

U.S.N.

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January 2024 Semester End Main Examinations

Programme: B.E.

Branch: Institutional Elective

Course Code: 21MA7IENMT

Course: NUMBER THEORY

Semester: VII

Duration: 3 hrs.

Max Marks: 100

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.			UNIT - I	CO	PO	Marks
	1	a)	State any six basic properties of congruence.	CO1	PO1	06
		b)	Find the remainder when 2^{340} is divided by 341.	CO1	PO1	07
		c)	Apply Wilson's theorem to find the remainder when $15!$ is divided by 17.	CO1	PO1	07
			OR			
	2	a)	State and prove Fermat's little theorem.	CO1	PO1	06
		b)	Solve the linear Diophantine equation $49x + 81y = 47$.	CO1	PO1	07
		c)	Solve the system of linear congruences $x \equiv 3 \pmod{11}$, $x \equiv 5 \pmod{19}$ and $x \equiv 10 \pmod{29}$.	CO1	PO1	07
			UNIT - II			
	3	a)	Define the ϕ , σ and τ function and hence evaluate ϕ , σ and τ for $n = 3000$.	CO2	PO1	06
		b)	Define a multiplication function. If f and g are both multiplicative, then prove that $f * g$ is multiplicative.	CO2	PO1	07
		c)	State Euler's theorem. Find the last two digits in the decimal representation of 3^{100} .	CO2	PO1	07
			UNIT - III			
	4	a)	Apply the algebra of indices to solve the congruence $8x^5 \equiv 3 \pmod{13}$.	CO3	PO1	06
		b)	Define primitive root. Find incongruent primitive roots modulo 17.	CO3	PO1	07
		c)	Apply Lucas' theorem to show that $n = 257$ is prime (Choose $x = 3$).	CO3	PO1	07
			UNIT - IV			
	5	a)	Define continued fraction and express $1001/45$ as a finite continued fraction.	CO4	PO1	06

	b)	Define the generalized law of quadratic reciprocity. Apply the same to compute the Jacobi Symbol $\left(\frac{221}{399}\right)$.	CO4	PO1	07
	c)	Define quadratic reciprocity and hence compute the Legendre symbol $\left(\frac{3797}{7297}\right)$.	CO4	PO1	07
		OR			
6	a)	Express finite simple continued fraction $[1; 2, 3, 4, 5]$ as a rational number.	CO4	PO1	06
	b)	Find the infinite continued fraction expansion for ' $\sqrt{22}$ '.	CO4	PO1	07
	c)	Solve the quadratic congruence $25x^2 + 70x + 37 \equiv 0 \pmod{13}$.	CO4	PO1	07
		UNIT - V			
7	a)	Describe briefly about Mordell's equation.	CO5	PO1	06
	b)	Using the fact that $3 + 2\sqrt{2}$ yields the least solution of $x^2 - 2y^2 = 1$, find two new solutions.	CO5	PO1	07
	c)	Write 15795 as the sum of four squares.	CO5	PO1	07
