U.S.N.										

B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

July 2023 Semester End Main Examinations

Programme: B.E. | **Semester: VIII**
Branch: Institutional Elective | **Duration: 3 hrs.**
Course Code: 21MA8IELIA | **Max Marks: 100**
Course: Linear Algebra | **Date: 06.07.2023**

Instructions: 1. Answer any FIVE full questions, choosing one full question from each unit.
2. Missing data, if any, may be suitably assumed.

Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice.

		UNIT - I	*CO*	*PO*	**Marks**
1	a)	Let R^+ be the set of all positive real numbers. Define vector addition as $u+v=uv\ \forall u,\ v\in R^+$ and scalar multiplication by $k.u=u^k\ \forall u\in R^+,\ k\in R$. Show that R^+ is vector space over the field of numbers.	*CO1*	*PO1*	**6**
	b)	Consider the vector space $P_2(t)$ of polynomials of degree ≤ 2. The set $S=\{p_1, p_{2,} p_3\}$ where $p_1(t)=t+1$, $p_2(t)=t-1$ and $p_3(t)=(t-1)^2$ form a basis S of $P_2(t)$. Prove that given polynomials are independent and hence find the coordinate vector of the polynomial $v=t^2-3t+2$.	*CO1*	*PO1*	**7**
	c)	Find the basis and dimension of the row space and the column space of the matrix $\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$.	*CO1*	*PO1*	**7**
		UNIT - II			
2	a)	Find the matrix representation with respect to the bases $B=\{(1,2),\ (2,0)\}$ and $S=\{(1,1,0),(0,1,1),(0,1,-1)\}$ in R^2 and R^3 respectively of the linear transformation $T:R^2\rightarrow R^3$ defined by $T(a,b)=(4a+b,3a,2a-b)$.	*CO1*	*PO1*	**6**
	b)	The linear transformation $T:R^3\rightarrow R^4$ is defined by $T(x,y,z)=(x+2y+z,\ 2x+4y+2z,\ 3x+7y+6z, 2x+5y+5z)$. Find the bases for the image space and the null space and hence verify Rank-Nullity theorem.	*CO1*	*PO1*	**7**

		c)	Define singular and non-singular transformations. Let $G:R^2 \to R^2$ be the linear transformation defined by $F(x,\ y)=(2x+y,\ \ 3x+2y)$. Show that G is non-singular and hence find F^{-1} if it exists.	*CO1*	*PO1*	**7**
			UNIT - III			
	3	a)	Apply Cayley-Hamilton theorem to find A^{-1} of the matrix $A=\begin{bmatrix}1 & 0 & 3\\ 2 & 1 & -1\\ 1 & -1 & 1\end{bmatrix}$.	*CO2*	*PO1*	**6**
		b)	Obtain the Eigen space for the linear transformation defined by $T\ (x,y,z)=(2x+y+z,\ y-z\ ,\ 2y+4z)$.	*CO2*	*PO1*	**7**
		c)	Find the characteristic and minimal polynomial of the matrix $A=\begin{bmatrix}4 & -1 & 0 & 0 & 0\\ 1 & 2 & 0 & 0 & 0\\ 0 & 0 & 3 & 1 & 0\\ 0 & 0 & 0 & 3 & 1\\ 0 & 0 & 0 & 0 & 3\end{bmatrix}$.	*CO2*	*PO1*	**7**
			OR			
	4	a)	Find the Eigen values and corresponding Eigen vectors for the linear transformation $T:R^3 \to R^3$ defined by $T(a,b,c)=(2a-c,\ 2a+b-2c,\ -a+2c)$.	*CO2*	*PO1*	**6**
		b)	Find the modal matrix P such that the matrix $A=\begin{bmatrix}7 & 2\\ -4 & 1\end{bmatrix}$ is diagonalizable and hence find A^6.	*CO2*	*PO1*	**7**
		c)	Determine all possible Jordan canonical form for the linear operator $T:V \to V$ whose characteristic polynomial is $f(t)=(t+7)^3(t-2)$. In each case find the minimal polynomial.	*CO2*	*PO1*	**7**
			UNIT - IV			
	5	a)	Find a basis of the subspace W of R^4 that is orthogonal to $u_1=(1,-1,3,2)$ and $u_2=(3,-4,6,1)$.	*CO3*	*PO1*	**6**
		b)	Construct an orthogonal matrix P whose first row is $\left(\frac{2}{3},\ \frac{1}{3},\ \frac{2}{3}\right)$ in R^3.	*CO3*	*PO1*	**7**

		c)	Find the matrix of the inner product $\langle f, g\rangle = \int_{-1}^{1} f(t)*g(t)dt$ with respect to the basis $S=\{-1, t, -t^2\}$ in the polynomial space $P_2(t)$, where the matrix of the inner product is defined as $A=[a_{ij}]=\langle f_i, f_j\rangle$.	CO3	PO1	**7**
			OR			
6	a)		Solve the system of equations $AX=b$ by the method of least squares where $A=\begin{bmatrix}1 & -3\\2 & 6\\7 & -3\\3 & 4\end{bmatrix}$ and $b=\begin{bmatrix}1\\3\\2\\1\end{bmatrix}$ and hence find the least square error.	CO3	PO1	**10**
	b)		Obtain the QR factorization of the matrix $A=\begin{bmatrix}1 & -1 & -1\\1 & 0 & 0\\1 & -1 & 0\\0 & 1 & -1\end{bmatrix}$.	CO3	PO1	**10**
			UNIT - V			
7	a)		Find the matrix of transformation that reduces the quadratic form $x^2+3y^2+3z^2-2yz$ into canonical form and hence discuss its nature.	CO3	PO1	**10**
	b)		Find the singular value decomposition of $A=\begin{bmatrix}-3 & 1\\6 & -2\\6 & -2\end{bmatrix}$.	CO3	PO1	**10**

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