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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

February / March 2023 Semester End Main Examinations

Programme: B.E.

Branch: Institutional Elective

Course Code: 21MA7OENMT

Course: NUMBER THEORY

Semester: VII

Duration: 3 hrs.

Max Marks: 100

Date: 22.02.2023

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

UNIT - I

1	a) State and prove Wilson's theorem.	6
	b) Compute the least residue of $13^{18} + 19^{12} \pmod{247}$.	7
	c) Solve the polynomial congruence $x^3 - 5x + 1 \equiv 0 \pmod{27}$	7

OR

2	a) Find the remainder when 3^{287} is divided by 23.	6
	b) State and prove Chinese remainder theorem.	7
	c) Solve the Diophantine equation $170x - 455y = 625$.	7

UNIT - II

3	a) Compute $\sigma(6120)$, $\tau(6120)$, $\mu(672)$, $\Phi(675)$.	6
	b) Let f and g be both multiplicative functions, then prove that $f * g$ is multiplicative.	7
	c) If $n = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \cdots p_r^{k_r}$ is canonical representation, then prove that $\Phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_r}\right)$.	7

UNIT - III

4	a) Compute: $\text{ord}_{13}5$ and $\text{ord}_{13}7$.	6
	b) Using Lucas' theorem, show that $n = 1117$ is prime (choose $x=2$).	7
	c) Solve the congruence: $11^{3x} \equiv 5 \pmod{13}$.	7

UNIT - IV

5	a) Solve the quadratic congruence: $3x^2 + 5x + 9 \equiv 0 \pmod{11}$.	6
	b) Using generalized law of quadratic reciprocity, evaluate (i) $\left(\frac{59}{131}\right)$ (ii) $\left(\frac{71}{73}\right)$	7
	c) Define finite continued fraction and hence express $\left(\frac{225}{157}\right)$ as the same.	7

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

OR

6	a)	Find the value of Jacobi symbol $\left(\frac{32}{15}\right)$ and $\left(\frac{22}{105}\right)$.	6
	b)	Define infinite simple continued fraction and express e as the same.	7
	c)	Define quadratic residues and non residues .Find the quadratic residues and non residues of an integer 18.	7

UNIT - V

7	a)	Write a note on Pythagorean triangle.	6
	b)	Using the fact that $5 + 2\sqrt{6}$ yields the least solution of $x^2 - 6y^2 = 1$, find two new solutions.	7
	c)	Write 15,795 as the sum of four squares	7
