U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

January / February 2025 Semester End Main Examinations

Programme: B.E. **Semester: VII**
Branch: Institutional Elective **Duration: 3 hrs.**
Course Code: 21MA7IENMT **Max Marks: 100**
Course: NUMBER THEORY

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	*CO*	*PO*	**Marks**
1	a)	Find the least positive integer that leaves the remainder 3 when divided by 7, 4 when divided by 9, 8 when divided by 11.	*1*	*1*	**6**
	b)	Twenty-three weary travelers entered the outskirts of a lush and beautiful Forest. They found 63 equal heaps of plantains and seven single fruits, and divided them equally. Find the number of fruits in each heap.	*1*	*1*	**7**
	c)	Solve the polynomial congruence $x^3+3x+5\equiv 0 \pmod 9$.	*1*	*1*	**7**
		OR			
2	a)	Explain briefly about Fermat Numbers.	*1*	*1*	**6**
	b)	State Fermat's Little theorem and hence find the remainder when $2^{1000000}$ is divided by 7.	*1*	*1*	**7**
	c)	State and prove Pythagorean Equation.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Find the remainder when 17^{6666} is divided by 10 using Euler's theorem.	*2*	*1*	**6**
	b)	Evaluate: $(i)\phi(5040)$ $(ii)\tau(6120)$ $(iii)\sigma(360)$ $(iv)\mu(672)$ (v) $(\tau*\sigma)(4)$.	*2*	*1*	**7**
	c)	Define: (i) Summation function, (ii) Sum-of-Divisors function, (iii) Number-of-Divisors function, (iv) Mobius function and (v) Euler's Totient function.	*2*	*1*	**7**
		OR			
4	a)	If $n=p_1^{k_1}.p_2^{k_2}.p_3^{k_3}...........p_r^{k_r}$ be the canonical decomposition of positive integer n, then prove that $\phi(n)=n\left(1-\frac{1}{p_1}\right)\left(1-\frac{1}{p_2}\right)-----------\left(1-\frac{1}{p_r}\right)$.	*2*	*1*	**6**
	b)	If $h=f*g$, where f and g are multiplicative functions, then prove that $h(mn)=h(m)h(n)$. Illustrate it with $m=2$ and $n=3$.	*2*	*1*	**7**

		c)	State and prove Euler's theorem.	2	1	**7**
			UNIT - 3			
	5	a)	Solve the congruence $11x \equiv 7 \pmod{18}$ using the indices to base 5.	3	1	**6**
		b)	Using the corollary of Lucas' theorem, verify that n =1213 is prime by taking $x = 5$.	3	1	**7**
		c)	Define order of a positive integer. Compute $\text{ord}_{21} 5$ and $\text{ord}_{13} 7$.	3	1	**7**
			OR			
	6	a)	Find the incongruent solutions of the congruence $x^3 - 1 \equiv 0 \pmod{13}$.	3	1	**6**
		b)	Define primitive root and hence verify that 2 is a primitive root modulo 9.	3	1	**7**
		c)	Show that 2 is not a primitive root modulo any Fermat prime f_n, where $n \geq 2$.	3	1	**7**
			UNIT - 4			
	7	a)	Define: (i) The Legendre symbol, (ii) The Jacobi symbol, (iii) Law of Quadratic Reciprocity.	4	1	**6**
		b)	Find the continued fraction expression for e.	4	1	**7**
		c)	Solve the Quadratic congruence $3x^2 - 4x + 7 \equiv 0 \pmod{13}$.	4	1	**7**
			OR			
	8	a)	State and prove Euler's Criterion.	4	1	**6**
		b)	Evaluate the Legendre Symbols i) $\left(\frac{125}{17}\right)$ ii) $\left(\frac{15625}{17}\right)$.	4	1	**7**
		c)	Define Finite continued fraction and express $\frac{225}{157}$ as a finite simple continued fraction.	4	1	**7**
			UNIT - 5			
	9	a)	Describe briefly about Pell's equation.	5	1	**6**
		b)	Express 64,125 as sum of four squares.	5	1	**7**
		c)	Using the fact that $5 + 2\sqrt{6}$ yields the least solution of $x^2 - 6y^2 = 1$, find two new solutions.	5	1	**7**
			OR			
	10	a)	Explain briefly about Mordell's equation.	5	1	**6**
		b)	Using the fact that $1 + \sqrt{2}$ is the least solution of $x^2 - 2y^2 = -1$, find a new solution.	5	1	**7**
		c)	Express (i) 333 (ii) 1170 (iii) 5850 (iv) 1225 and (v) 229 as sum of two squares.	5	1	**7**

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