

# B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

## February / March 2023 Semester End Main Examinations

**Programme: B.E.**

**Branch: Institutional Elective**

**Course Code: 21MA70ENMT**

**Course: NUMBER THEORY**

**Semester: VII**

**Duration: 3 hrs.**

**Max Marks: 100**

**Date: 22.02.2023**

**Instructions:** 1. Answer any FIVE full questions, choosing one full question from each unit.  
2. Missing data, if any, may be suitably assumed.

### UNIT - I

- 1 a) State and prove Wilson's theorem. 6
- b) Compute the least residue of  $13^{18} + 19^{12} \pmod{247}$ . 7
- c) Solve the polynomial congruence  $x^3 - 5x + 1 \equiv 0 \pmod{27}$  7

### OR

- 2 a) Find the remainder when  $3^{287}$  is divided by 23. 6
- b) State and prove Chinese remainder theorem. 7
- c) Solve the Diophantine equation  $170x - 455y = 625$ . 7

### UNIT - II

- 3 a) Compute  $\sigma(6120)$ ,  $\tau(6120)$ ,  $\mu(672)$ ,  $\Phi(675)$ . 6
- b) Let  $f$  and  $g$  be both multiplicative functions, then prove that  $f * g$  is multiplicative. 7
- c) If  $n = p_1^{k_1} \cdot p_2^{k_2} \cdot p_3^{k_3} \cdots p_r^{k_r}$  is canonical representation, then prove that  $\Phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \cdots \left(1 - \frac{1}{p_r}\right)$ . 7

### UNIT - III

- 4 a) Compute:  $\text{ord}_{13} 5$  and  $\text{ord}_{13} 7$ . 6
- b) Using Lucas' theorem, show that  $n = 1117$  is prime (chose  $x=2$ ). 7
- c) Solve the congruence:  $11^{3x} \equiv 5 \pmod{13}$ . 7

### UNIT - IV

- 5 a) Solve the quadratic congruence:  $3x^2 + 5x + 9 \equiv 0 \pmod{11}$ . 6
- b) Using generalized law of quadratic reciprocity, evaluate (i)  $\left(\frac{59}{131}\right)$  (ii)  $\left(\frac{71}{73}\right)$  7
- c) Define finite continued fraction and hence express  $\left(\frac{225}{157}\right)$  as the same. 7

**Important Note:** Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

**OR**

- |   |    |  |          |
|---|----|--|----------|
| 6 | a) | Find the value of Jacobi symbol $\left(\frac{32}{15}\right)$ and $\left(\frac{22}{105}\right)$ .           | <b>6</b> |
|   | b) | Define infinite simple continued fraction and express e as the same.                                       | <b>7</b> |
|   | c) | Define quadratic residues and non residues .Find the quadratic residues and non residues of an integer 18. | <b>7</b> |

**UNIT - V**

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|---|----|---|----------|
| 7 | a) | Write a note on Pythagorean triangle.   | <b>6</b> |
|   | b) | Using the fact that $5 + 2\sqrt{6}$ yields the least solution of $x^2 - 6y^2 = 1$ , find two new solutions. | <b>7</b> |
|   | c) | Write 15,795 as the sum of four squares   | <b>7</b> |

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B.M.S.C.E. - ODD SEM 2022-23