FATIGUE LOADING

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Introduction to Fatigue failure:

Some machine elements are subjected to static loads and for such elements failure theories are used to predict failure (Yielding or Fracture). However, most metallic structures like aircrafts, ships, bridges, automobiles, and machine elements like- gears, axles, shafts, bearings, cams and followers, are subjected to varying or fluctuating stresses. Fluctuating stresses (repeated over long periods of time) will cause a part to fail (fracture) at a stress level much smaller than the ultimate strength or even yield strength. Unlike static loading where failure can be detected before it happens, fatigue failures are sudden and therefore catastrophic.
Fatigue failures are similar to brittle fracture and the fracture surfaces are perpendicular to the load axis. **More than 85% of the failures are due to fatigue loading. Fatigue failure is due to crack formation and propagation.**

Mechanical failures were observed to take place in metals and materials subject to repetitive stresses well below their yield strength. The theory came to be that the metal became "tired" or "fatigued", hence the term "fatigue" or "metal fatigue". A typical Railway axle failure, which was the origin of a different school of thought in terms of material failure mechanism, is shown in Fig.1. The axle, although made of steel, behaved like a brittle material in the way it failed. The failure was sudden and catastrophic and the material did not show any evidence of yielding; the two halves of the axle had absolutely no deformation and did not have any microstructural changes.

Fig.1 Railway Axle failure- origin of fatigue loading concepts

Between 1852 and 1870, the first systematic fatigue tests were carried out on specifically designed laboratory specimens by August Wohler, a German railway engineer, simulating the loading conditions of a railway axle. These tests enabled Wohler to relate his experimental results to the stresses in locomotive axles. In 1870, Wohler compiled a report of his experimental work which contained several conclusions known as Wohler's laws.

Rotating beam fatigue testers are one of the oldest methods used to determine a material's fatigue behaviour. A sample is placed in the machine and a force is applied via a bending moment using weights hung off the sample. The force induces a surface stress that will be tensile on one side of the sample (generally the top) and compressive on the opposite side. When the test is started, the sample will rotate at the desired rate and this rotation will cause the surfaces to interchange so that each surface experiences alternating tensile and compressive stresses. This is illustrated in Fig.2.

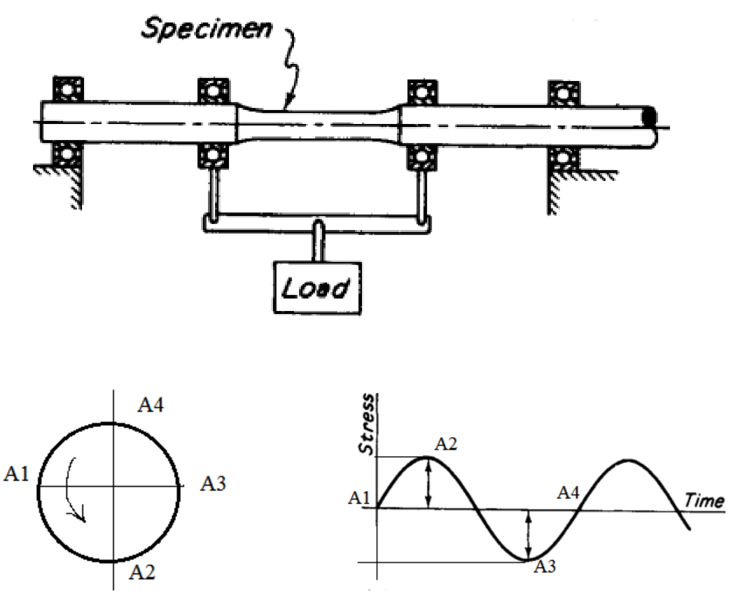


Fig.2 Rotating beam Bending test carried out by Wohler

<u>Stages in Fatigue failure:</u>

Stage-1: Crack Nucleation

Fatigue cracks usually start at locations of high stress concentrations (notches) such as sudden changes in cross section, sharp corners, cracks, blowholes, inclusions, welding defects, scratches, inspection stamps, holes etc. Localized yielding and slip along grain boundaries lead to micro-cracks.

Stage-2: Crack propagation

-Macro crack development, orderly crack growth

Stage 3: Unstable crack leading to failure

- Remaining material cannot support stress, which will lead to rapid fracture.

The fatigue failure stages is shown in the Fig.3

Fig.3 Mechanism of fatigue failure

Fundamentals of Fatigue Loading:

A typical fatigue load is shown in Fig. 4.

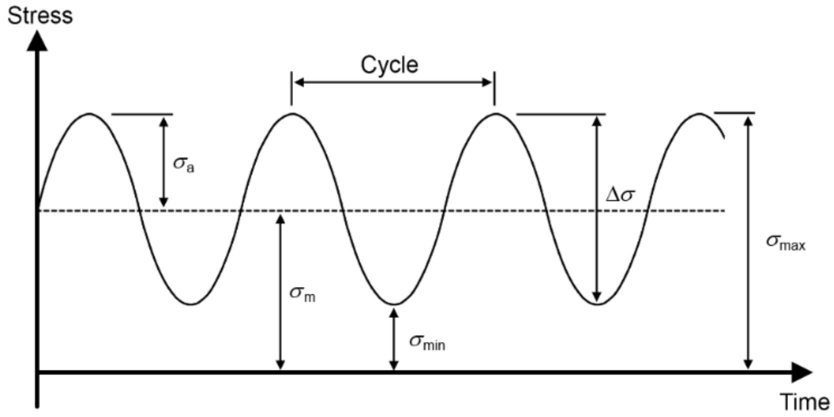


Fig.4 A typical Fatigue load

The stress varies between a *maximum stress*, σ_{max}, and a *minimum stress*, σ_{min}, during a load cycle. In the field of fatigue, the variation in stress is often defined using the *stress amplitude*, σ_a, and the mean stress, σ_m . Further, variables defining the *stress range*, $\Delta\sigma$, and the *R*-value are frequently used to describe a stress cycle.

Maximum Stress= σ_{max}

Minimum Stress= σ_{min}

Mean Stress, Average stress $\sigma_m = (\sigma_{max} + \sigma_{min}) / 2$

Variable stress, Alternating stress $\sigma_a = (\sigma_{max} - \sigma_{min}) /2$

R= Stress Ratio = $\sigma_{min} / \sigma_{max}$

Stress Range= $\Delta\sigma = \sigma_{max} - \sigma_{min}$

A= Amplitude Ratio = σ_a / σ_m

MEAN STRESS
A stress component always there on the member.

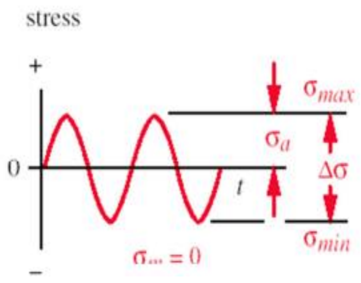
VARIABLE COMPONENT
Superimposed on the Mean stress component to obtain Cyclic/ Fluctuating stress.

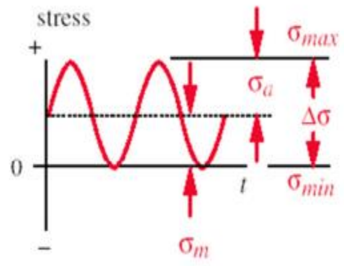
$\sigma_{max} = \sigma_m + \sigma_a$

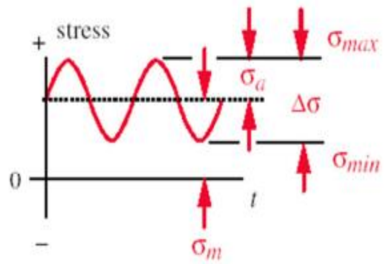
$\sigma_{min} = \sigma_m - \sigma_a$

Different types of cyclic loads:

Different types of cyclic/ fluctuating stresses are shown in Fig. 5.



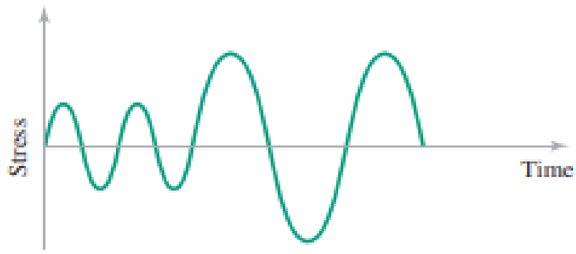




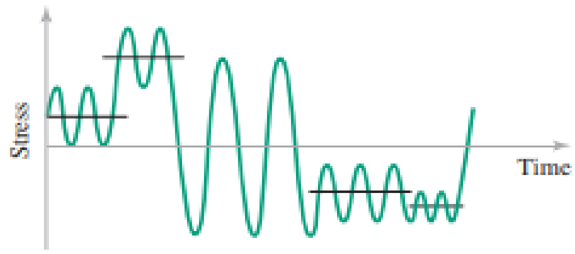
Completely reversed stress
$\sigma_m = 0$, R= -1

Released Tension
$\sigma_m = 0.5\ \sigma_{max}$, R= 0 , $\sigma_{min} = 0$

Fluctuating Stresses
Non Zero mean stress



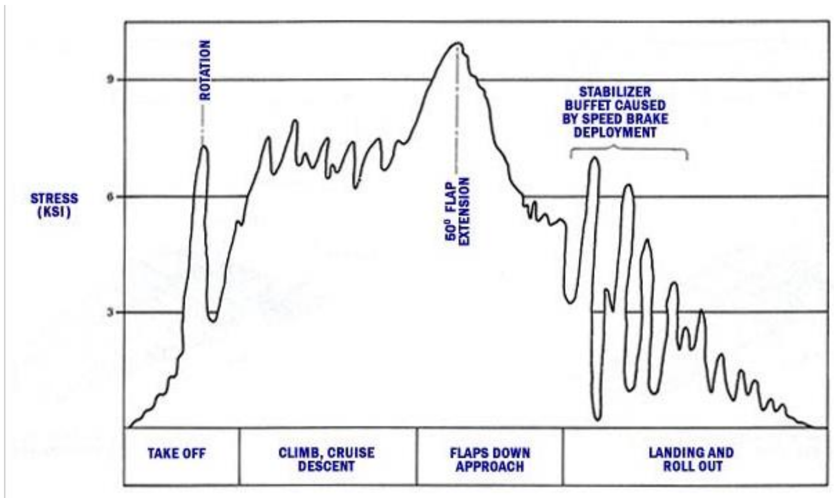
(a)



(b)

Zero mean and changing Amplitude stress

Changing mean and changing Amplitude stress



A quasi-random stress–time pattern that might be typical of an operational aircraft during any given mission.

Fig.4 Types of Cyclic loads

FATIGUE TESTING:

In stress based fatigue tests, multiple samples of identical size, shape and composition are subjected to different levels of stress amplitude, σ_a, and the number of cycles to failure, N, is measured for each sample.

Various types of instruments and machines are used to apply cyclic loading and include rotating bend and cantilever bend machines, servo-hydraulic or servo-electric axial push-pull testing systems, and electric motor driven torsion fatigue testers.
The resulting S-N data for each identical specimen is plotted on either a log-log or semi log graph. Regression is used to fit a curve through the points resulting in an S-N diagram.
A typical rotating bending test machine and the test specimen is shown in Fig. 5.
Most fatigue experiments are performed with $\sigma_m = 0$ *(e.g. rotating beam tests).*

Rotating Beam Fatigue Testing Machine:

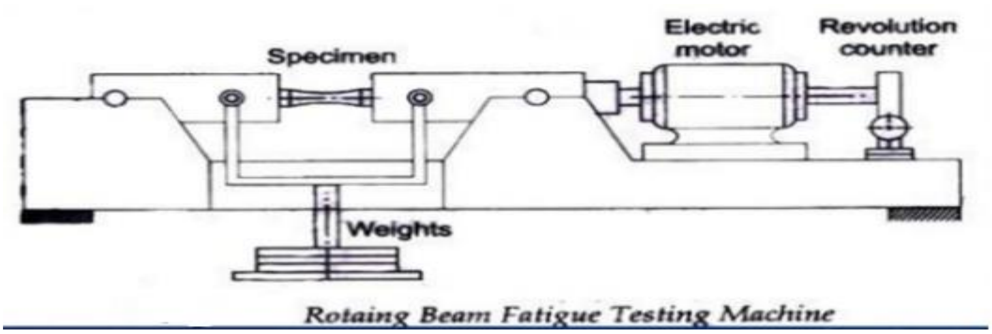


Fig.5. Rotating Bending test machine and Test specimen for

Engineering fatigue data is usually plotted as **S-N curve.** Here S is the stress and N the number of cycles to failure (usually fracture). The x-axis is plotted as log (N). It should be noted that the stress values plotted are nominal values and ***does not take into account local stress concentrations.***

Typically the stress value chosen for the stress is low ($< \sigma_y$) and hence S-N curves deal with fatigue failure at a large number of cycles ($>10^4$cycles). These are the **high cycle fatigue** tests. The results of the test are plotted on a log-log sheet and a typical S-N diagram is shown in Fig.6. As obvious, if the magnitude of alternating stress increases the fatigue life decreases.

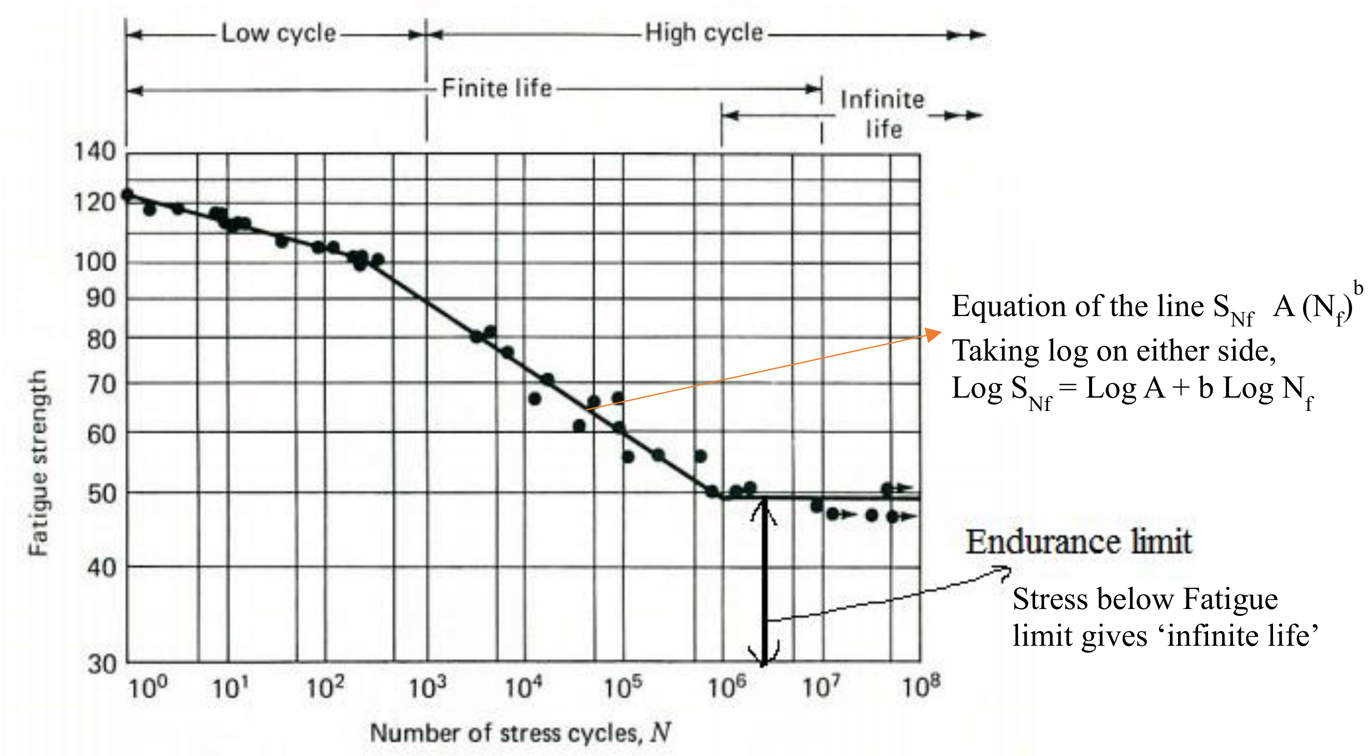
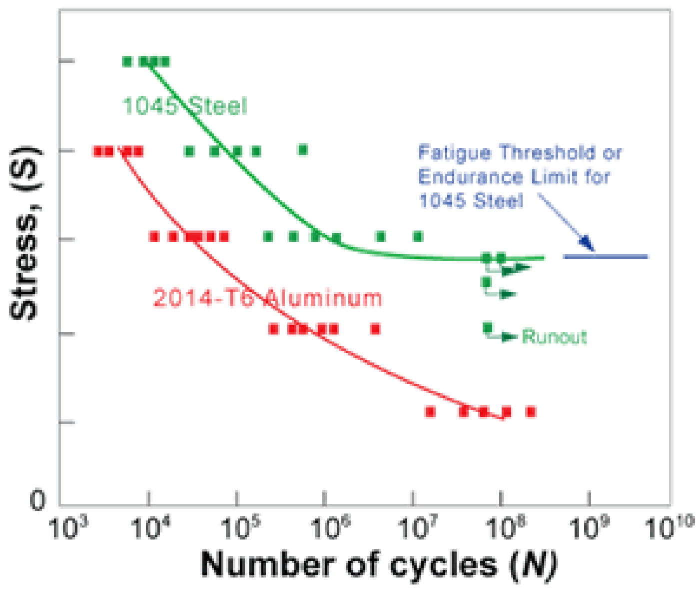


Fig.6. S-N diagram



- Steel, Ti show fatigue limit or Endurance Limit (σ_{en})
- Al, Mg, Cu show no fatigue limit.

Fig.7. S-N diagram for steel and Aluminum

Broadly two kinds of S-N curves can be differentiated for two classes of materials.

- Those where a stress below a threshold value gives a 'very long' life; this stress value is called the *Fatigue Limit / Endurance limit*. Steel and Ti come under this category.
- Those where a decrease in stress increases the fatigue life of the component, but no distinct fatigue life is observed. Al, Mg, Cu come under this category. (Fig.7)

- For these materials, the fatigue response is specified as **fatigue strength/endurance strength,** which is defined as the stress level at which failure will occur for some specified number of cycles (e.g., 10^8 cycles).
- ***For Aluminum, 100 MPa @ 5X10^8 cycles.***

Low Cycle Fatigue and High Cycle fatigue:

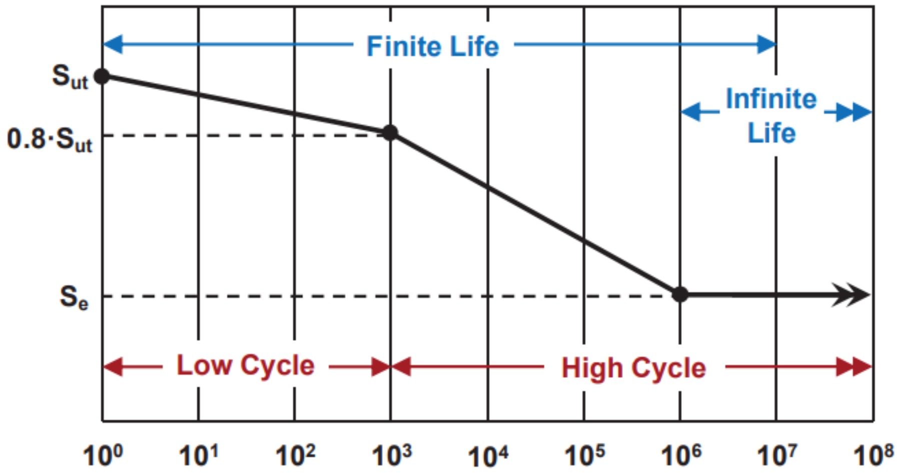


Fig.8.Low and high cycle fatigue

Low-Cycle fatigue: Domain associated with high loads and short service life. Significant plastic strain occurs during each cycle. Low number of cycles to produce failure. (Fig.8)

Number of cycles to produce fatigue failure: $1<N<10^3$

High-cycle fatigue: For low stress levels wherein deformations are totally elastic, longer lives result. This is called **high-cycle fatigue** in as much as relatively large numbers of cycles are required to produce fatigue failure. Domain associated with low loads and long service life. Strains are mostly confined to the elastics range.

High number of cycles to produce fatigue failure. $N>10^3$

Endurance Strength Modification Factors:

The most important deviations that occur in design situation compared to standard test conditions are:

- **Load variations**
- **Size variations**
- **Surface finish differences**
- **Temperature differences**
- **Reliability**
- **Other miscellaneous-effects- corrosive environment, fretting, residual stresses, plating, metal spraying etc.**

To account for these conditions a variety of modifying factors, each of which is intended to account for a single effect, is applied to the endurance limit value of test specimen obtained under laboratory conditions.

If σ_{en}= Un-corrected Endurance limit in reversed bending of the highly polished test specimen under laboratory testing conditions,
Then, Corrected Endurance limit = $(A \times B \times C)\ \sigma_{en}$
Where 'A' is the load correction factor, 'B' is the size correction factor,
and 'C' is the surface correction factor.

<u>Effect of Loading on Endurance Limit— Load Correction Factor (A)</u>

The endurance limit (σ_{en}) of a material as determined by the rotating beam method is for reversed bending load.

There are many machine members which are subjected to loads other than reversed bending loads. Thus the endurance limit will also be different for different types of loading.

The endurance limit depends upon the type of loading and may be modified as discussed below:

A = Load correction factor for the type of loading.
Its value is usually taken as 1 for reversed or rotating bending load
= for the reversed axial load, its value may be taken as 0.7.

= for the reversed torsional or shear load, its value may be taken as

0.5 to 0.6 for ductile materials and 0.8 for brittle materials.

<u>Effect of Size on Endurance Limit—Size Correction Factor (B)</u>

The rotating beam specimen is small with 7.5 mm diameter. The larger the machine part, the greater the probability that a flaw exists somewhere in the component. The chances of fatigue failure originating at any one of these flaws are more. The endurance limit, therefore, reduces with increasing the size of the component.

Diameter (d) (mm)	B
$d \leq 7.5$	1.00
$7.5 < d \leq 50$	0.85
$d > 50$	0.75

Values of size factor

<u>Effect of Surface Roughness on Endurance Limit—Surface Correction Factor (C)</u>

The surface of the rotating beam specimen is polished to mirror finish. The final polishing is carried out in the axial direction to smooth out any circumferential scratches. This makes the specimen almost free from surface scratches and imperfections.

Fatigue properties are very sensitive to surface conditions, Fatigue initiation normally starts at the surface since the maximum stress is at the surface.

The factors which affect the surface of a fatigue specimen can be roughly divided into three categories:

- *Surface roughness*
- *Changes in surface properties*
- *Surface residual stress*

Different surface finishes produced by different machining processes can appreciably affect fatigue performance. **Polished surface** normally known as 'par bar' which is used in laboratory, gives the best fatigue strength. The surface correction factors can be obtained by Fig.9 for different surface conditions of the components.

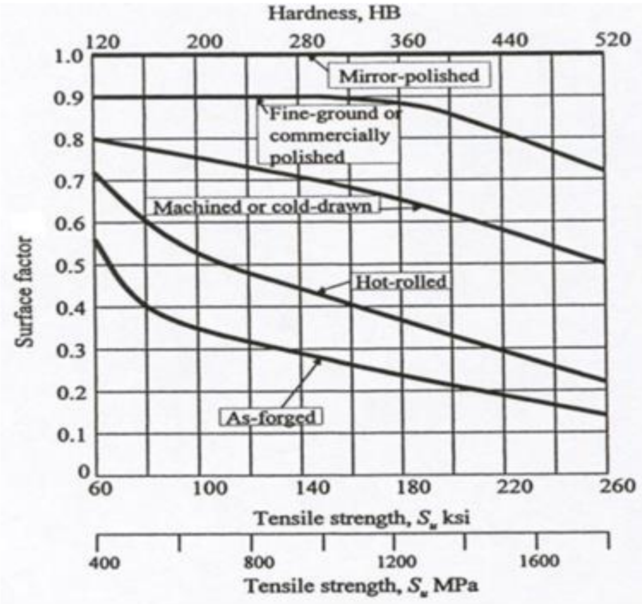


Fig.9. Surface Correction Factor based on surface finish and ultimate strength

TABLE 2.2.

The average values of the surface correction factor C

Ultimate stress σ_u, MN/m^2 (Kgf/mm^2)	*C* for Machine surface (Cold drawn)	*C* for Hot rolled surface
410 (42)	0.91	0.72
480 (49)	0.90	0.68
550 (56)	0.88	0.62
620 (63)	0.86	0.58
690 (70)	0.85	0.55
760 (77)	0.84	0.52
20 (84)	0.82	0.48
1030 (105)	0.78	0.38
1370 (140)	0.72	0.30

(Reference: Data Hand Book-Mahadevan and Balaveera Reddy)

Temperature: This factor accounts for reductions in fatigue life which occur when the operating temperature of the part differs from room temperature (the testing temperature). Upto 450°C, the correction factor is taken as 1.
Reliability: This factor accounts for the scatter of test data. Generally for 50% reliability, the correction factor is 1.

A number of other factors can act to reduce the fatigue resistance of a part.

These include tensile residual stresses, corrosion, plating, metal spraying, cyclic frequency and other factors. Factors that reduce fatigue resistance must be accounted for when designing parts. Surface treatments such as shot peening can induce compressive residual stresses and increase the fatigue resistance of a part (cracks don't open and grow well in compressive stress fields).

Electroplating, especially chromium plating, while improves corrosion resistance and/or the looking of surface finish, generally **decreases** the fatigue limit of steel.

Grinding is a necessary process to improve surface finish, abrade hard materials, and tighten the tolerance. However, it often introduces surface tension and the heat generated in the grinding process might temper the previously quench hardened components.

Forging refines the grain structure and improves physical properties of the metal. Nevertheless, forging can cause decarburization (loss of surface carbon atoms) which is harmful to fatigue life.

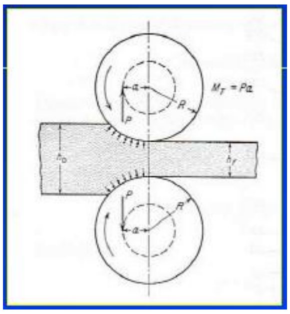
Hot rolling can also cause decarburization (loss of surface carbon atoms), a damaging loss regarding the fatigue life.

Carburizing and nitriding produce higher strength and hardness at the surface and thus improves fatigue life.

Commercial methods introducing favourable compressive stresses:

- **Surface rolling** - Compressive stress is introduced in between the rollers during sheet rolling
- **Shot peening** - Projecting fine steel or cast-iron shot against the surface at high velocity
- **Polishing** - Reducing surface scratches
- **Thermal stress** - Quenching or surface treatments introduce volume change giving compressive stress.

Surface rolling and shot peening operations are shown in Fig. 10.



Surface rolling

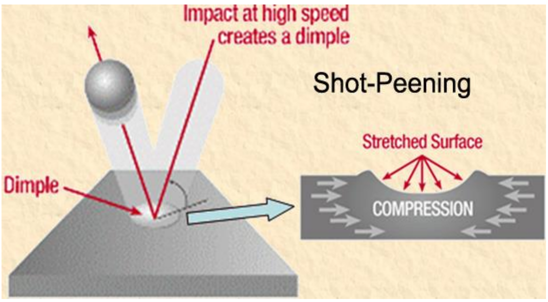


Fig. 10 surface rolling and Shot peening.

Relation between Endurance limit in reversed bending and ultimate tensile strength:

For Steels: $\sigma_{en} = 0.5\ \sigma_u$ **for** $\sigma_u < 1400$ **MPa**
$= 700$ **MPa for** $\sigma_u > 1400$ **MPa**
For Cast iron: $\sigma_{en} = 0.4\ \sigma_u$
For Aluminium: $\sigma_{en} = 0.4\ \sigma_u$
For Copper alloys: $\sigma_{en} = 0.4\ \sigma_u$

NOTCH SENSITIVITY: The term "NOTCH" is referred to any discontinuity in shape or non-uniformity of material. A notch is a stress raiser, and at the notch, the local stresses will be much higher than the nominal stresses. The notch serves as a starting point for fatigue crack.

The notch sensitivity of a material is a measure of how sensitive a material is to notches or geometric discontinuities. Notch sensitivity is defined as the degree to which the theoretical effect of stress concentration is actually reached in a notch.

- **Different types of Notches:**
  - Metallurgical Notches- inclusions, blowholes, quenching cracks etc.
  - Mechanical Notch- grooves, holes, threads, keyways, fillets, serrations, surface indentations etc.
  - Service notch- chemical or corrosion pits, scuffing, fretting, impact indentations, etc.

It is observed that actual reduction in endurance limit of a material due to stress concentration is less than the amount indicated by the theoretical Stress concentration factor is K_t.

To take into this reduction in stress concentration, Fatigue stress concentration factor K_{ft} is introduced in Fatigue analysis.

$$K_{ft} = \frac{Endurance\ limit\ of\ a\ notch\ free\ specimen}{Endurance\ limit\ of\ a\ notched\ specimen}$$

Notch sensitivity is defined as the susceptibility of the material to succumb to the damaging effects of stress raising notches in fatigue loading. The notch sensitivity factor 'q' is defined as

$$q = \frac{Increase\ of\ actual\ stress\ over\ nominal\ stree}{Increase\ of\ theoretical\ stress\ over\ nominal\ stress}$$

$$\mathbf{q = (K_{ft} - 1) / (K_t - 1)}$$

where 'q' is the notch sensitivity index, K_{ft} is the fatigue notch factor and K_t is the stress-concentration factor.

A material is said to be fully notch sensitive if q approaches a value of 1.0; it is not notch sensitive if the ratio approaches 0. Notch sensitivity index q can be obtained from charts shown in Fig.11.

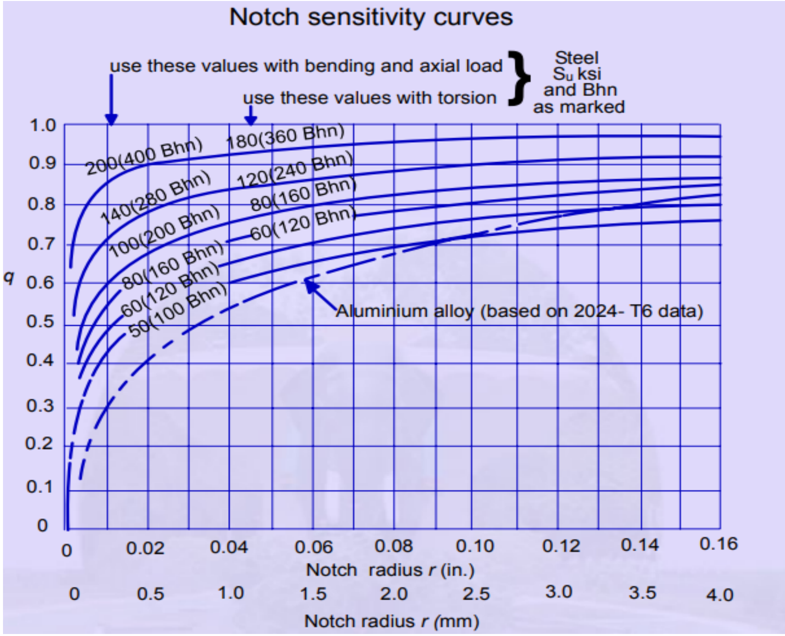


Fig. 11. Notch sensitivity index 'q' based on notch radius

Effect of Mean Stresses on Fatigue Life:

When a component is subjected to fluctuating stresses, there is a mean stress component as well as alternating stress component as illustrated in the Fig.12.

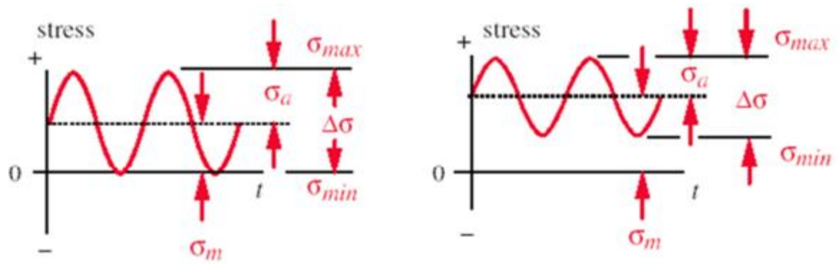


Fig.12. Fluctuating stresses

Mean stress has effect on fatigue life when present in combination with alternating component. In general, compressive mean stresses are beneficial and tensile mean stresses are detrimental to fatigue life as shown in the Fig.13.

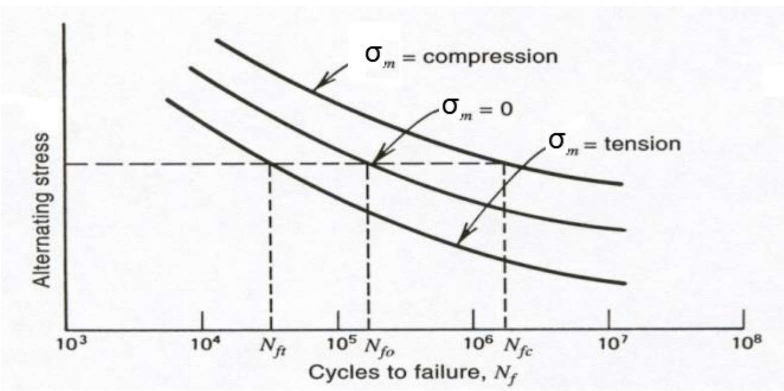


Fig.13. Effect of mean stress on fatigue life

When the component is subjected to both mean stress and alternating stress, the actual failure occurs at different scattered points as shown in Fig.14. There exists a border, which divides the safe region from unsafe region for various combinations of σ_m and σ_a.

This border can be: (i) Soderberg's line
(ii) Goodman's line
(iii) Gerber's parabola

Fig.15 illustrates the way failure prediction is made using the above criteria.

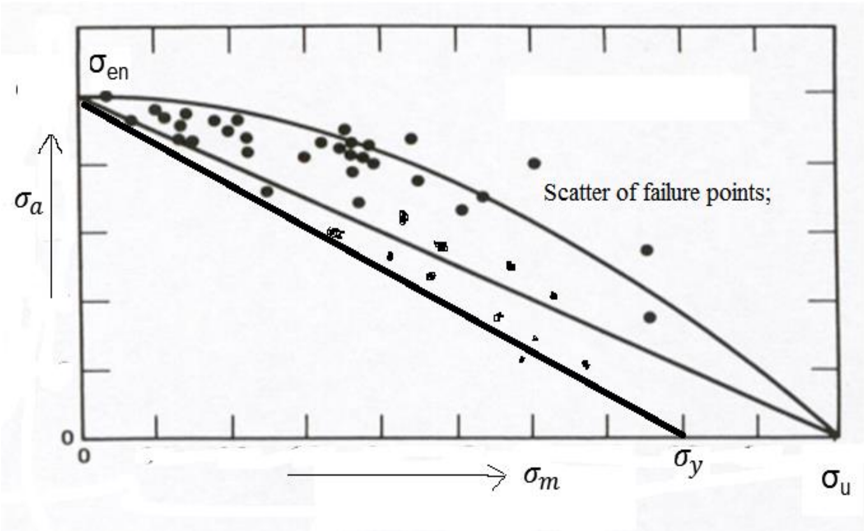


Fig.14 Failure locus with different ratios of Alternating stress and Mean stress

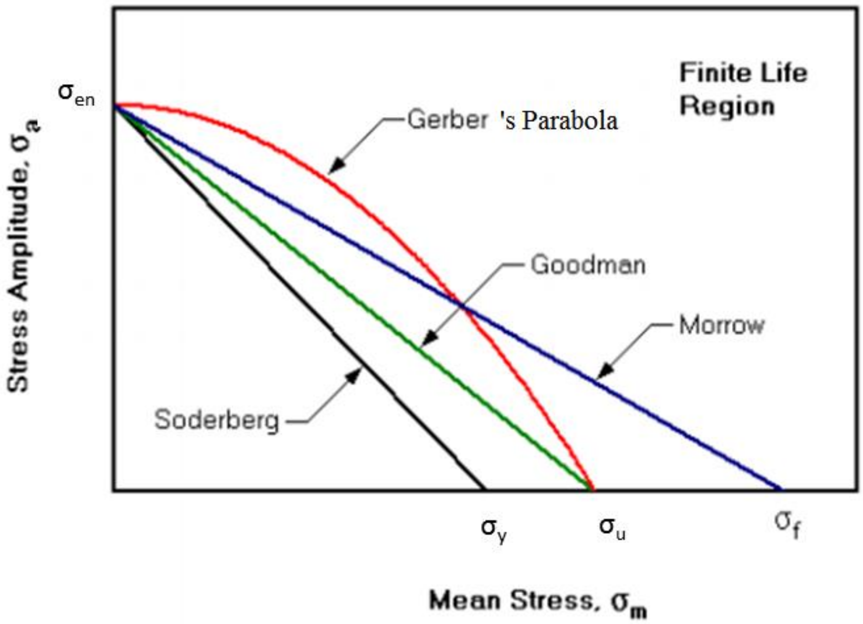
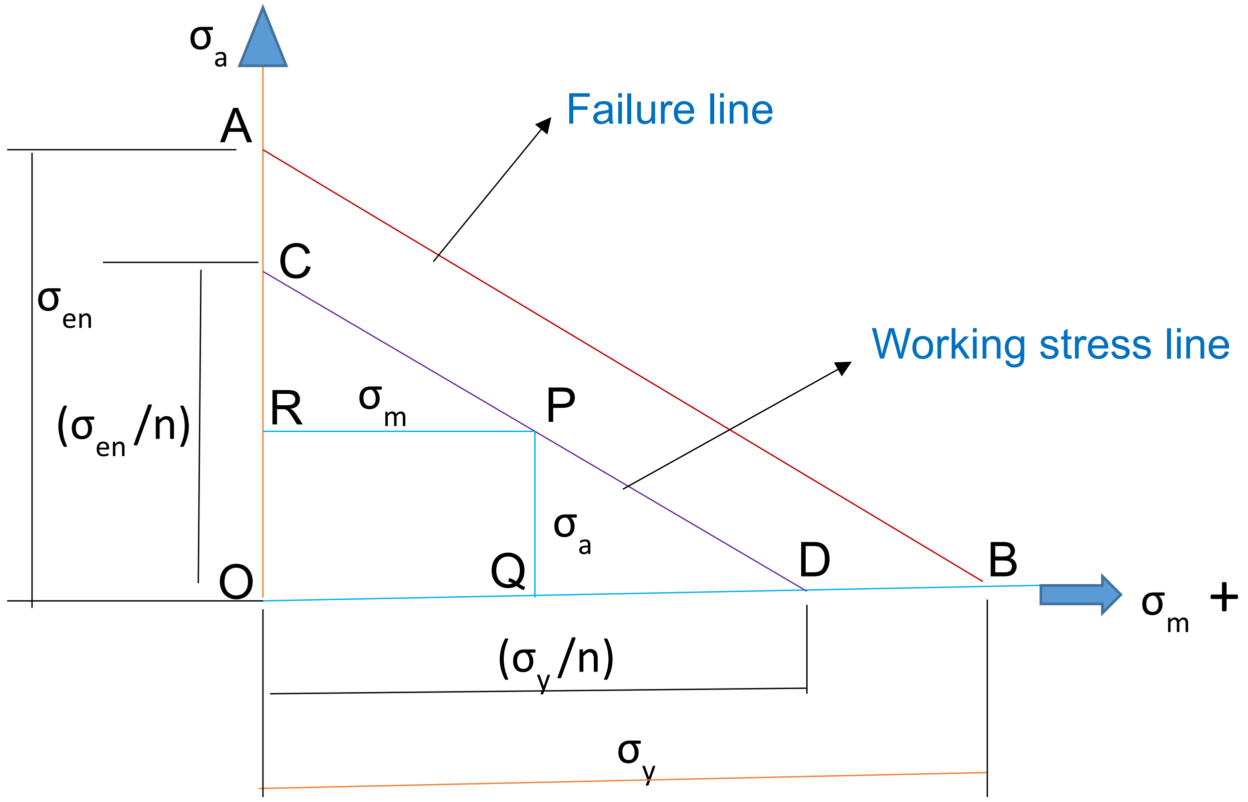


Fig.15 Failure prediction using different criteria

SODERBERG'S RELATION:

Soderberg's relation is based on yield strength of the material whereas all other failure relations for dynamic loading are based on ultimate strength of the material.

This theory proposes that designs for fluctuating normal stress states should be based on a limiting condition defined by a straight line drawn from the endurance limit on the vertical axis to the yield stress on the horizontal axis in the first quadrant.



All mean stresses are plotted on X axis and alternating stresses are plotted on Y axis.
When σ_a =0, the component fails when the stress value reaches yield strength σ_y.
When σ_m =0, the component fails when the stress value reaches endurance strength σ_{en}.
The line joining σ_y and σ_{en} indicates the failure line for different combinations of σ_a and σ_m . This theory proposes that designs for fluctuating stress should be based on limiting condition defined by a straight line drawn from the endurance limit on vertical axis to the yield point on the horizontal axis in the first quadrant. The working stress line is drawn considering the Factor of safety 'n' as shown in the diagram.

Comparing the similar triangles DQP and DOC

$$\frac{QP}{OC} = \frac{QD}{OD} = \frac{OD-OQ}{OD} = 1 - \frac{OQ}{OD}$$

$$\frac{QP}{OC} + \frac{OQ}{OD} = 1$$

$$\frac{\sigma_a}{\sigma_{en}/n} + \frac{\sigma_m}{\sigma_y/n} = 1$$

$$\frac{n\sigma_a}{\sigma_{en}} + \frac{n\sigma_m}{\sigma_y} = 1$$

$$\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

Considering the fatigue stress concentration factor, correction factors for type of loading, size and surface, the Goodman's equation for ***ductile materials*** can be modified and written as:

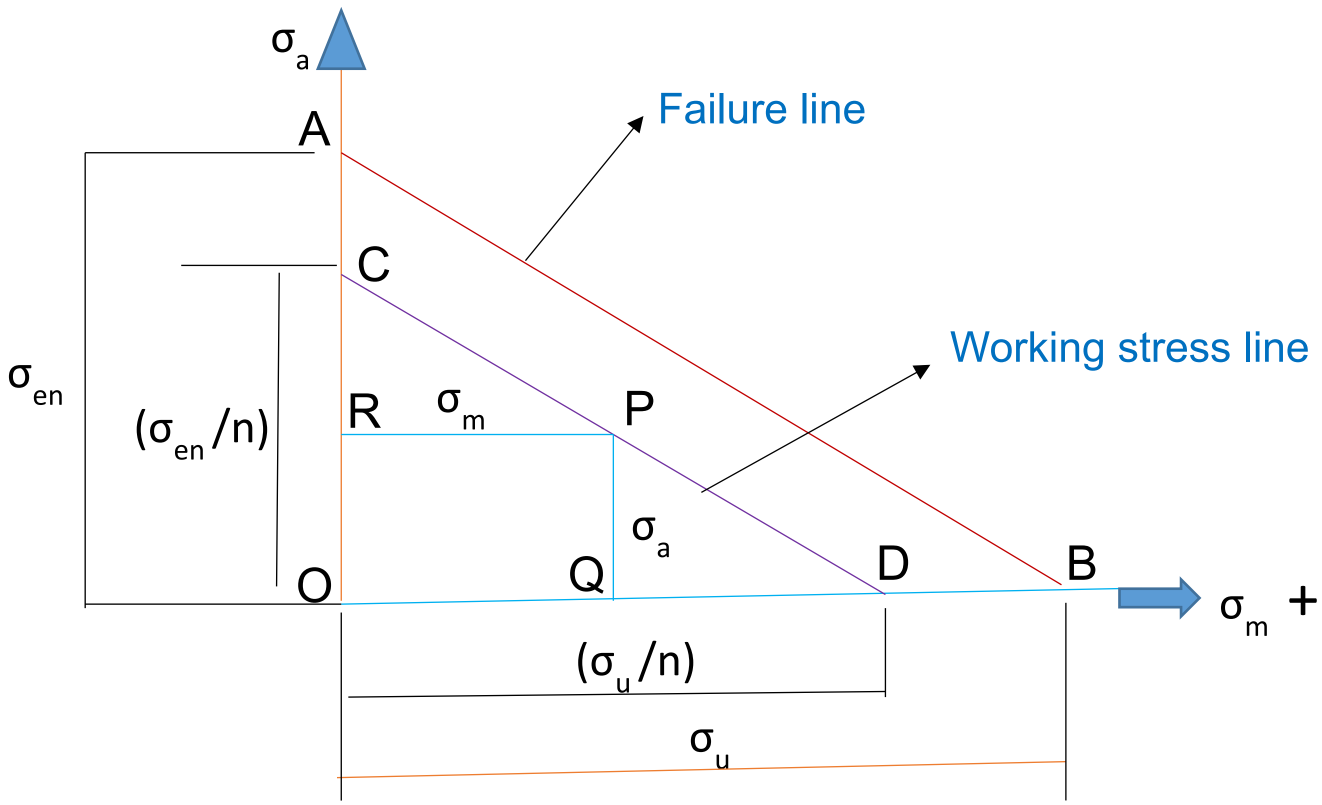
$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

The Soderberg's equation for ***brittle materials*** can be modified and written as:

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{k_t\sigma_m}{\sigma_y} = \frac{1}{n}$$

k_t is considered in brittle materials for the fact that stress concentration is serious in brittle materials even in static loading. Mean stress being a stress component whose magnitude does no vary with time,

Goodman's Relation:



This theory proposes that designs for fluctuating normal stress states should be based on a limiting condition defined by a straight line drawn from the endurance limit on the vertical axis to the Ultimate tensile strength on the horizontal axis in the first quadrant.

The derivation can be made on similar lines to that of Soderberg's Relation.

Considering the fatigue stress concentration factor, correction factors for type of loading, size and surface, the Goodman's equation for ***ductile materials*** can be modified and written as:

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

The Goodman's equation for ***brittle materials*** can be modified and written as:

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{k_t\sigma_m}{\sigma_u} = \frac{1}{n}$$

Problem 1:

Determine the magnitude of the load 'P' for a simply supported beam of 400 mm length if the load at the mid span varies cyclically from 2P to 4P. Size of the beam is 50 mm diameter. The endurance limit for reversed bending is 350 MPa and yield point stress in tension is 520 MPa. Take size factor as 0.85, and surface correction factor as 0.9. Use a design factor of safety of 1.9. There is no keyway present in the critical section on the shaft.

Solution:

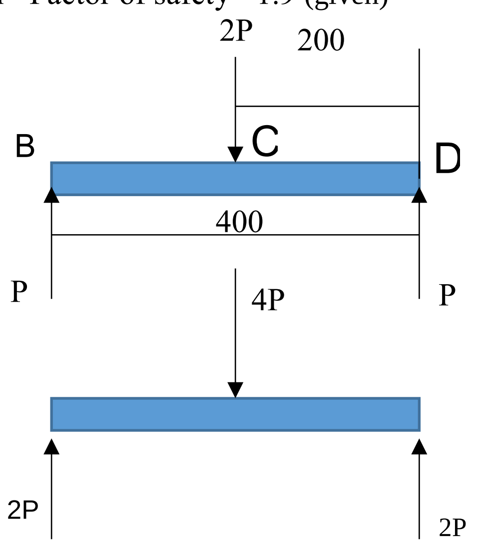
Since there is no keyway present in the critical section, the effect of stress concentration can be neglected. $k_t = K_{tf} = 1$.

A= Correction factor for the type of loading= 1 (as the member is subjected to Bending stress)

B= Size factor= 0.85 (given)

C= Surface correction factor = 0.9 (given)

n= Factor of safety= 1.9 (given)



Bending moment at B=0, at D=0. The bending moment at C varies from minimum to maximum as the bending load varies from minimum to maximum.

M_{max} = Maximum Bending moment at C= 200 X 2P= 400P N-mm
M_{min} = Minimum Bending moment at C= 200 X P= 200P N-mm

$$M_m = \frac{M_{max} + M_{min}}{2} \qquad M_m = \frac{400P + 200P}{2} = 300P$$

$$M_a = \frac{M_{max} - M_{min}}{2} \qquad M_a = \frac{400P - 200P}{2} = 100P$$

$$\sigma_m = \frac{32M_m}{\pi d^3} \qquad \sigma_m = \frac{32 \times 300P}{\pi d^3} = 0.0244P$$

$$\sigma_a = \frac{32M_a}{\pi d^3} \qquad \sigma_a = \frac{32 \times 100P}{\pi d^3} = 0.00815P \text{ MPa}$$

Substituting the values in the Soderberg's equation,

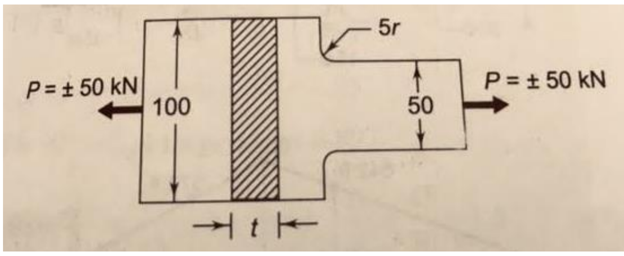
$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1}{1 \times 0.85 \times 0.9}\frac{0.00815P}{350} + \frac{0.0244P}{520} = \frac{1}{1.9}$$

P = 6806 N

Problem 2:

A component machined from a steel plate made of 45C8 is shown in the Fig. It is subjected to a completely reversed axial loading of 50 kN. The factor of safety is 2. Size factor is 0.85 Determine the plate thickness for an infinite life. Take the notch sensitivity factor as 0.8. Yield strength in tension is 315 MPa. The Ultimate Tensile strength is 610 MPa.



Solution:

Consider the smaller section of the plate for design as the stress is maximum in that section. The plate is subjected to completely reversed axial loading.

σ_u =Ultimate tensile strength of 45C8- 610 MPa.

σ_{en} = Endurance strength in reversed bending= 0.5x σ_u = 0.5 x 610 =305 MPa

A= Correction factor for the type of loading =0.7 (for axial loading)

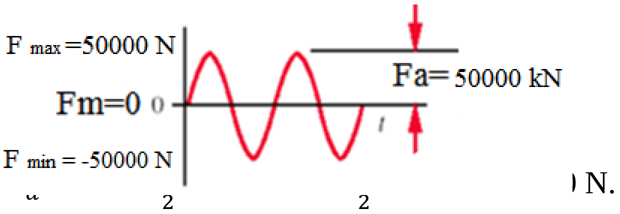
B= Size factor = 0.85 (size factor)

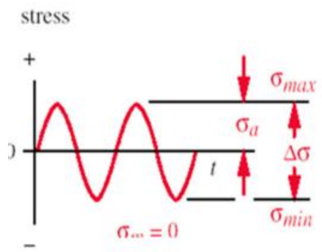
C= Surface correction factor =0.86 from table 2.2 (for 630 MPa, for machined plate)

n= Factor of safety = 2 (given)

σ_y = 315 MPa (given)

q= notch sensitivity index= 0.8 (given)





The mean axial load is zero and hence the mean axial stress is zero.

$\sigma_m = 0$ and $\sigma_a = \frac{F_a}{A} = \frac{50000\times4}{50\times t} = \frac{1000}{t}$

Determination of Fatigue stress concentration factor:

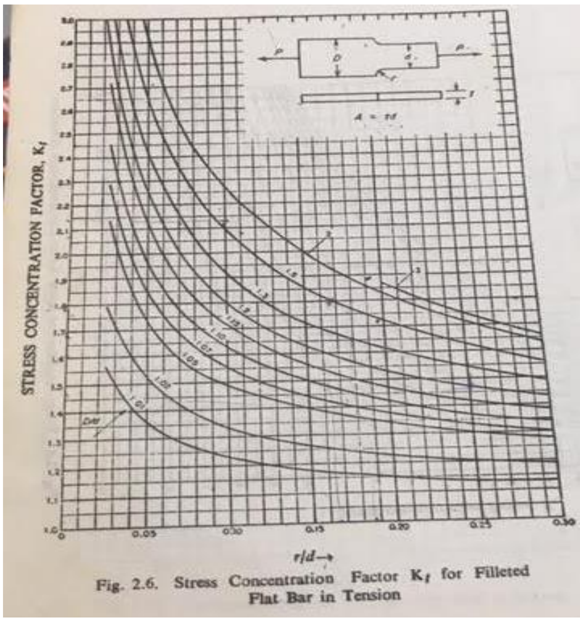


Fig. 2.6. Stress Concentration Factor K_t for Filleted Flat Bar in Tension

r/d= 5/50= 0.1

D/d= 100/50=2

Determine K_t from Fig.2.6 of Mahadevan;

K_t =2.25.

K_{tf} = 1+ q (K_t -1);

K_{tf} = 1+ 0.8 (2.25-1) = 2;

q is notch sensitivity index.

Substituting the values in the Soderberg's equation,

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{2}{0.7 \times 0.85 \times 0.86} \frac{1000}{t \times 315} + 0 = \frac{1}{2}$$

t = 24.81 mm= 25 mm

Problem 3:

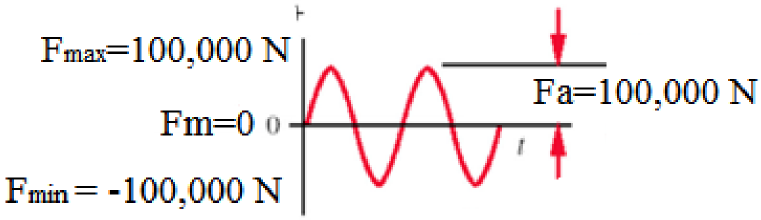
A steel connecting rod is subjected to a completely reversed axial load of 100 kN. Suggest a suitable size of the rod using a Factor of safety of 2. The ultimate tensile strength of the material is 1100 MPa. The yield strength is 930MPa. The correction factor for Loading may be taken as 0.85, and size factor of 0.85. Neglect the column action and effects of stress concentration.

Solution:

Connecting rods are subjected to completely reversed axial loading.
It is clearly stated that column action should be neglected, hence effect of bending stresses due to buckling need not be considered.
It has also been given that the effect of stress concentration has to be neglected.

Given: Correction factor for type of loading (Axial) = **A=0.85**
Correction factor for size=**B=0.85**
Correction factor for surface = **C= 0.76** (from Table 2.2, Mahadevan, for σ_u =1100 MPa, for cold drawn steel)

Factor of safety= **n = 2**
Neglecting Stress concentration, **K_{tf} =1**



The mean axial load is zero and hence the mean axial stress is zero.

$$\sigma_m = 0$$

Substituting the values in the Soderberg's equation,

$$\frac{K_{tf}}{ABC} \frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1}{0.7 \times 0.85 \times 0.76} \frac{\sigma_a}{550} + 0 = \frac{1}{2}$$

$$\sigma_a = 151 \text{ MPa}$$

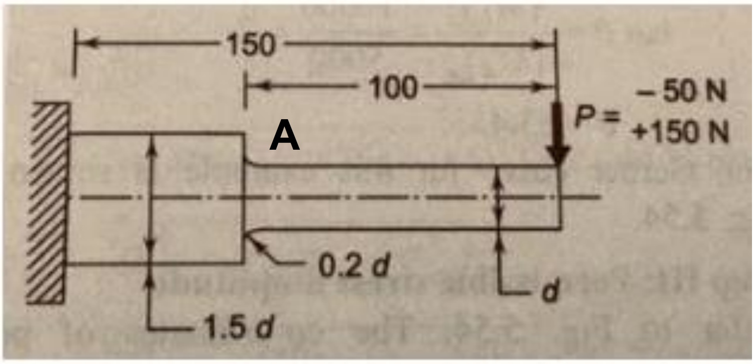
Let 'd' be the diameter of the connecting rod.

$$\sigma_a = \frac{F_a}{A} = \frac{100000 \times 4}{\pi d^2} = 151$$

d = 29.5 or 30 mm

Problem 4:

A cantilever beam made of cold drawn C40 steel is shown in Fig.2. The force P acting at the free end varies from -50 N to +150 N as shown. The factor of safety should be 2. Notch sensitivity index at the fillet cross section is 0.9. Determine the diameter 'd' at the fillet cross section using (i) Soderberg's relation (ii) Goodman's relation.



Solution:

Note: The stresses will be maximum at the section where there is change of cross section. Point A at the fillet experiences maximum bending stress.

σ_u =Ultimate tensile strength of C40- (from Table 1.8, Mahadevan, page 418)
= 600 MPa. (570-667 MPa range)
σ_{en} = Endurance strength in reversed bending= 0.5x σ_u = 0.5x 600 =300 MPa
σ_y = Yield strength in tension= 324 MPa
n= Factor of safety= 2

Correction factor for type of loading =**A= 1 (bending)**
Correction factor for size= **B=0.85**
Correction factor for surface = **C= 0.86** (from Table 2.2, Mahadevan, for σ_u =600 MPa, for cold drawn steel)

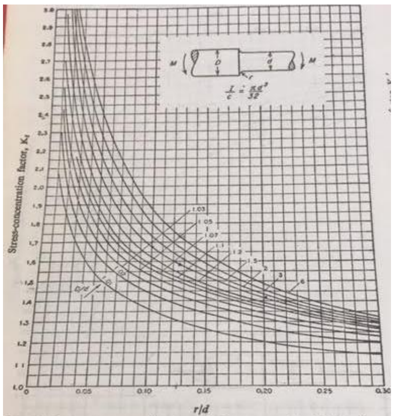


Fig. 2.16. Stress-concentration factor K_t for a stepped shaft in bending

Determination of Fatigue stress concentration factor:

r/d= 0.2d/d= 0.2,
D/d= 1.5d/d= 1.5
Determine K_t from Fig.2.16 of Mahadevan;
K_t =1.43.
K_{tf} = 1+q (K_t -1);
q is notch sensitivity index=0.9 (given).
K_{tf} = 1+0.9(1.43-1)= 1.387;

M_{max} = +150 x 100= 15000 N-mm
M_{min} = -50 x 100= **-** 5000 N-mm

$$M_m = \frac{M_{max} + (-M_{\min})}{2} \qquad M_m = \frac{15000 - 5000}{2} = 5000 \text{ N-mm}$$

$$M_a = \frac{M_{max} - (-M_{\min})}{2} \qquad M_a = \frac{15000 - (-5000)}{2} = 10000 \text{ N-mm}$$

$$\sigma_m = \frac{32M_m}{\pi d^3} \qquad \sigma_m = \frac{32 \times 5000}{\pi d^3} = \frac{50955}{d^3}$$

$$\sigma_a = \frac{32M_a}{\pi d^3} \qquad \sigma_a = \frac{32 \times 10000}{\pi d^3} = \frac{101910}{d^3}$$

Substituting the values in the Soderberg's equation,

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_y} = \frac{1}{n}$$

$$\frac{1.387}{1 \times 0.85 \times 0.86}\frac{101910}{d^3 \times 300} + \frac{50955}{d^3 \times 324} = \frac{1}{2}$$

d = 11.90 mm

Substituting the values in the Goodman's equation,

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

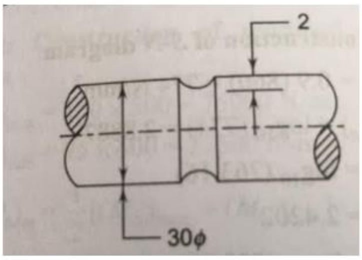
$$\frac{1.387}{1 \times 0.85 \times 0.86}\frac{101910}{d^3 \times 300} + \frac{50955}{d^3 \times 600} = \frac{1}{2}$$

d = 11.34 mm

Soderberg's relation is more conservative in its approach and gives a higher value of the diameter.

Problem 6:

A **polished** **steel bar shown in Fig. is subjected to an axial tensile stress that varies from zero** F_{max}**. The radius of the groove is 3 mm. The outer diameter of the bar is 30 mm. The notch sensitivity factor at the groove is 0.95. The ultimate tensile strength of the bar is 1250 MPa. The endurance limit in reversed bending is 600 MPa. Find the maximum force the bar can carry for an infinite life based on Goodman's Criterion with a factor of safety of 2.**



Solution:

σ_u =Ultimate tensile strength of the bar = 1250 MPa

σ_{en} = Endurance strength in reversed bending= 600MPa

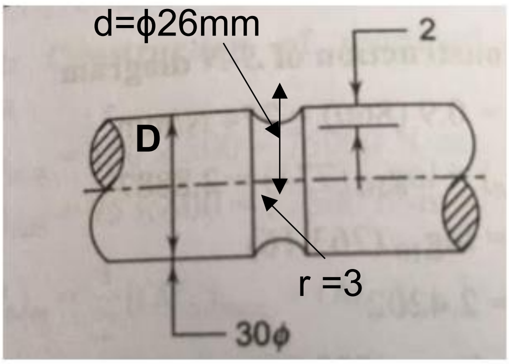
Correction factor for type of loading (Axial)= **A=0.7**

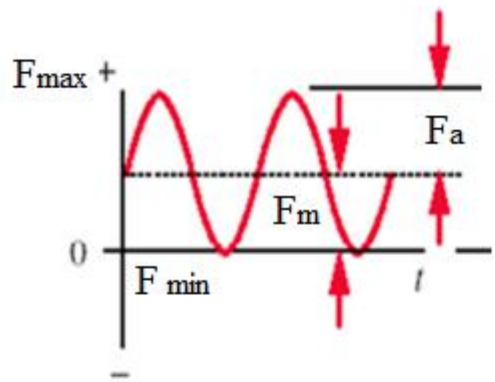
Correction factor for size=**B=0.85**

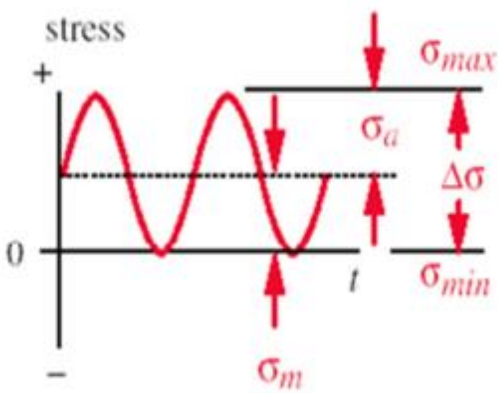
Correction factor for surface = **C= 1 as the bar has a polished surface.**

Factor of safety= **n = 2**

Determination of Fatigue stress concentration factor:
r/d= 3/26= 0.115, D/d= 30/26= 1.15
Determine K_t from Fig.2.9 of Mahadevan;
K_t =1.85.
K_{ft} = 1+q(K_t -1); K_{ft} = 1+0.95(1.85-1)= 1.807;
q is notch sensitivity index.







$$F_{min} = 0$$

$$F_a = F_m = \frac{F_{max} - F_{min}}{2} = \frac{F_{max}}{2} = 0.5\, F_{max}$$

$$\sigma_m = \sigma_a = \frac{0.5F_{max}}{A} = \frac{0.5F_{max} \times 4}{\pi \times d^2} = \frac{0.5F_{max} \times 4}{\pi \times 26^2} = 0.00094\, F_{max}$$

Substituting the values in the Goodman's equation,

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

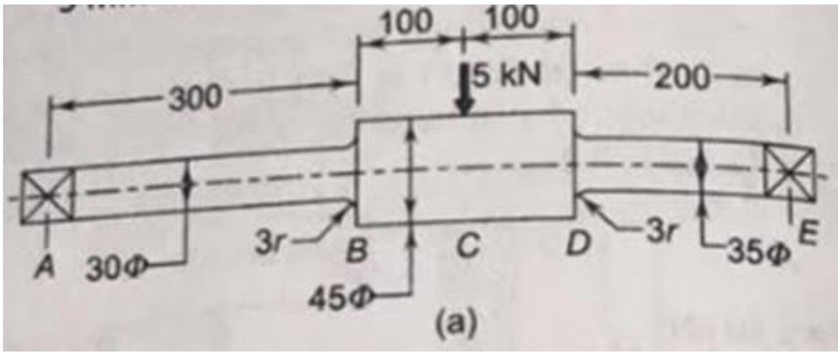
$$\frac{1.807}{0.7 \times 0.85 \times 1}\frac{0.00094\, F_{max}}{600} + \frac{0.00094\, F_{max}}{1250} = \frac{1}{2}$$

$$\boldsymbol{F_{max}} = \mathbf{90876\ N}$$

The maximum load that the bar can carry is 90876 N.

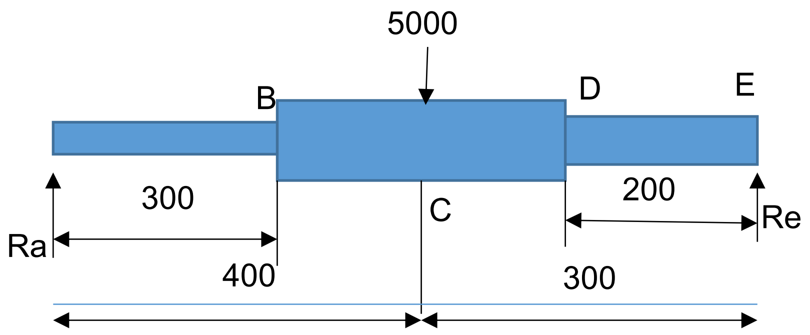
Problem 6:

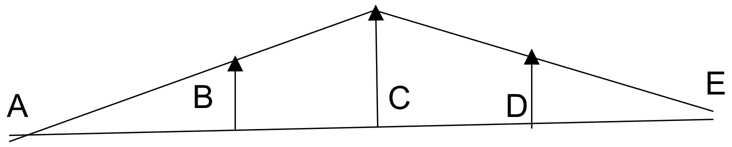
A rotating shaft, subjected to a non-rotating force of 5 kN is shown in Fig.4. The shaft is machined from plain carbon steel with an ultimate tensile strength of 500 MPa. The equivalent notch radius can be taken as 3 mm. Determine the factor of safety of this member based on the Goodman's criterion. Take the notch sensitivity index at the fillet as 0.9. Comment on the result.



Solution:

Note: Failure possibilities must be investigated at three sections- B, C, and D.
At section C, although the bending moment is maximum, the diameter is more and there is no stress concentration.
At section D, the diameter is more and bending moment is lower compared to Section B.
Section B is prone to failure and considered for design.





Bending moment diagram

Taking moments about 'A':

5000 x 400 -Re x700=0

Re= 2857 N, Ra= 5000-2857=2143 N

Bending Moment at B=2143 x 300= 642900 N-mm

Bending Moment at C=2143 x 400= 571400 N-mm

Bending Moment at D=2857 x 200= 642900 N-mm

Since the shaft is rotating and the load is stationary, the load on the shaft is of completely reversed bending type. $\sigma_m = 0$

A= 1 (for bending)

B= 0.85 (size factor)

C= 0.89 from table 2.2 (for 500 MPa, for machined shaft)

σ_u = 500 MPa

σ_{en} = Endurance strength in reversed bending= 0.5x σ_u = 0.5x500 =250 MPa

Determination of Fatigue stress concentration factor:

r/d= 3/30= 0.1, D/d= 45/30= 1.5

Determine K_t from Fig.2.16 of Mahadevan; K_t =1.67.

K_{tf} = 1+q (K_t -1); K_{tf} = 1+0.9 (1.67-1)= 1.60;

q is notch sensitivity index.

$$M_a = \frac{M_{max} - M_{min}}{2} \qquad M_a = \frac{642900 - (-642900)}{2} = 642900 \text{ N-mm}$$

$$\sigma_a = \frac{32 M_a}{\pi d^3} \qquad \sigma_a = \frac{32 \times 642900}{\pi \times 30^3} = 242.6\ MPa$$

Substituting the values in the Goodman's equation,

$$\frac{K_{tf}}{ABC}\frac{\sigma_a}{\sigma_{en}} + \frac{\sigma_m}{\sigma_u} = \frac{1}{n}$$

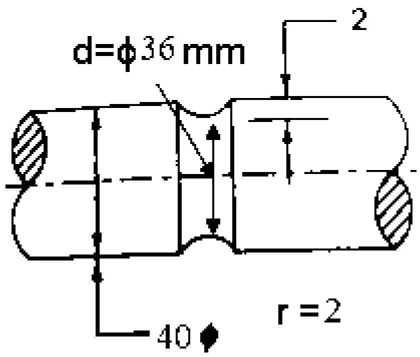
$$\frac{1.60}{1 \times 0.85 \times 0.89}\frac{242.6}{250} + 0 = \frac{1}{n}$$

n = 0.467 < 1

The F.S. is less than 1. This component will not have infinite life instead will have a finite life less than a million cycles.

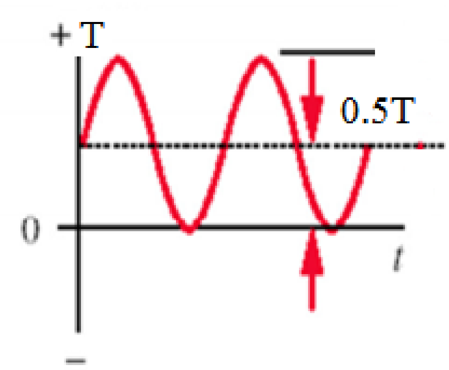
Problem 7:

A 40 mm diameter steel shaft has σ_y =413 MPa σ_{en} = 336 MPa. For a factor of safety of 2, what (i) repeated (ii) reversed torques can be sustained by the shaft indefinitely? The shaft has a groove machined on it. The radius of the groove is 2 mm and the diameter at the bottom of the groove is 36 mm. Take size factor =0.85 and surface factor=1.



A= 0.6 (for Torsion)
B= 0.85 (size factor, given)
C= 1 (given)
σ_{en} = Endurance strength in reversed bending= 336 MPa (given)
σ_y = Yield strength = 413 MPa
$\boldsymbol{\tau_y = 0.5 \times \sigma_y}$ **= 0.5 x 413= 206.5 MPa**

(i) Let T be the released torque acting on the component



$T_m = (T_{max} + T_{min}) / 2 = T/2 = 0.5T$

$T_a = (T_{max} - T_{min}) / 2 = T/2 = 0.5T$

Mean shear stress= $\tau_m = \frac{16T_m}{\pi d^3} = \frac{16\times 0.5\,T}{3{,}14\times 36^3} = 5.46\text{x } 10^{-5}$ MPa

$Alternating\ shear\ stress = \tau_a = 5.46\text{x } 10^{-5}$ MPa

Determination of Fatigue stress concentration factor:
r/d= 2/36= 0.05, D/d= 40/36= 1.33
Determine K_t from Fig.2.13 of Mahadevan; K_t =1.85.

$K_{tf} = 1+q(K_t - 1)$; $K_{tf} = 1 + 1\ (1.85-1) = 1.85$;
Assuming 'q' notch sensitivity index =1 .

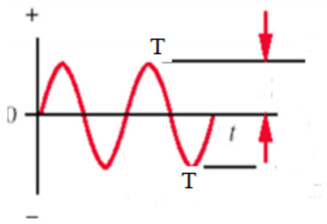
Substituting the values in the Soderberg's equation,

$$\frac{K_{tf}}{ABC}\frac{\tau_a}{\sigma_{en}} + \frac{\tau_m}{\tau_y} = \frac{1}{n}$$

$$\frac{1.85}{0.6\times 0.85\times 1}\frac{5.46\times 10^{-5}\ \times T}{413} + \frac{5.46\times 10^{-5}\ \times T}{206.5} = \frac{1}{2}$$

T= 684931 N-mm

(ii) Let 'T' be the completely released torque acting on the component



$T_m = (T_{max} + T_{min}) / 2 = 0$

$T_a = (T_{max} - T_{min}) / 2 = 2T/2 = T$

$\tau_m = 0$ $\qquad \tau_a = \frac{16T_a}{\pi d^3} = \frac{16\times T}{3{,}14\times 36^3} = 1.09\text{x}10^{-4}$ T MPa

Substituting the values in the Soderberg's equation,

$$\frac{K_{tf}}{ABC}\frac{\tau_a}{\sigma_{en}} + \frac{\tau_m}{\tau_y} = \frac{1}{n}$$

$$\frac{1.85}{0.6 \times 0.85 \times 1}\frac{1.09 \times 10^{-4} \times T}{413} = \frac{1}{2}$$

T= 522466 N-mm

STRESSES DUE TO COMBINED LOADING:

When a machine part is subjected to both variable normal stress and a variable shear stress, then it is designed by using the following two theories of combined stresses:

- Maximum shear stress theory.
- Maximum normal stress theory.

In practice, the machine elements are subjected to combined bending and torsional stresses which are cyclic. In case of two dimensional stresses, each of the stresses may have two components –mean and variable. In such cases, equivalent normal stress component and equivalent shear stress component are calculated.

Combined Axial and bending fatigue loading:

$$\sigma_{eq-n(axial)} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right)\frac{K_{tf}\ \sigma_a}{A \times B \times C}$$

$$\sigma_{eq-n(bending)} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right)\frac{K_{tf}\ \sigma_a}{A \times B \times C}$$

When a component is subjected to combined axial and bending fatigue loading, then, the maximum equivalent normal stress is given by:

$$\sigma_{eq}\,max = \sigma_{eq-n(axial)} + \sigma_{eq-n(bending)}$$

Design will be based on maximum normal stress theory.

$$\sigma_{eq}\,max = \frac{\sigma_y}{n}$$

Combined bending and torsion fatigue loading:

The equivalent normal stress component is given by:

$$\sigma_{eq-n} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right)\frac{K_{tf}\ \sigma_a}{A \times B \times C}$$

The equivalent shear stress component is given by:

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right)\frac{K_{tf}\ \tau_a}{A \times B \times C}$$

Using Maximum shear stress theory, Maximum shear stress is calculated.

$$\tau_{eq}\ max = \sqrt{\left(\frac{1}{2}\ \sigma_{eq-n}\right)^2 + \left(\tau_{eq}{}^2\right)}$$

Design will be based on maximum shear stress theory.

$$\tau_{eq}\ max = \frac{\tau_y}{n}$$

Using Maximum normal stress theory, Maximum normal stress is calculated.

$$\sigma_{eq}\ max = \frac{1}{2}\sigma_{eq-n} + \sqrt{\left(\frac{1}{2}\ \sigma_{eq-n}\right)^2 + \left(\tau_{eq}{}^2\right)}$$

Design will be based on maximum normal stress theory.

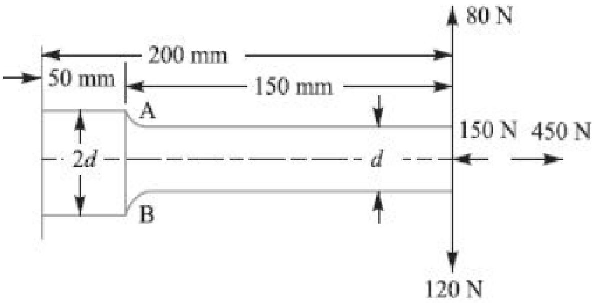
$$\sigma_{eq}\ max = \frac{\sigma_y}{n}$$

PROBLEMS ON COMBINED VARIABLE NORMAL STRESSES AND VARIABLE SHEAR STRESSES

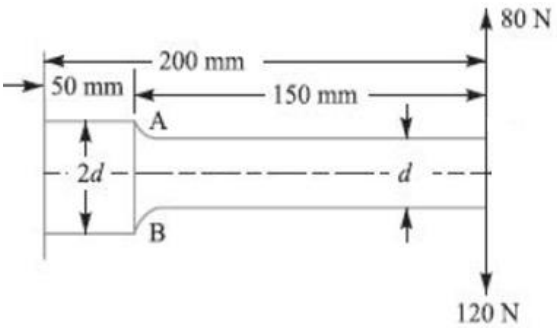
Problem 1:

A steel cantilever is 200 mm long. It is subjected to an axial load which varies from 150 N (compression) to 450 N (tension) and also a transverse load at its free end which varies from 80 N up to 120 N down. The cantilever is of circular cross-section. It is of diameter 2d for the first 50mm and of diameter 'd' for the remaining length. Determine its diameter taking a factor of safety of 2. Assume the following values:

Yield stress = 330Mpa
Endurance limit in reversed loading = 300Mpa
Correction factors =0.7 in reversed axial Loading = 1.0 in reversed Bending
Stress concentration factor = 1.44 for bending
= 1.64 for axial loading
Size effect factor = 0.85
Surface effect factor = 0.90
Notch sensitivity index = 0.90



Solution:



Consider bending load on the component: The section at the fillet is critical from the point of view of design as it experiences Max bending stress due to smaller diameter and change in cross section (point A).

$$M_{max} = 120 \times 150 = 18000\ N - mm$$

$$M_{min} = \ - 80 \times 150 = -12000\ N - mm$$

$$M_m = \frac{M_{max} + M_{min}}{2} \quad M_m = \frac{18000 - 12000}{2} = 3000\ N - mm$$

$$M_a = \frac{M_{max} - M_{min}}{2} \quad M_a = \frac{18000 + 12000}{2} = 15000\ N - mm$$

$$\sigma_m = \frac{32 \times M_m}{\pi \times d^3} \qquad \sigma_m = \frac{32 \times 3000}{\pi \times d^3} = \frac{30573}{d^3} \text{ MPa}$$

$$\sigma_a = \frac{32 M_a}{\pi d^3} \qquad \sigma_a = \frac{32 \times 15000}{\pi d^3} = \frac{152866}{d^3} \text{ MPa}$$

A=1 for bending, B=0.85, C=0.90, q=0.9 (given)

Determination of Fatigue stress concentration factor:
K_{tf} = 1+q(K_t -1); K_{tf} = 1+0.9 (1.44-1)=1.396 (K_t is given as 1.44 in bending)
q is notch sensitivity index.

σ_y = Yield strength of the component = 330 MPa)
σ_{en}= Endurance strength in reversed bending= 300 MPa

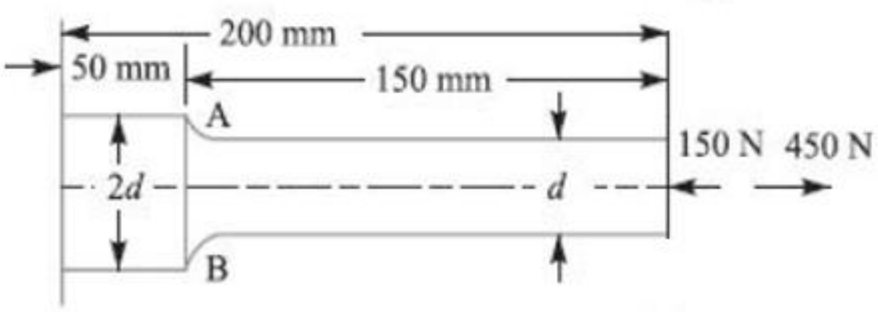
Equivalent normal stress in bending:

$$\sigma_{eq-n\,(bending)} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{K_{tf}\ \sigma_a}{A \times B \times C}$$

$$\sigma_{eq-n\ \text{bending}} = \frac{30573}{d^3} + \left(\frac{330}{300}\right) \frac{1.396}{1x0.85x0.9} \frac{152866}{d^3}$$

$$\sigma_{eq-n\ (bending)} = \frac{337424}{d^3} \text{ MPa}$$

Equivalent axial stress in Tension:



F_{max}= +450 N (Tensile)
F_{min}= - 150 N (Compressive)

$$F_m = \frac{F_{max}+F_{min}}{2} = \frac{450-150}{2} = 150 \text{ N}$$

$$F_a = \frac{F_{max}-F_{min}}{2} = \frac{450+150}{2} = 300 \text{ N}$$

$$\sigma_m = \frac{F_m}{A} \qquad \sigma_m = \frac{4 \times F_m}{\pi d^2} \qquad \sigma_m = \frac{4\times150}{\pi d^2} = \frac{191}{d^2} \text{ MPa}$$

$$\sigma_a = \frac{F_a}{A} \qquad \sigma_a = \frac{4 \times F_a}{\pi d^2} \qquad \sigma_a = \frac{4\times300}{\pi d^2} = \frac{382}{d^2} \text{ MPa}$$

A=0.7 for axial loading, B=0.85, C=0.90, q=0.9 (given)

Determination of Fatigue stress concentration factor:
K_{tf} = 1+q(K_t -1); K_{tf} = 1+0.9 (1.64-1)=1.576 (K_t is given as 1.64 in axial loading)
q is notch sensitivity index.

$$\sigma_{eq-n(axial)} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right)\frac{K_{tf}\ \sigma_a}{A \times B \times C}$$

$$\sigma_{eq-n\ \text{axial}} = \frac{191}{d^2} + \left(\frac{330}{300}\right)\frac{1.576}{1x0.85x0.9}\frac{382}{d^2}$$

$$\sigma_{eq-n\ axial} = \frac{1056.6}{d^2} \text{ MPa}$$

$$\sigma_{eq\ max} = \sigma_{eq-n\ bending} + \sigma_{eq-n\ axial} = \frac{\sigma_y}{n}$$

$$\frac{337424}{d^3} + \frac{1056}{d^2} = \frac{330}{2}$$

Solving the above equation, we get d = 12.9 mm or 13 mm

Problem 2:

A hot rolled steel shaft is subjected to a torsional moment that varies from 330 N.m clockwise to 110 N.m counterclockwise and an applied bending moment at a critical section varies from 440 N.m to -220 N.m. The shaft is of uniform cross section and no key way is present at the critical section. Determine the required shaft diameter. The material has an ultimate strength of 550 MN/m^2 and yield strength of 410 MN/m^2. Take the endurance limit as half the ultimate strength, factor of safety of 2, Load factor of 0.55, size factor of 0.85 and a surface finish factor of 0.62.

Solution:

σ_u =Ultimate tensile strength of the bar = 550 MPa
σ_{en} = Endurance limit in reversed bending= 0.5 x550 = 275 MPa
σ_y = Yield strength in Tension= 410 MPa.

$K_{ft} = K_t = 1$ since there is no stress concentration;

Equivalent shear stress in Torsion:

$$T_m = \frac{T_{(\max)} + T_{(min)}}{2} = \frac{330 + (-110)}{2} = 110\ N.m = 110\ \times 10^3 \text{ N-mm}$$

$$T_a = \frac{T_{(\max)} - T_{(min)}}{2} = \frac{330 - (-110)}{2} = 220\ N.m = 210 \times 10^3 \text{ N-mm}$$

Mean shear stress, $\tau_m = \frac{16T_m}{\pi d^3} = \frac{16(110 \times 10^3)}{\pi d^3} = \frac{560 \times 10^3}{d^3}$ MPa

Variable shear stress, $\tau_a = \frac{16T_a}{\pi d^3} = \frac{16(220 \times 10^3)}{\pi d^3} = \frac{1120 \times 10^3}{d^3}$ MPa

$\boldsymbol{\tau_y}$=**Yield strength in shear is 0.5$\boldsymbol{\sigma_y = 0.5 \times 410 = 205\ MPa}$.**

A=0.55 for bending,   B=0.85,   C=0.62,   (given)

$$\tau_{eq} = \tau_m + \left(\frac{\tau_y}{\sigma_{en}}\right) \frac{K_{tf}\,\tau_a}{ABC}$$

$$\tau_{eq} = \frac{560 \times 10^3}{d^3} + \left(\frac{205}{275}\right) \frac{1}{0.55 \times 0.85 \times 0.62} \frac{1120 \times 10^3}{d^3}$$

$$\boldsymbol{\tau_{eq} = \frac{3440 \times 10^3}{d^3}} \textbf{ MPa}$$

For bending moment,

$$M_m = \frac{M_{(\max)} + M_{(min)}}{2} = \frac{440 + (-220)}{2} = 110 N.m = 110 \times 10^3 \text{ N-mm}$$

$$M_a = \frac{M_{(\max)} - M_{(min)}}{2} = \frac{440 - (-220)}{2} = 330 N.m = 330 \times 10^3 \text{ N-mm}$$

Mean bending stress, $\sigma_m = \frac{32M_m}{\pi d^3} = \frac{32(110 \times 10^3)}{\pi d^3} = \frac{1120 \times 10^3}{d^3}$ MPa

and Variable bending stress, $\sigma_a = \frac{32 \times M_a}{\pi d^3} = \frac{32(330 \times 10^3)}{\pi d^3} = \frac{3360 \times 10^3}{d^3}$ MPa

A=1 for bending,   B=0.85,   C=0.62   (given)

$$\sigma_{eq-n} = \sigma_m + \left(\frac{\sigma_y}{\sigma_{en}}\right) \frac{K_{tf}\,\sigma_a}{ABC}$$

$$\sigma_{eq-n} = \frac{110 \times 10^3}{d^3} + \left(\frac{205}{275}\right) \frac{1 \times 330 \times 10^3}{1 \times 0.85 \times 0.62 \times d^3}$$

$$\boldsymbol{\sigma_{eq-n} = \frac{10626}{d^3}\ \text{MPa}}$$

A. Determine the diameter of the shaft using the Maximum shear stress theory.

$$\tau_{eq}\,max = \sqrt{\left(\frac{1}{2}\,\sigma_{eq-n}\right)^2 + \left(\tau_{eq}{}^2\right)}$$

$$\boldsymbol{\tau_{eq}\,max} = \sqrt{\left(\frac{1}{2}\ \frac{10626\times10^3}{d^3}\right)^2 + \left(\frac{3440\times10^3}{d^3}\right)} = \boldsymbol{\frac{6177\times10^3}{d^3}}$$

$$\tau_{eq}\,max = \frac{\tau_y}{n} \qquad \boldsymbol{\frac{6177\times10^3}{d^3}} = \frac{205}{2}$$

d=39.2 mm

B. Determine the diameter of the shaft using the Maximum normal stress theory.

$$\sigma_{eq}\,max = \frac{1}{2}\sigma_{eq-n} + \sqrt{\left(\frac{1}{2}\,\sigma_{eq-n}\right)^2 + \left(\tau_{eq}{}^2\right)}$$

$$\sigma_{eq}\,max = \frac{1}{2}\frac{10626}{d^3} + \sqrt{\left(\frac{1}{2}\ \frac{10626 \times 10^3}{d^3}\right)^2 + \left(\frac{3440 \times 10^3}{d^3}\right)}$$

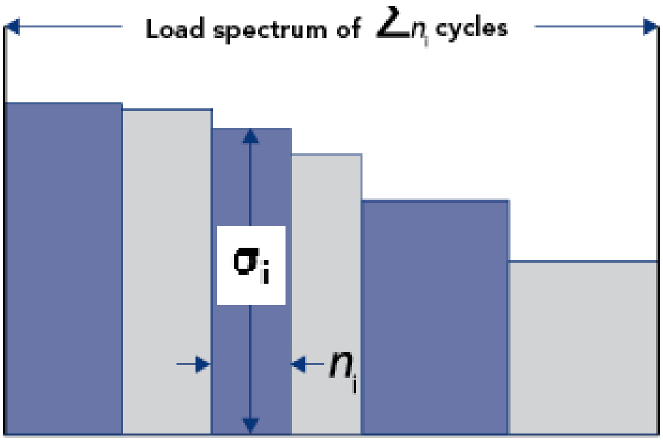
$$\sigma_{eq}\,max = \frac{\sigma_y}{n} = \frac{400}{2}$$

$$\frac{11490\times10^3}{d^3} = 200$$

d= 38.5 mm, choose the bigger value; d= 39.2 mm or 40 mm

Cumulative Fatigue Damage:

In certain applications, the mechanical component is subjected to different stress levels for different parts of work cycle. The life of such a component is determined by MINER'S rule. Suppose that a component is subjected to completely reversed stresses σ_1 for n1 cycles, σ_2 for n2 cycles and so on. Let N1 be the number of stress cycles before fatigue failure when only σ_1 is acting. One stress cycle will consume (1/N1) of the failure life and since there are n1 such cycles at this stress level, the percentage damage of fatigue life will be (1/N1)n1 or (n1/ N1). Similarly, the proportionate damage at stress level σ_2 is (n2/ N2).



Cumlative number of applied cycles

According to this hypothesis, the rupture occurs when the sum of fractions of damage (C) , defined only by the consumed cycles (ni/Ni), at various load levels, reaches unity.

$$C=\frac{n1}{N1}+\frac{n2}{N2}+\frac{n3}{N3}+---\frac{nx}{Nx}=1;$$ this is known as Miner's equation.

Sometimes, the number of cycles n1, n2, n3---- at stress cycles σ_1, σ_2, σ_3 are unknown. Suppose that α1, α2, α3 ---- are the portions of the total life that will be consumed by the stress levels σ_1, σ_2, σ_3 --- etc.

Let N be the total life of the component.

Then, n1= α1 N, n2= α2 N, n3= α3 N.

[If a part is stressed for 3,000 cycles at a stress level which would cause failure in 100,000 cycles, 3 percent of the fatigue life would be expended.

$$\therefore \alpha 1=\frac{3000}{100000}=0.03$$

Repeated stress at another stress level would consume another similarly calculated portion of the fatigue life.]

Substituting the values in Miner's equation,

$$\frac{\alpha 1}{N1}+\frac{\alpha 2}{N2}+\frac{\alpha 3}{N3}+\text{-----------}\frac{\alpha x}{Nx}=\frac{1}{N}$$

Also,

$\alpha 1$ + $\alpha 2$ + $\alpha 3$ +----- αx = 1

With the help of the above equation, the life of the components subjected to different stress levels can be determined.

Problem:

Tests show that the median life of bearings operating at high frequency to be $2x10^8$ cycles under 1 kN load and $3x10^7$ cycles under 2 kN load. How many cycles the we can expect the bearing to last if 1 kN load operates 90% of the time and 2 kN load operates during the remaining 10% of the time?

Solution:

Let the total number of cycles (life) be N.

α1= Portion of the total life consumed by 1 kN load = 0.9

The number of cycles n1 for 1kN load= α1xN= 0.9N

α2 = Portion of the total life consumed by 2 kN load = 0.1

The number of cycles n2 for 2kN load= α2xN= 0.1N

According to the Miners Equation,

$$\frac{\mathbf{n1}}{\mathbf{N1}}+\frac{\mathbf{n2}}{\mathbf{N2}}+\frac{\mathbf{n3}}{\mathbf{N3}}+---\frac{\mathbf{nx}}{\mathbf{Nx}}=\mathbf{1}$$

In this case, $\frac{\mathbf{n1}}{\mathbf{N1}}+\frac{\mathbf{n2}}{\mathbf{N2}}$= 1

Where N1= Failure life of the bearing when only 1 kN load acts= **$2x10^8$ cycles**

N2= Failure life of the bearing when only 2 kN load acts= **$3x10^7$ cycles**

Substituting the above values in Miners equation,

$$\frac{0.9N}{2\times 10^8}+\frac{0.1N}{3\times 10^7}=1$$

N= 1.3 $x10^8$ cycles

PROBLEMS FOR PRACTICE:

1) **Determine the diameter of a circular rod made of ductile material with a fatigue strength (complete stress reversal), σ_e = 265 MPa and a tensile yield strength of 350 MPa. The member is subjected to a varying axial load from Wmin = -300 x 10^3 N to Wmax = 700 x 10^3 N and has a stress concentration factor = 1.8. Use factor of safety as 2.0.**

2) **A circular bar of 500 mm length is supported freely at its two ends. It is acted upon by a central concentrated cyclic load having a minimum value of 20 kN and a maximum value of 50 kN. Determine the diameter of bar by taking a factor of safety 1.5, size effect of 0.85, surface finish factor of 0.9. The material properties of bar at given by: ultimate strength of 650 MPa, yield strength of 500 MPa and endurance strength of 350 MPa.**

3.A simply supported beam has a concentrated load at the centre which fluctuates from a value of P to 4 P. The span of the beam is 500 mm and its cross-section is circular with a diameter of 60 mm. Taking for the beam material an ultimate stress of 700 MPa, a yield stress of 500 MPa, endurance limit of 330 MPa for reversed bending, and a factor of safety of 1.3, calculate the maximum value of P. Take a size factor of 0.85 and a surface finish factor of 0.9.

4. A connecting rod is subjected to an axial load that fluctuates between 120 kN in tension to 60 kN in compression. The material has a yield strength of 360 MPa, and normal endurance stress of 300 MPa. Taking the factor of safety as 2.1, find a suitable diameter of the connecting rod.

5. A round rod of diameter 1.2d is reduced to a diameter d with a fillet radius of 0.1d. This stepped rod is to sustain a twisting a twisting moment that fluctuates between +2.5 kN-m and +1.5 kN-m together with a bending moment that fluctuates between +1kN-m to – 1kN-m. The rod is made of carbon steel (σ_y=330 MPa and σ_u=620 MPa) Determine the diameter 'd' of the rod. Take the load factor =1 for bending 0.6 for torsion. Size and surface finish factor= 0.85. Factor of safety=2.