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B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

June 2025 Semester End Main Examinations

Programme: B.E. **Semester: VII**
Branch: Institutional Elective **Duration: 3 hrs.**
Course Code: 21MA7IENMT **Max Marks: 100**
Course: NUMBER THEORY

Instructions: 1. All questions have internal choices.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - 1	***CO***	***PO***	**Marks**
1	a)	State and prove Chinese remainder theorem.	*1*	*1*	**6**
	b)	If a cock is worth five coins, a hen three coins, and three chicks together one coin, how many cocks, hens, and chicks, totaling 100, can be bought for 100 coins?	*1*	*1*	**7**
	c)	Solve the polynomial congruence $x^3 - 5x + 1 \equiv 0 (\mathrm{mod}\, 27)$.	*1*	*1*	**7**
		OR			
2	a)	Write any six properties of congruence.	*1*	*1*	**6**
	b)	Find the smallest positive integer n such that $3^2 / n$, $4^2 / n+1$ and $5^2 / n+2$.	*1*	*1*	**7**
	c)	Show that $f_5 = 2^{2^5} + 1$ is divided by 641.	*1*	*1*	**7**
		UNIT - 2			
3	a)	Define each of the following with an example i) Perfect prime, ii) Mersenne prime, iii) Dirichlet Product.	*2*	*1*	**6**
	b)	Compute i) $(\tau * \sigma)(4)$, ii) $\phi(1976)$, iii) $\mu(945)$, iv) $\tau(6120)$.	*2*	*1*	**7**
	c)	Find the remainder when 3^{256} is divided by 100.	*2*	*1*	**7**
		OR			
4	a)	Define σ, ϕ, τ functions and compute σ, ϕ and τ for $n = 3000$.	*2*	*1*	**6**
	b)	Define a multiplicative function. If f and g are both multiplicative, then prove that $f * g$ is multiplicative.	*2*	*1*	**7**
	c)	State Euler's theorem. Find the remainder when 245^{1040} is divided by 18.	*2*	*1*	**7**
		UNIT - 3			
5	a)	Solve the congruence $11x^3 \equiv 2 (\mathrm{mod}\ 23)$ using indices to base 5.	*3*	*1*	**6**
	b)	Apply corollary of Lucas' theorem to show that $n = 127$ is a prime by choosing $x = 3$.	*3*	*1*	**7**

		c)	Using the fact that 5 is a primitive root modulo 54, find the remaining incongruent primitive roots.	*3*	*1*	**7**
			OR			
	6	a)	Show that 2 is not a primitive root modulo any Fermat prime f_n, where $n \geq 2$.	*3*	*1*	**6**
		b)	Using Lucas' theorem, show that $n = 773$ is a prime by choosing $x = 3$	*3*	*1*	**7**
		c)	Solve the congruence $2x^4 \equiv 5 \pmod{13}$ using indices to base 2.	*3*	*1*	**7**
			UNIT - 4			
	7	a)	Solve the quadratic congruence $3x^2 - 4x + 7 \equiv 0 (\bmod 13)$.	*4*	*1*	**6**
		b)	Using the generalized law of quadratic reciprocity, evaluate Jacobi symbol (221/399).	*4*	*1*	**7**
		c)	Define continued fraction and represent finite continued fraction [1; 2, 3, 4, 5] as a rational number.	*4*	*1*	**7**
			OR			
	8	a)	Determine whether 10 and 7 are quadratic residues of 13.	*4*	*1*	**6**
		b)	Write the properties of Legendre symbol. Compute Jacobi symbol (15625/17).	*4*	*1*	**7**
		c)	Find the infinite continued fraction expansion for 'e'.	*4*	*1*	**7**
			UNIT - 5			
	9	a)	Describe briefly about Catalan's Conjecture.	*5*	*1*	**6**
		b)	Describe sum of four square and write 29,791 as the sum of four square.	*5*	*1*	**7**
		c)	Using the fact that $1+\sqrt{2}$ is the least solution of $x^2 - y^2 = -1$, find a new solution.	*5*	*1*	**7**
			OR			
	10	a)	Describe Fermat's last theorem with an example.	*5*	*1*	**6**
		b)	Using the fact that $3+2\sqrt{2}$ yields the least solution of $x^2 - y^2 = 1$, find two new solutions.	*5*	*1*	**7**
		c)	Write 64,125 as sum of four square.	*5*	*1*	**7**

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