks
			UNIT - II			
2	a)		3% of the product produced by a machine is found to be defective. Find the probability that first defective occurs in the (i) 5th item inspected (ii) first five inspected Also, compute mean and variance of *X*.	CO1	PO1	**06**
	b)		A warehouse receives orders for a particular product on a regular basis. When an order is placed, customers can order 1, 2, 3 or 4 units of the product. Historical data suggests that the size of any given order is equally likely to be of any of the four sizes. Let *X* denote size of an order. (i) Find the mean and standard deviation of customer order size. (ii) Find the probability that a customer orders at least two units.	CO1	PO1	**07**
	c)		Obtain the limiting probability density function of t-distribution when $n \to \infty$.	CO1	PO1	**07**
			OR			

	3	a)	The daily consumption of diesel in a petrol bunk is in excess of 20,000 liters is a gamma variate with parameters $\alpha = 2$, $\lambda = 1/10,000$. The bunk has a daily stock of 30,000 liters. Find the probability that the stock is inadequate on a particular day.	CO1	PO2	**06**
		b)	Lots of 40 components each are called acceptable if they contain no more than 3 defectives. The procedure for sampling the lot is to select 5 components at random (without replacement) and to reject the lot if a defective is found. What is the probability that exactly one defective is found in the sample if there are 3 defectives in the entire lot? Also, find the expected value and the variance of the number of defectives in the sample.	CO2	PO2	**07**
		c)	Derive an expression for the mean and variance of a χ^2-distribution	CO1	PO1	**07**
			UNIT - III			
	4	a)	The weights of 1500 ball bearings are normally distributed with a mean of 635 gms and standard deviation of 1.36gms. If 300 random samples of size 36 are drawn from this population, determine the mean and standard deviation of the sampling distribution of means if sampling is done (i) with replacement (ii) without replacement.	CO1	PO1	**06**
		b)	Suppose that it is known that the amount of uranium dug in a uranium mine during one week is a random variable with mean 20 kg. (i) Apply Markov's inequality to find a bound on the probability that week's production is at least 24kg? (ii) If the variance of the amount of uranium dug in the mine equals 16. Applying Chebyshev's inequality to find a lower bound on the probability that week's production will be between 8kg and 32kg?	CO2	PO2	**07**
		c)	A continuous random variable X has the following probability distribution $f(x) = 3xe^{-5x}, \ x > 0$ (i) Find the moment generating function for X. (ii) Find the mean and variance for X.	CO2	PO2	**07**
			UNIT - IV			
	5	a)	The number of traffic accidents in Berkeley follows a Poisson distribution with parameter λ, California in 10 randomly chosen no rainy days in 1998 is as follows: 4, 0, 6, 5, 2, 1, 2, 0, 4, 3. Find the maximum likelihood estimator of parameter λ.	CO1	PO1	**06**
		b)	Suppose that when the value μ is transmitted at location A then the value received at location B is normal with mean μ and variance σ^2 but with σ^2 being unknown. If 9 successive values are, 5, 8.5, 12, 15, 7, 9, 7.5, 6.5, 10.5, compute a 95% confidence interval for μ.	CO2	PO2	**07**
		c)	Suppose $X_1, X_2, ..., X_n$ are independent $Uniform[0,\theta]$ random variables. Let $\hat{\theta} = \dfrac{2(X_1 + X_2 + ... + X_n)}{n}$ be an estimator of θ. Find the bias, variance and risk of $\hat{\theta}$.	CO1	PO1	**07**

6	a)	Suppose that 100 randomly selected families and 200 randomly selected singles were asked what type of car they drove: sport, sedan, hatchback, truck, van/SUV. The results are shown in Table. Apply chi-square test for homogeneity, to test whether families and singles have the same distribution of cars? Test at a level of significance of 0.05.		CO2	PO1	**06**

	Sport	Sedan	Hatchback	Truck	Van/SUV
Family	5	15	35	17	28
Single	45	65	37	46	7

	b)	An experiment designed to compare two treatments against corrosion yielded the following data in pieces of wire subjected to the two treatments.		CO2	PO2	**07**

Treatment 1	65.2	67.1	69.4	78.2	74	80.3
Treatment 2	59.4	72.1	68	66.2	58.5	-

Apply Mann-Whitney test to test whether treatment 1 is better than treatment 2. Use 1% level of significance.

	c)	An auto rental firm is using 15 identical motors that are adjusted to run at a fixed speed to test 3 different brands of gasoline. Each brand of gasoline is assigned to exactly 5 of the motors. Each motor runs on 10 gallons of gasoline until it is out of fuel. The following represents the total mileages obtained by the different motors:		CO2	PO2	**07**

Gas 1	220	251	226	246	260
Gas 2	244	235	232	242	225
Gas 3	252	272	250	238	256

Apply one-way ANOVA test to test the hypothesis that the average mileage obtained is not affected by the type of gas used. Use the 5% level of significance.

		OR				
7	a)	The following data represent the number of hours of flight training received by 18 student pilots from a certain instructor prior to their first solo flight: 9, 12, 18, 14, 12, 14, 12, 10, 16, 11, 9, 11, 13, 11, 13, 15, 13, 14. At the 0.05 level of significance to test the instructor's claim that the median time required before his student's solo is 12 hours of flight training by applying Wilcoxon-signed-rank test.		CO2	PO2	**06**
	b)	Two sample polls of votes for 2 candidates A and B are taken from residents of different areas. The results are given below. Examine whether the nature of the area is related to voting preferences in this election by applying Chi Square test for independence at 1% level of significance.		CO2	PO2	**07**

Votes for / Area	A	B
Rural	620	380
Urban	550	450

	c)	In a study of nutrition cares in nursing homes, it is found that among 55 patients with hypertension, 24 were on sodium restricted diets. Of 149 patients without hypertension, 36 were on sodium restricted diets. Can we conclude that in the sampled populations, the proportion of patients on sodium restricted diets is higher amongst the patients with hypertension than among the patients without hypertension?		CO2	PO1	**07**

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