

# B.M.S. College of Engineering, Bengaluru-560019

Autonomous Institute Affiliated to VTU

## January / February 2025 Semester End Main Examinations

Programme: B.E.

Semester: VI

Branch: Mechanical Engineering

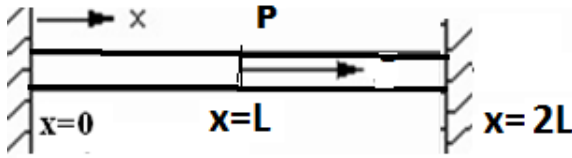
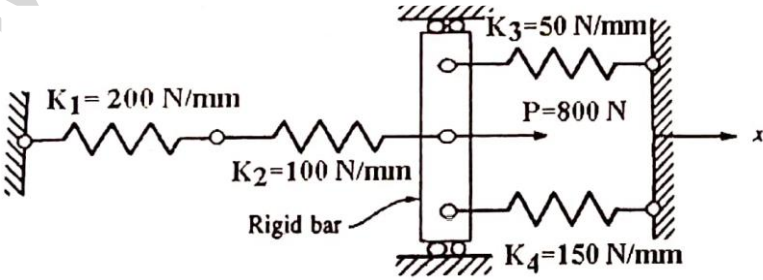
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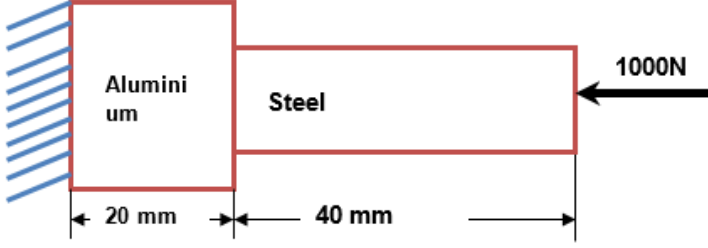
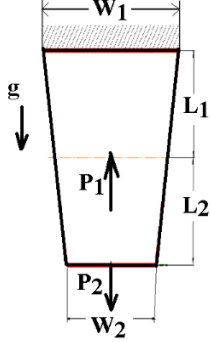
Course Code: 22ME5PCMFEE / 20ME6DCMFEE

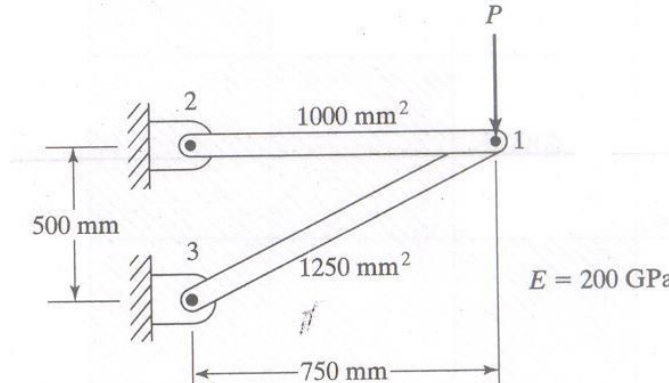
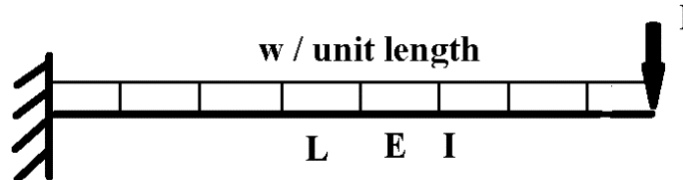
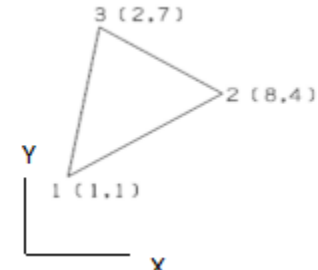
Max Marks: 100

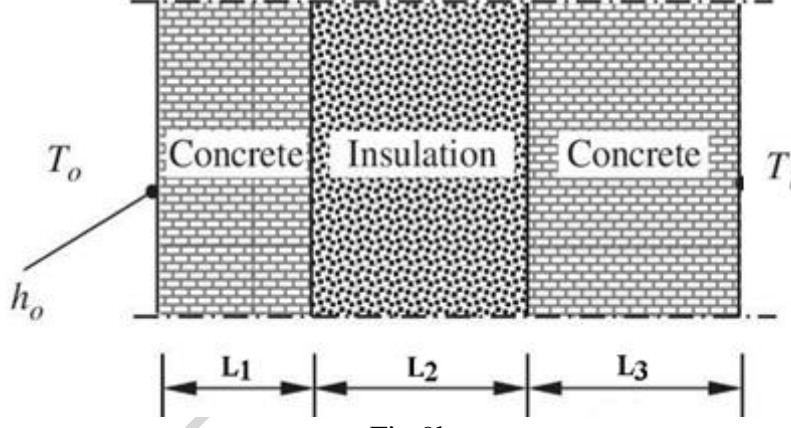
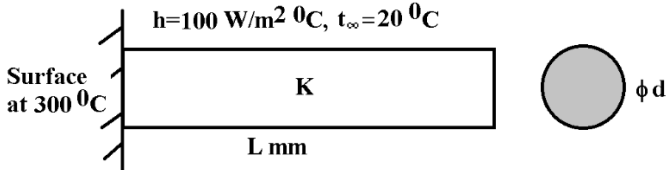
Course: Modelling and Finite Element Analysis

**Instructions:** 1. Answer any FIVE full questions, choosing one full question from each unit.  
2. Missing data, if any, may be suitably assumed.

| Important Note: Completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages. Revealing of identification, appeal to evaluator will be treated as malpractice. |   |    | UNIT - I  | CO  | PO         | Marks |
|--|---|----|---|-----|------------|-------|
|  | 1 | a) | Write the following in the matrix form<br>1. Equilibrium equations in 3D<br>2. Strain-displacement relations in 3D<br>3. Stress-strain relations in 3D<br>4. Stress-strain relations for Plane stress condition   | CO1 | PO1<br>PO2 | 08    |
|  |   | b) | Using the Rayleigh-Ritz method obtain expressions for displacement and stress for the uniform bar shown in the fig 1. What is the maximum displacement if $P = A = L = E = 1$ ?<br><br>Fig. 1 | CO1 | PO1<br>PO2 | 12    |
|  |   |    | OR  |     |            |       |
|  | 2 | a) | Determine the nodal displacements for the spring system shown in fig 2a using principle of minimum potential energy.<br><br>Fig 2a  | CO1 | PO1<br>PO2 | 10    |
|  |   | b) | Evaluate the following integral with suitable Gauss quadrature. Verify the answer with analytical solution.<br>$I = \int_0^3 (1 + 2r + 3r^2 + 4r^3) dr$   | CO1 | PO1<br>PO2 | 06    |

|   |    |   |            |            |    |
|---|----|---|------------|------------|----|
|   | c) | With suitable examples, differentiate essential and non-essential boundary conditions.  | CO1        | PO1<br>PO2 | 04 |
|   |    | <b>UNIT - II</b>  |            |            |    |
| 3 | a) | Formulate the element stiffness matrix for a 2-noded bar element with 1-dof at each node.   | CO2        | PO1<br>PO3 | 08 |
|   | b) | <p>Determine displacement field, support reactions and stresses for the stepped bar shown in fig 3b. <math>A_{Al} = 40 \text{ mm}^2</math>, <math>E_{Al} = 70 \times 10^3 \text{ N/mm}^2</math>, <math>A_{St} = 20 \text{ mm}^2</math>, <math>E_{St} = 200 \times 10^3 \text{ N/mm}^2</math>.</p>  <p style="text-align: center;">Figure 3b</p>   | CO2        | PO1<br>PO3 | 12 |
|   |    | <b>OR</b>   |            |            |    |
| 4 | a) | Derive shape functions for 2-noded bar element in natural coordinate system.  | CO2        | PO1<br>PO3 | 06 |
|   | b) | <p>For the problem shown in figure 4b, determine the nodal displacements, element stresses and reactions. Thickness of the plate = 5 mm, <math>W_1 = 50 \text{ mm}</math>, <math>W_2 = 30 \text{ mm}</math>. Density = <math>8000 \text{ kg/m}^3</math> and <math>E = 100 \text{ GPa}</math>. Consider <math>L_1 = 1.5 \text{ m}</math>, <math>L_2 = 1.5 \text{ m}</math>, <math>P_1 = 70 \text{ kN}</math>, and <math>P_2 = 70 \text{ kN}</math>.</p>  <p style="text-align: center;">Figure 4b</p> | CO4<br>CO6 | PO1<br>PO2 | 14 |
|   |    | <b>UNIT - III</b>   |            |            |    |
| 5 | a) | Derive Hermitian shape functions for a 2-noded beam element and sketch their variation.   | CO<br>CO3  | PO1<br>PO2 | 08 |
|   | b) | For the pin-jointed configuration shown in figure 5b, determine the element stiffness matrix, displacement vector, and stresses in the elements if $P = 10 \text{ kN}$ .  | CO<br>CO3  | PO1<br>PO2 | 12 |

|   |    |   |            |            |           |
|---|----|---|------------|------------|-----------|
|   |    |  <p>Fig 5b</p>  |            |            |           |
|   |    | <b>OR</b>   |            |            |           |
| 6 | a) | Derive the element stiffness matrix for 2-noded plane truss element.  | CO2        | PO1        | <b>06</b> |
|   | b) | For the beam loaded as shown in fig 6b, determine the nodal unknowns and support reactions. $E=200 \text{ GPa}$ , $I=10 \times 10^{-4} \text{ m}^4$ . Take $L=3\text{m}$ , $w=10\text{N/m}$ , $P=50\text{kN}$ .                         | CO2<br>CO3 | PO2        | <b>14</b> |
|   |    |  <p>Fig. 6b</p>  |            |            |           |
|   |    | <b>UNIT - IV</b>  |            |            |           |
| 7 | a) | Sketch 2D constant strain element indicating the degrees of freedom. Obtain expressions for<br>i) Shape functions,<br>ii) Jacobian, and<br>iii) Strain-displacement matrix  | CO2<br>CO3 | PO2        | <b>12</b> |
|   | b) | For the triangular element shown in fig 7b, determine the strains $\epsilon_x$ , $\epsilon_y$ and $\gamma_{xy}$ given displacements $q_1 = 0.001$ , $q_2 = -0.004$ , $q_3 = 0.003$ , $q_4 = 0.002$ , $q_5 = -0.002$ and $q_6 = 0.005$ . | CO2<br>CO3 | PO2        | <b>08</b> |
|   |    |  <p>Fig.7b</p>   |            |            |           |
|   |    | <b>OR</b>   |            |            |           |
| 8 | a) | Formulate shape functions for 3-noded quadratic bar element,  | CO4        | PO1<br>PO2 | <b>06</b> |

|  |    |    |   |     |            |    |
|--|----|----|---|-----|------------|----|
|  |    |    |   |     |            |    |
|  |    | b) | Sketch 9-noded quadrilateral element in local coordinates. Identify nodal coordinates and write the shape functions for the same.   | CO4 | PO1        | 08 |
|  |    | c) | With suitable illustrations, discuss iso, sub and super-parametric elements.  | CO4 | PO1        | 06 |
|  |    |    | <b>UNIT - V</b>   |     |            |    |
|  | 9  | a) | Derive shape functions for a 2-noded one-dimensional heat transfer element in global coordinates.   | CO4 | PO1<br>PO2 | 06 |
|  |    | b) | <p>Consider a wall built up of concrete and thermal insulation. The outdoor temperature is <math>T_o = -15^\circ\text{C}</math>, and the temperature inside is <math>T_i = 30^\circ\text{C}</math>. The wall is subdivided into three layers as shown in fig 9b. The thermal conductivity for concrete is <math>K_c = 2 \text{ W/m}^\circ\text{C}</math> and that of the insulator is <math>K_i = 0.05 \text{ W/m}^\circ\text{C}</math>. Convection heat transfer is occurring at outer surface with convection coefficient of <math>h_o = 24 \text{ W/m}^2 \text{ }^\circ\text{C}</math>. Obtain the temperature distribution in the wall. Take <math>L_1 = 5 \text{ cm}</math>, <math>L_2 = 25 \text{ cm}</math>, <math>L_3 = 50 \text{ cm}</math>.</p>  <p style="text-align: center;">Fig 9b</p> | CO4 | PO1<br>PO2 | 14 |
|  |    |    | <b>OR</b>   |     |            |    |
|  | 10 | a) | Derive shape functions for 2-noded one-dimensional heat transfer element in natural coordinate system.  | CO4 | PO1<br>PO2 | 06 |
|  |    | b) | <p>Determine the temperature distribution and amount of heat transfer in a rectangular fin as shown in figure 10b. Use two 2-noded 1-D heat transfer elements. Also obtain the temperature at mid-point of the 1<sup>st</sup> element. Assume that the end face of the fin is insulated.</p> <p><math>L = 0.05 \text{ m}</math>, <math>d = 0.01 \text{ m}</math>, <math>K = 50 \text{ W/m}^\circ\text{C}</math></p> <p>Surrounding;<br/><math>h = 100 \text{ W/m}^2 \text{ }^\circ\text{C}</math>, <math>t_\infty = 20^\circ\text{C}</math></p>  <p style="text-align: center;">Fig 10b</p>   | CO4 | PO1<br>PO2 | 14 |

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