

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January 2024 Semester End Main Examinations

Programme: B.E.

Branch: Institutional Elective

Course Code: 21MA7IENMT

Course: NUMBER THEORY

Semester: VII

Duration: 3 hrs.

Max Marks: 100

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I			CO	PO	Marks
1	a)	State any six basic properties of congruence.	CO1	PO1	06
	b)	Find the remainder when 2^{340} is divided by 341.	CO1	PO1	07
	c)	Apply Wilson's theorem to find the remainder when $15!$ is divided by 17.	CO1	PO1	07
OR					
2	a)	State and prove Fermat's little theorem.	CO1	PO1	06
	b)	Solve the linear Diophantine equation $49x + 81y = 47$.	CO1	PO1	07
	c)	Solve the system of linear congruences $x \equiv 3 \pmod{11}$, $x \equiv 5 \pmod{19}$ and $x \equiv 10 \pmod{29}$.	CO1	PO1	07
UNIT - II					
3	a)	Define the \emptyset , σ and τ function and hence evaluate \emptyset , σ and τ for $n = 3000$.	CO2	PO1	06
	b)	Define a multiplication function. If f and g are both multiplicative, then prove that $f * g$ is multiplicative.	CO2	PO1	07
	c)	State Euler's theorem. Find the last two digits in the decimal representation of 3^{100} .	CO2	PO1	07
UNIT - III					
4	a)	Apply the algebra of indices to solve the congruence $8x^5 \equiv 3 \pmod{13}$.	CO3	PO1	06
	b)	Define primitive root. Find incongruent primitive roots modulo 17.	CO3	PO1	07
	c)	Apply Lucas' theorem to show that $n = 257$ is prime (Choose $x = 3$).	CO3	PO1	07
UNIT - IV					
5	a)	Define continued fraction and express $1001/45$ as a finite continued fraction.	CO4	PO1	06

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

	b)	Define the generalized law of quadratic reciprocity. Apply the same to compute the Jacobi Symbol $\left(\frac{221}{399}\right)$.	CO4	PO1	07
	c)	Define quadratic reciprocity and hence compute the Legendre symbol $\left(\frac{3797}{7297}\right)$.	CO4	PO1	07
		OR			
6	a)	Express finite simple continued fraction $[1; 2, 3, 4, 5]$ as a rational number.	CO4	PO1	06
	b)	Find the infinite continued fraction expansion for ' $\sqrt{22}$ '.	CO4	PO1	07
	c)	Solve the quadratic congruence $25x^2 + 70x + 37 \equiv 0 \pmod{13}$.	CO4	PO1	07
		UNIT - V			
7	a)	Describe briefly about Mordell's equation.	CO5	PO1	06
	b)	Using the fact that $3 + 2\sqrt{2}$ yields the least solution of $x^2 - 2y^2 = 1$, find two new solutions.	CO5	PO1	07
	c)	Write 15795 as the sum of four squares.	CO5	PO1	07
