UNIT – 1

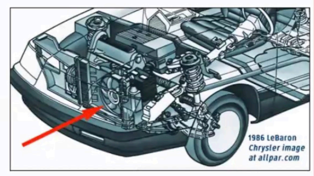
Introduction to Turbomachinery: Definition, Parts of a Turbomachine; Classification; Dimensionless parameters and their significance (No Derivation only Discussion); Specific speed, General Analysis of Turbomachines: Euler equation and its alternate forms-components of energy transfer **(05+1T) Hours**

Examples of Turbomachines.

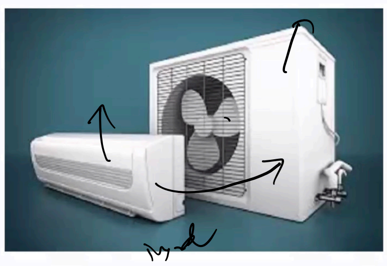
Vacuum cleaner

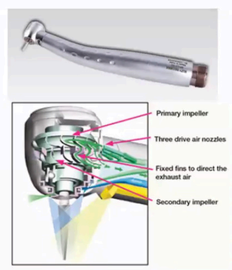
Fan

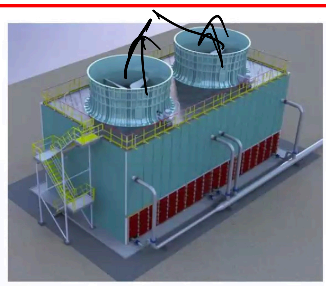
Computer cooling fan



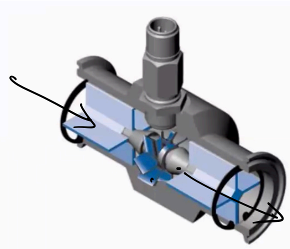
Radiator cooler



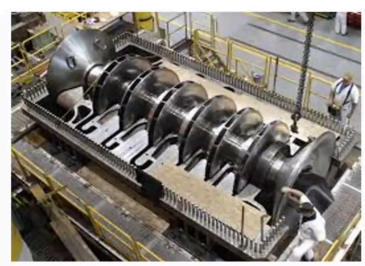




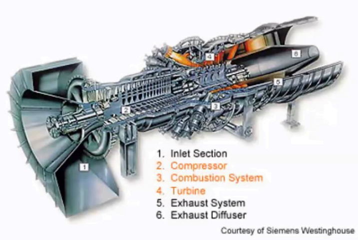
Cooling tower



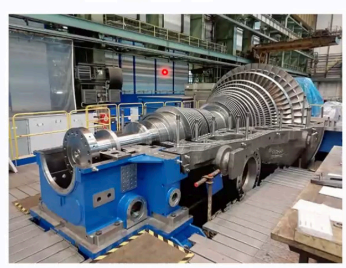
Turbine flow meter



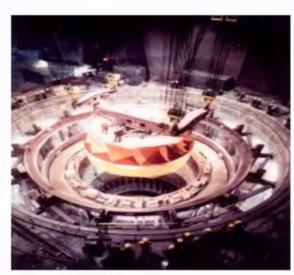
Centrifugal Industrial Compressor



Gas Turbine Power Plant

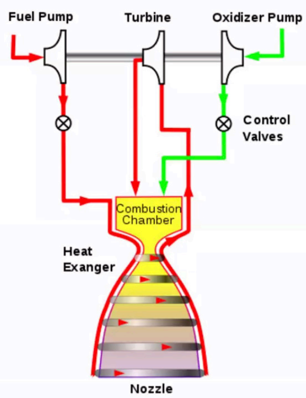


Steam Turbine Power Plant



Hydraulic Turbine

Rocket Engine



Tidal turbine

Wind turbine

Classification of Turbomachines

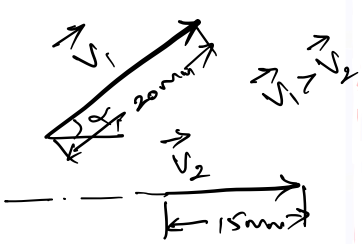
Turbomachines → *Function* / *Flow Direction* / *Flow Physics*

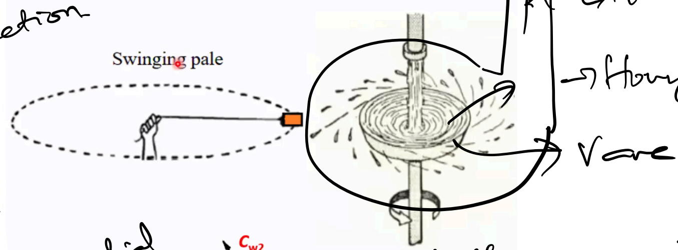
"Turbo" or "turbinis" is of Latin origin

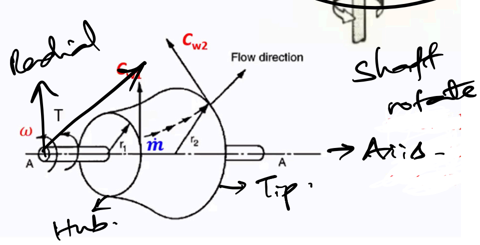
Implies that which spins or whirls around

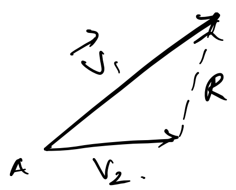
- *Function*
  - Power Absorbing Device → Compressor & Pump (Blower, Fan)
  - Power Generating Device → Turbine
    - Hydraulic
    - Steam
    - Gas.
- *Flow Direction*
  - Axial flow machines
  - Centrifugal / Radial flow machines
  - Mixed flow machines
- *Flow Physics*
  - Compressible flow machines
    - Air/Wind
    - Gas
    - Refrigerants
    - Steam → (Superheated)
  - Incompressible flow machines
    - (Wet steam)
    - Water

Vectors → $\vec{V}$ → magnitude → Direction









Turbomachine: It is a device/machine, in which there is a energy transfer. between, → Continuous.

a) Fluid to rotor → Turbines. → Power producing

b) Rotor to fluid → Fan, Blower, Compressor, Pump → Power absorbing

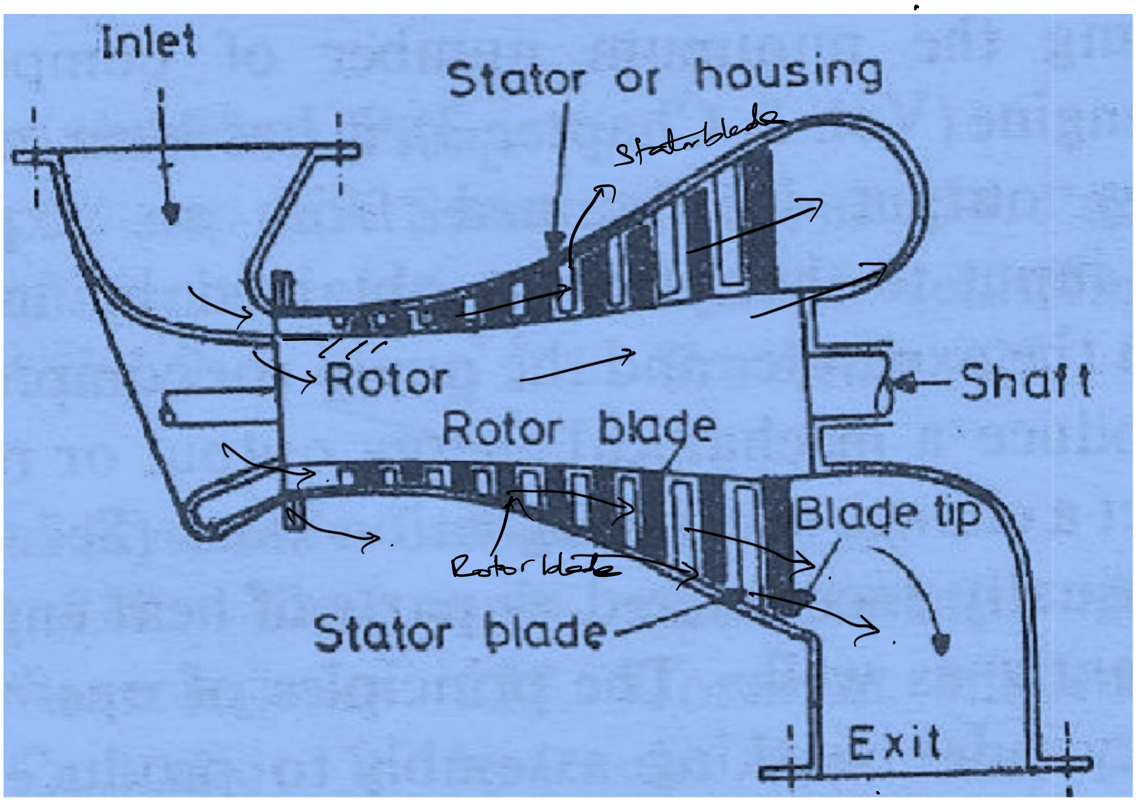
a) O/p → Power Kw. → Rotor will have blades/Vanes.

b) O/p → Pressure head/rise. → —— " ——

Definition of a Turbomachine:

Turbomachine is a device in which there is continuous energy transfer between the fluid to rotor OR rotor to fluid which leads to generation of power OR. pressure rise.

Parts of a Turbomachine.



1) Rotor → Cylindrical / Conical. → Alloy.

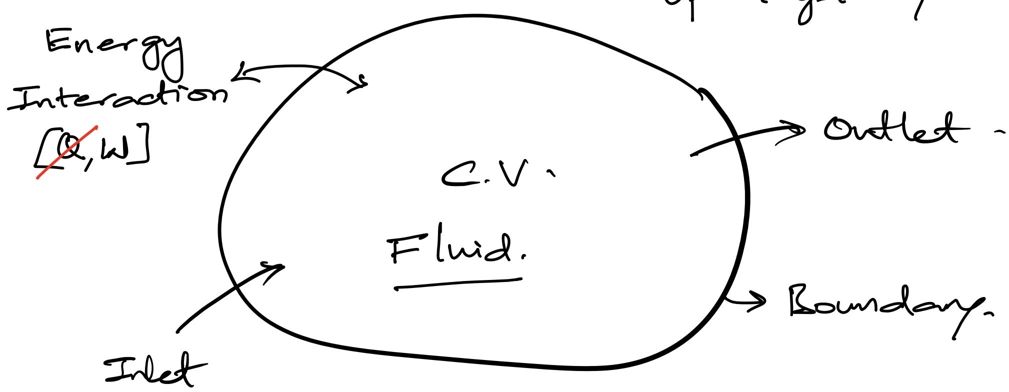
2) Blades
- Rotor blades
- Stator blades

} Aerodynamic design.

3) Casing/Housing → Converts the kinetic energy to pressure head.

Apply I-law of thermodynamics to Turbomachines.

Open system / Control Volume.



Energies	Flow energy	Head.

1) Pressure Energy. $(P.v) \Rightarrow \frac{P}{\rho}$.

2) Kinetic Energy $\Rightarrow \frac{V^2}{2}$

3) Potential Energy $\Rightarrow Zg$.

4) Enthalpy. $\rightarrow h = f(T)$. → Hot Gases, Steam.

$h = u + pv$.

1) Pressure head $\Rightarrow \rho g h = \gamma h$.

2) Kinetic head $\rightarrow \frac{V^2}{2g}$.

3) Potential head $\rightarrow \frac{Zg}{g} = Z$.

$v = \frac{1}{\rho}$.

Steady flow Energy Equation

$$\cancel{\dot{Q}}^{0} + \dot{m}\left[u_1 + p_1 v_1 + \frac{\vec{V}_1^2}{2} + Z_1\right] = \dot{m}\left[u_2 + p_2 v_2 + \frac{\vec{V}_2^2}{2} + Z_2\right] + \dot{W}$$

$$W = \dot{m}\left[(u_1 - u_2) + (p_1 v_1 - p_2 v_2) + \left(\frac{\vec{V}_1^2 - \vec{V}_2^2}{2}\right) + (Z_1 - Z_2)\right] \longrightarrow (1)$$

If $Z_1 = Z_2$, [Horizontal position]

$$W = \dot{m}\left[(u_1 - u_2) + (p_1 v_1 - p_2 v_2) + \frac{\vec{V}_1^2 - \vec{V}_2^2}{2}\right] \longrightarrow (2)$$

If $h = u + pv$. [Gas turbine, Steam turbine].

$$W = m\left[(h_1 - h_2) + \frac{\vec{V}_1^2 - \vec{V}_2^2}{2}\right] \longrightarrow (3)$$

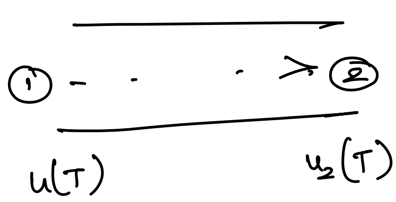
For an incompressible fluid.

$v_1 = \frac{1}{\rho}$ $\rho_1 = \frac{1}{v}$ V :

$v = \frac{V}{\rho}$ $u_1 - u_2$ = negligible.

Density changes are negligible

ΔU & ΔPE are neglected.



$$W = m\left[\cancel{(u_1 - u_2)} + \left(\frac{P_1 - P_2}{\rho}\right) + \frac{V_1^2 - V_2^2}{2} + \cancel{(Z_1 - Z_2)}\right].$$

$$W = \dot{m}\left[(P_1 - P_2) + \rho\frac{(V_1^2 - V_2^2)}{2}\right] \longrightarrow (3)$$

$$W = \dot{m}\left[\underbrace{\left(P_1 + \frac{\rho \vec{V}_1^2}{2}\right)} - \underbrace{\left(P_2 + \frac{\rho \vec{V}_2^2}{2}\right)}\right] \longrightarrow (4)$$

Stagnation/Total pressure $P_0 = p + \frac{\rho \vec{V}^2}{2}$

$$W = \dot{m}\left[P_{0_1} - P_{0_2}\right] \rightarrow (5)$$

$\because$ $P_{0_1} = P_1 + \frac{\rho \vec{V}_1^2}{2}$

$P_{0_2} = P_2 + \frac{\rho V_2^2}{2}$

Application of II law of thermodynamics.

1) Efficiency → $\eta = \frac{O/p}{I/p.} = \frac{W}{Q.}$

2) Entropy. → 'ds' $= \frac{\delta Q}{T}\Big|_{rev.}$ → property

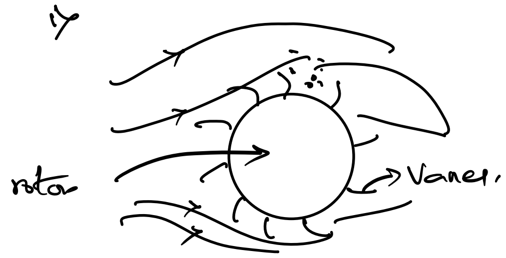
3) Absolute temperature → 'T'

I law → ideal.

II law → $\underbrace{\text{Losses}}$

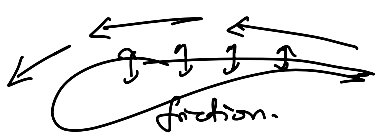
Actual Conditions.

Losses in Turbomachines.

1) 

Eddies, Turbulence, viscosity. $\underbrace{\hspace{8em}}$

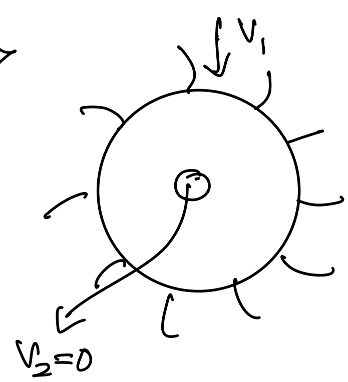
Internal losses. (take away energy)

2) 

Skin friction. → Vane surfaces.

Rotor

Rotor shaft.

3) 

Kinetic Energy Losses.

$$\frac{V_1^2}{2} \longrightarrow \frac{V_2^2}{2} \simeq 0$$

inlet Exit

4) Mechanical friction → Bearing.

5) Leakage $Q_1 \rightarrow Q_2 < Q_1$

inlet Exit

$\Delta Q = \underbrace{Q_1 - Q_2}_{\text{Lost.}}$

Efficiencies — Turbine.

Energy available at the fluid. (1) $\rightarrow$ **Rotor Losses.** $\rightarrow$ Energy available at the rotor. (2) $\rightarrow$ **Frictional losses** $\rightarrow$ Energy at the shaft (3).

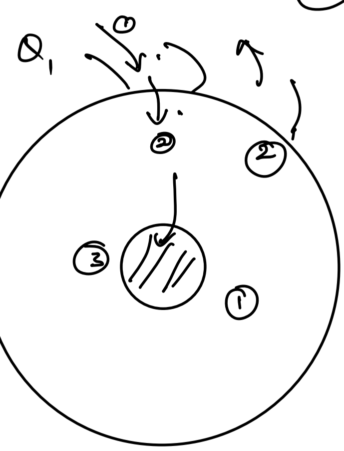
$\Delta \dot{m} = m_1 - m_2$

$m_2 = m_1 - \Delta \dot{m}$

$\Delta Q = Q_1 - Q_2$

$Q_2 = Q_1 - \Delta Q$

ΔQ (Lost)



$\rightarrow Q_2.$

$\Delta Q = Q_1 - Q_2$

1) Hydraulic (or) Blade (Vane) efficiency $\rightarrow$ $\eta_{hyd} = \dfrac{(2)}{(1)}$

2) Mechanical efficiency $\rightarrow$ $\eta_{mech} = \dfrac{(3)}{(2)}$

3) Volumetric efficiency $\rightarrow$ $\eta_{vol} = \dfrac{Q_1 - \Delta Q}{Q_1} = \dfrac{\dot{m} - \Delta \dot{m}}{\dot{m}}$

4) Overall efficiency $\eta_{overall} = \eta_{hyd} * \eta_{mech} * \eta_{vol}.$

Compressor/pump/fan/Blower.

Energy supplied to the shaft. (1) $\rightarrow$ **Frictional losses.** $\rightarrow$ Energy available at the rotor (2) $\rightarrow$ **Rotor Losses.** $\rightarrow$ Energy available @ the fluid. (3).

1) Hydraulic (or) Blade efficiency.

$$\eta_{hyd} = \frac{(3)}{(2)}$$

2) Mechanical efficiency

$$\eta_{mech} = \frac{2}{1}$$

3) Volumetric efficiency.

$$\eta_{vol} = \frac{Q_{exit}}{Q_{entry}} \left[\frac{Q-\Delta Q}{Q} \text{ (OR) } \frac{\dot{m}-\Delta\dot{m}}{\dot{m}} \right]$$

4) Overall efficiency

$$\eta_{overall} = \eta_{hyd} \times \eta_{mech} \times \eta_{vol}$$

Model studies of Turbomachines.

Variables (or) Parameters of Turbomachine analysis

1) Diameter of the machine → 'D' → 'm' → 'L'

2) Discharge through the machine → 'Q' → 'm^3/s' → L^3T^{-1}

3) Speed of the machine → 'N' → rpm → T^{-1}

✓ 4) Head under which the machine is working → 'H' → 'm' → 'L'

5) Density of the fluid → 'ρ' → 'kg/m^3' → ML^{-3}

6) Dynamic viscosity of fluid → 'μ' → $N\text{-}s/m^2$ → $ML^{-1}T^{-1}$

7) Power → 'P' → J/s (or) W → ML^2T^{-3}.

✓ 8) Acceleration due to gravity → 'g' → m/s^2 → LT^{-2}.

9) Young's modulus → 'E' → N/m^2 → $ML^{-1}T^{-2}$.

⊗ 'gH' is combined as a single variable.

$\therefore$ Total number of variables $m = 8$.

Number of non-dimensional units $n = 3$.

$\therefore$ according to Buckingham's π-theorem, the number of π terms. $m - n = 8 - 3 = 5$ π terms.

⊗ $\pi_1, \pi_2, \pi_3, \pi_4$ & $\pi_5 \rightarrow$ as Home work.

Choose the repeating variables. $\rightarrow \rho, D, N$. $\quad u = \frac{\pi D N}{60}$

$\therefore \pi_1 = \rho^{a_1} D^{b_1} N^{c_1} Q \longrightarrow \pi_1 = \frac{Q}{N D^3} \rightarrow$ Discharge coefficient.

$\pi_2 = \rho^{a_2} D^{b_2} N^{c_2} gH \longrightarrow \pi_2 = \frac{gH}{N^2 D^2} \rightarrow$ Head coefficient.

$\pi_3 = \rho^{a_3} D^{b_3} N^{c_3} \mu \longrightarrow \pi_3 = \frac{\mu}{\rho N D^2} = \frac{\mu}{\rho u D} = \frac{1}{Re}$.

$\pi_4 = \rho^{a_4} D^{b_4} N^{c_4} P \longrightarrow \pi_4 = \frac{P}{\rho N^3 D^5} \rightarrow$ Power coefficient.

$\pi_5 = \rho^{a_5} D^{b_5} N^{c_5} E \longrightarrow \pi_5 = \frac{E}{\rho N^2 D^2} = \frac{E/\rho}{N^2 D^2} \rightarrow$ Compressibility factor.

Discussion:

① $\pi_1 = \frac{Q}{N D^3} \rightarrow$ ①

$Q = A\vec{V} \rightarrow \vec{V} = \frac{Q}{A} = \frac{Q}{D^2}$. $\quad A = \frac{\pi D^2}{4}$.

$\pi_1 = \frac{Q/D^2}{ND}$ $\begin{matrix}\rightarrow \text{Fluid (Absolute) velocity} \\ \rightarrow \text{Rotor (or) Blade velocity}\end{matrix}$ $\quad u = \frac{\pi D N}{60}$ $\Big\}$ $\pi_1 = \frac{\vec{V}}{u}$. $\quad \dot{m} = \rho A \vec{V}$

✓ $\vec{V} \propto u \longrightarrow Q \propto N D^3$.

$$Q = A\vec{V} = \frac{\pi D^2}{4}\vec{V} \rightarrow Q \propto \vec{V}D^2$$

$$Q \propto u D^2$$

$$\vec{V} = \sqrt{2gH} \rightarrow \vec{V}^2 = 2gH \rightarrow \vec{V}^2 \propto H$$

$$\left(\frac{Q}{D^2}\right)^2 \propto H$$

$$\left(\frac{ND^3}{D^2}\right)^2 \propto H$$

$$N^2 D^2 \propto H$$

$$\vec{V}^2 \propto H \propto N^2 D^2 \propto u^2$$

② $\pi_2 = \frac{gH}{N^2 D^2} = \frac{gH}{u^2} \rightarrow$ ②. $\pi_1 = \frac{Q}{ND^3}$.

$\therefore \pi_2 = \frac{gH}{\left(\frac{Q/D^2}{\pi_1}\right)^2}$ → Head available $u = \underline{ND} = \frac{Q}{\pi_1 D^2} \rightarrow \frac{Q/D^2}{\pi_1}$

$\left(\frac{Q/D^2}{\pi_1}\right)$ → Kinetic energy.

③ $\pi_4 = \frac{P}{\rho N^3 D^5} = \frac{P}{\rho N^3 D^3 D^2} = \frac{P}{\rho u^3 D^2}$.

→ Power developed/absorbed.

$$\pi_4 = \frac{P}{\rho D^2 \left(\frac{Q/D^2}{\pi_1}\right)^3}$$ → Kinetic energy.

Density.

Unit quantities:

i) Unit speed 'N_u' : It is the speed of the turbine working under unit head of given working fluid.

$\pi_1 = \frac{\vec{V}}{u} \rightarrow u \propto \vec{V}$ (OR) $\vec{V} \propto u$. $\vec{V} = \sqrt{2gH} \rightarrow \vec{V} \propto \sqrt{H}$.

$\vec{V} \propto u \propto \sqrt{H} \rightarrow$ ①

$u = \frac{\pi D N}{60} \rightarrow u \propto N$. $\vec{V} \propto u \propto N \propto \sqrt{H}$.

$N \propto \sqrt{H}$

$N = C_1 \sqrt{H} \rightarrow$ ②

@ $H = 1m$, $N = N_u$ $\therefore$ (2) becomes.

$$N_u = C_1\sqrt{1}$$

$$C_1 = N_u \rightarrow (3)$$

$$\therefore N = N_u\sqrt{H}$$

$$N_u = \frac{N}{\sqrt{H}} \rightarrow (4)$$

(2) Unit discharge 'Q_u': It is the discharge from a turbomachine working under unit head of the working fluid.

$$Q = A\vec{V} \rightarrow Q \propto \vec{V} \propto \sqrt{H} \rightarrow (1)$$

$$Q \propto \sqrt{H}$$

$$Q = C_2\sqrt{H} \rightarrow (2)$$

@ $H = 1m$; $Q = Q_u$ $\therefore$ $Q_u = C_2\sqrt{1}$

$$C_2 = Q_u \rightarrow (3)$$

$$Q = Q_u\sqrt{H}$$

$$Q_u = \frac{Q}{\sqrt{H}} \rightarrow (4)$$

(3) Unit Power 'P_u': It is the power developed by the turbomachine working under unit head of the working fluid.

$P = \gamma Q H$ (OR) $P = wQH$ $\rightarrow$ $P \propto QH \rightarrow (1)$ $\because Q \propto \sqrt{H}$

$$P \propto \sqrt{H}.H$$

$$P \propto H^{3/2}.$$

$$P = C_3 H^{3/2} \rightarrow (2)$$

@ $H = 1m$; $P = P_u$

$$P_u = C_3 1^{3/2}.$$

$$C_3 = P_u \rightarrow (3)$$

$$P = P_u H^{3/2}.$$

$$P_u = \frac{P}{H^{3/2}} \rightarrow (4)$$

$$N_u = \frac{N}{\sqrt{H}}$$

$$Q_u = \frac{Q}{\sqrt{H}}$$

$$P_u = \frac{P}{H^{3/2}}$$

Specific speed of a turbomachine. 'N_s'

Independent of the size 'D'

Turbine: 'N_{sT}'

$$N_{sT} = \frac{\pi_4}{\pi_2} \quad \begin{matrix}\to \text{Power Coefficient} \\ \to \text{Head coefficient.}\end{matrix}$$

$$N_{sT} = \frac{\left(\dfrac{P}{\rho N^3 D^5}\right)^{1/2}}{\left(\dfrac{gH}{N^2 D^2}\right)^{5/4}} \to \text{Eliminate 'D'}$$

$$= \frac{\dfrac{P^{1/2}}{\rho^{1/2} N^{3/2} \cancel{D^{5/2}}}}{\dfrac{g^{5/4} H^{5/4}}{N^{5/2} \cancel{D^{5/2}}}}$$

$$= \frac{\dfrac{P^{1/2}}{\rho^{1/2} N^{3/2}}}{\dfrac{g^{5/4} H^{5/4}}{N^{5/2}}}$$

$$N_{sT} = \frac{N\sqrt{P}}{\sqrt{\rho}\,(gH)^{5/4}} = \frac{\overset{\text{rad/s}}{N}\sqrt{P/\rho}}{(gH)^{5/4}}$$

Nondimensional Specific speed.

Independent of ρ and g.

$$N_{sT} = \frac{\overset{\text{rpm}}{N}\sqrt{P}}{H^{5/4}} \quad \text{rpm.}$$

Dimensional specific speed.

Compressor/pump: 'N_{sp}'

$$N_{sp} = \frac{\pi_1}{\pi_2} \quad \begin{matrix}\to \text{Discharge coefficient} \\ \to \text{Head coefficient.}\end{matrix}$$

$$N_{sp} = \frac{\left(\dfrac{Q}{N D^3}\right)^{1/2}}{\left(\dfrac{gH}{N^2 D^2}\right)^{3/4}} \to \text{Eliminate 'D'}$$

$$= \frac{\dfrac{Q^{1/2}}{N^{1/2} \cancel{D^{3/2}}}}{\dfrac{g^{3/4} H^{3/4}}{N^{3/2} \cancel{D^{3/2}}}}$$

$$= \frac{\dfrac{Q^{1/2}}{N^{1/2}}}{\dfrac{(gH)^{3/4}}{N^{3/2}}}$$

$$N_{sp} = \frac{\overset{\text{rad/s}}{N}\sqrt{Q}}{(gH)^{3/4}} \to \text{Non dimensional specific speed.}$$

Independent of 'g'

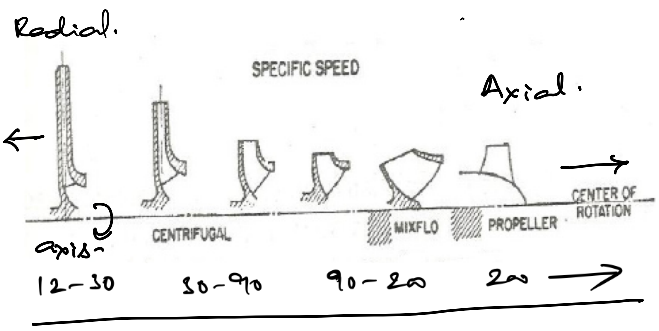
$$N_{sp} = \frac{\overset{\text{rpm}}{N}\sqrt{Q}}{H^{3/4}} \quad \text{rpm.} \to \text{Dimensional specific speed.}$$

Specific speed values for Turbines & Compressor/pump.

SN	Turbo machine (Power absorbing)	Ω (Nsp Ks) (Non dimensional)	Equation
1	Centrifugal pump (Slow – fast speed)	0.24 – 1.8	$\Omega = \frac{N\sqrt{Q}}{(gH)^{3/4}}$ → where N = radians / sec $Q = m^3/sec$ H = meters $g = m/sec^2$
2	Mixed flow pump	1.8 – 4.0	
3	Axial flow pump, Propeller pump	13.2 – 5.7	
4	Radial flow compressor, blower, etc.	0.4 – 1.4	
5	Axial flow blower, compressor, etc.	1.4 – 20	

SN	Turbo machine (Power generating)	Ω (Nsp) (Non dimensional)	Equation
6	Pelton turbine Single Jet Double Jet Multi Jet	0.02 – 0.19 0.1 – 0.3 0.14 – 3.9	$\Omega = \frac{N\sqrt{P/\rho}}{(gH)^{5/4}}$ Where, N = radians / sec P = watts $\rho = kg/m^3$ $g = m/sec^2$ H = meters
7	Francis turbine Radial flow – slow speed Mixed flow – medium Mixed flow – fast	0.39 – 0.65 0.65 – 1.2 1.2 – 2.3	
8	Propeller turbine – axial	1.6 – 3.6	
9	Kaplan turbine – axial	2.7 – 5.4	
10	Axial flow steam and gas turbines	0.35 – 1.9	

SN	Pumps	$Ns = \frac{N\sqrt{Q}}{H^{3/4}}$ rpm
1	Centrifugal pump Slow – Fast speed	12 – 95
2	Mixed flow pump	95 – 210
3	Axial flow pump Propeller pump	172 -320



SN	Water Turbines (Power generating)	$Ns = \frac{N\sqrt{P}}{H^{5/4}}$ rpm	Equation
1	Pelton turbine Single Jet Double Jet Multi Jet	3 – 30 17 – 50 24 – 70	$Ns = \frac{N\sqrt{P}}{H^{5/4}}$ rpm Where, N in rpm P in kW H in meters
2	Francis turbine Radial flow – slow speed Mixed flow – medium Mixed flow – fast	60 – 102 102 – 188 188 – 368	
3	Propeller turbine axial fixed blades	256 – 578	
4	Kaplan turbine axial adjustable blades	428 – 856	

Static and Stagnation properties:

1. Static state of a fluid is the state described by its various properties without the effect of velocity. These properties are measured by the static probes.

2. When the fluid is moving it has two quantities

   a. Static part → No Kinetic energy.

   b. Dynamic part → Kinetic energy.

Stagnation state of a moving fluid stream is the state attained by the fluid when it is suddenly brought to rest (isentropically) without any heat transfer [or] workdone. During this process the Kinetic energy of the fluid gets converted to internal energy.

$$\left.\begin{aligned} T_0 &= T + \frac{\vec{V}^2}{2C_p} \\ h_0 &= h + \frac{\vec{V}^2}{2} \\ P_0 &= p + \frac{\rho\vec{V}^2}{2} \end{aligned}\right\}$$

Stagnation values.

↓

Static term + Dynamic term.

Example problem on stagnation value calculation

① A stream of gases at a point of entry to a turbine has a static temperature of 1050 K, pressure 600 kpa and velocity 150 m/s. Find the total temperature and total pressure of the gases. Also find the difference b/n static and stagnation enthalpies. Take the value of C_p as 1.005 KJ/kgK, $\gamma = 1.4$, $R = 287$ J/kgK

Sol: Given: $P = 600$ kpa; $T = 1050$ K; $V = 150$ m/s.

Density $\rho = \frac{P}{RT} = \frac{600 \times 10^3}{287 \times 1050} = 1.99$ kg/m³.

Stagnation temperature $T_0 = T + \frac{V^2}{2C_p}$.

$$= 1050 + \frac{150^2}{2 \times 1.005 \times 10^3}$$

$$= 1061.2 K$$

Stagnation pressure $P_0 = P + \frac{\rho V^2}{2}$

$$= 600 \times 10^3 + \frac{1.99 \times 150^2}{2}$$

$$= 622 \text{ Kpa.}$$

Stagnation enthalpy $h_0 = C_p T_0$

$\boxed{\begin{array}{l} h = f(T) \\ u = f(T). \end{array}}$

$= 1.005 \times 1061.2$

$= 1065.4 \ KJ/Kg.$

Static enthalpy $h = C_p T$

$= 1.005 \times 1050$

$= 1054.3 \ KJ/Kg$

Difference in stagnation to static enthalpy.

$h_0 - h = 1065.4 - 1054.3 = 11.1 \ KJ/Kg.$

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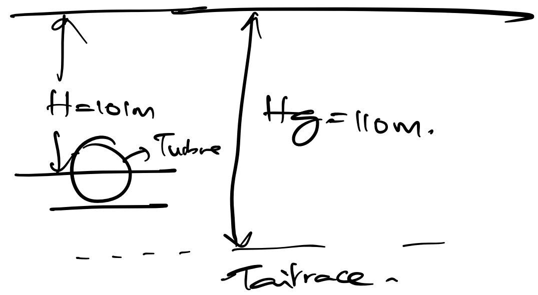
1. A flow rate of $3m^3/s$ of water is available at a height of 110 m at a project site. Due to losses in the supply line, the head available at the inlet to powerhouse is estimated to be only 101 m of water. The leakage losses in the powerhouse are negligible. Mechanical losses account for 150 kW. Frictional losses in the rotor blades may be taken as 250 kW. The exit velocity of water from the turbines is 4.5 m/s. Calculate the hydraulic, mechanical, and overall efficiencies of the plant. The specific weight of water may be taken as $9810N/m^3$.

Sol: $Q = 3m^3/s$; $H_g = 110m$; $H = 101m$; mech loss = 150 kW; Fr. loss = 250 kW.

$V_2 = 4.5 m/s$. To find: η_{hyd}, η_{mech}, $\eta_{overall}$.



Available Power $P_a = w Q H$.

$= 9810 \times 3 \times 101$

$P_a = 2972.4 \text{ KW}$.

$$\left.\begin{aligned} P_{net} &= P_a - \text{Hydraulic losses.} \\ P_{rotor} &= P_{net} - \text{Exit losses.} \end{aligned}\right\}$$

$P_{rotor} = P_a - (\text{Hydraulic losses} + \text{Exit losses})$.

$$= 2972.4 - \left(250 + \frac{V_2^2}{2g} wQ\right)$$

$$= 2972.4 - \left(250 + \frac{4.5^2}{2 \times 9.81} \times \frac{9810 \times 3}{1000}\right)$$

$P_{rotor} = 2692.02 \text{ KW}$.

$P_{shaft} = P_{rotor} - \text{mech losses} = 2692.02 - 150 = 2542.02 \text{ KW}$.

i) Hydraulic efficiency $\eta_{hyd} = \dfrac{P_{rotor}}{P_a} = \dfrac{2692.02}{2972.4}$

$\eta_{hyd} = 90.5\%$.

ii) Mechanical efficiency $\eta_{mech} = \dfrac{P_{shaft}}{P_{rotor}} = \dfrac{2542.02}{2692.02}$

$\eta_{mech} = 94.42\%$.

iii) Overall efficiency $\eta_{overall} = \frac{P_{shaft}}{P_a} = \frac{254202}{2972.4}$

$\eta_{overall} = 85.5$

[(OR) $\eta_{over} = \eta_{mech} * \eta_{hyd}$]

2. A reservoir has a head of 40 m and the channel leaving from the reservoirs permits a flow rate of $34m^3/s$. If the rotational speed of the rotor is 150 rpm. Determine the dimensionless specific speed of the turbine.

Sol: $H = 40m$; $Q = 34\ m^3/s$; $N = 150$ rpm; To find N_{sT} =

$P = wQH$ $\qquad w = 9810\ N/m^3$.

$= 9810 * 34 * 40$

$P = 13.34 \times 10^3$ KW. → in KW.

$N_{sT} = \frac{N\sqrt{P/\rho}}{(gH)^{5/4}}$ $\qquad N = \omega = \frac{2\pi N}{60}$ (N → rpm.)

$= \frac{2 \times \pi \times 150}{60}$

$= 15.7$ rad/s.

$= \frac{15.7 * \sqrt{\frac{13.34 \times 10^3}{1000}}}{(9.81 * 40)^{5/4}}$

$N_{sT} = 0.0328$ (unit less).

[For non-dimensional specific speed,

N → rad/s.

P → KW.]

8. A small scale model of a hydraulic turbine runs at 360 rpm under a head of 22 m and produces 10 kW output.Determine its: i) Unit discharge ii) Unit speed iii) Unit power. Take total to total efficiency as 0.8

Sol: $N = 360$ rpm; $H = 22$ m; $P = 10$ KW; $\eta = 0.8$. $w = 9810\ N/m^3$

$$\eta = \frac{P}{wQH} \rightarrow Q = \frac{P}{\eta w H}$$

$$Q = \frac{10 \times 10^3}{0.8 * 9810 * 22}$$

$$Q = 0.0579\ m^3/s.$$

i) Unit discharge $Q_u = \frac{Q}{\sqrt{H}}$

$$= \frac{0.0579}{\sqrt{22}}$$

$$Q_u = 0.0124\ m^3/s.$$

ii) Unit speed, $N_u = \frac{N}{\sqrt{H}}$

$$= \frac{360}{\sqrt{22}}$$

$$N_u = 76.75\ \text{rpm}.$$

iii) Unit Power, $P_u = \frac{P}{H^{3/2}}$

$$= \frac{10 \times 10^3}{22^{3/2}}$$

$$P_u = 96.91\ W.$$

3. A small turbine runs at 600 rpm using water at $0.05 m^3/s$ at a head of 12 m with an overall efficiency of 87%. The turbine is accepted as a model for developing a prototype to be used in a powerhouse with an available head of 60 m and a flow rate of $6.6 m^3/s$. The speed selected for the prototype is 300 rpm. Calculate number of turbines required for the powerhouse. Assume same efficiency for the prototype.

Sol: Model: $\rightarrow N_m = 600 rpm;\ Q_m = 0.05\ m^3/s;\ H_m = 12 m.$

Prototype: $\rightarrow N_p = 300 rpm;\ Q_p = 6.6 m^3/s;\ H_p = 60 m$ $\Big\}\ \eta = 87\%.$

To find $\eta_{turbines}$. {Head coefficient & Power coefficient (X)}

Head coefficient $\pi_2 = \frac{gH}{N^2D^2}$.

$$\left(\frac{gH}{N^2D^2}\right)_m = \left(\frac{gH}{N^2D^2}\right)_p$$

$$\frac{12}{600^2 \times D_m^2} = \frac{60}{300^2 \times D_p^2}$$

$$\frac{D_p}{D_m} = 4.47 \rightarrow (1)$$

Power Coefficient $\pi_4 = \frac{P}{\rho N^3 D^5}$

$\left\{ \eta = \frac{P}{P_a} = \frac{P}{wQH} \rightarrow P = \eta \cdot wQH. \right\}$

$P_m = \eta \cdot w Q_m H_m.$

$= 0.87 \times 9810 \times 0.05 \times 12$

$P_m = 5.1$ kW.

$P_a = \eta w Q_p H_p.$

$= 0.87 \times 9810 \times 6.6 \times 60$

$P_a = 3379.74$ kW.

$$\left(\frac{P}{\rho N^3 D^5}\right)_m = \left(\frac{P}{\rho N^3 D^5}\right)_p.$$

$$\frac{5.1}{600^3 \times D_m^5} = \frac{P_p}{300^3 \times (4.47 D_m)^5}$$

$P_p = 1137.67$ kW.

$\therefore$ Number of turbines at the site.

$$n_{turb} = \frac{P_a}{P_p} = \frac{3379.74}{1137.67} = 2.97 \simeq 3 \; [\text{units}]$$

4 An axial flow pump with an impeller rotor diameter of 300 mm handles water at a rate of $162m^3/h$ running at 1500 rpm. The energy input is 125 J/kg and Total to Total efficiency is 0.75. If a geometrically similar pump has a diameter of 200 , mm running at 3000 rpm, Find its i) Flow rate ii) change in total pressure iii) input power.

Sol: Pump 1: $D_1 = 0.3m$; $Q_1 = 162\, m^3/h$; $N_1 = 1500$ rpm; $\eta = 0.75$; $P_1 = 125\, J/kg$.

Pump 2: $D_2 = 0.2m$; $N_2 = 3000$ rpm;

To find $\rightarrow Q_2, P_{o_2}, P_2$

[$\because$ Head coefficient & Discharge coefficient]

Discharge coefficient $\pi_1 = \frac{Q}{ND^3}$.

$$\left(\frac{Q}{ND^3}\right)_1 = \left(\frac{Q}{ND^3}\right)_2.$$

$$\left(\frac{162}{1500 \times 0.3^3}\right)_1 = \left(\frac{Q_2}{3000 \times 0.2^3}\right)_2.$$

$$Q_2 = 96 \; m^3/h.$$

Head coefficient $\pi_2 = \frac{gH}{N^2 D^2}$.

$$\left(\frac{gH}{N^2 D^2}\right)_1 = \left(\frac{gH}{N^2 D^2}\right)_2.$$

[Pressure

$\because P_2 = \eta \rho H_2$. bar

N/m^2]

$$\left(\frac{125}{1500^2 \times 0.3^2}\right)_1 = \left(\frac{H}{3000^2 \times 0.2^2}\right)_2.$$

$H_2 = 222\; J/kg$. [Energy $\frac{N\text{-}m}{kg}$]

$\because$ Power $P = \underset{N/m^3}{w}\, Q \underset{m}{H} = \underset{kg/m^3}{\rho}\, Q \underset{J/kg}{H}$

$$P_2 = \eta \rho Q_2 H_2 = \frac{0.75 \times 1000 \times 96 \times 222}{3600} = 4.45 \text{ kW}.$$

(seconds)

5 The quantity of water avaialble for a hydroelectric power station is $260m^3/s$ and a head of 1.73 m. If the speed of the turbines is to be 50 rpm and the efficiency is 82.5%. Find the number of turbines required. Assume $N_s = 760$.rpm.

Sol: $Q = 260\ m^3/s$; $H = 1.73\ m$; $N = 50$ rpm; $\eta = 0.825$

$$N_{ST} = \frac{N\sqrt{P}}{H^{5/4}} \Rightarrow 760 = \frac{50\sqrt{P}}{1.73^{5/4}}$$

$P \to$ kW.

$$\therefore P = 909.5\ \text{kW/turbine}.$$

Total power available $= \eta\, w Q H$

$$= 0.825 \times 9810 \times 260 \times 1.73$$

$$P_a = 3640\ \text{kW}.$$

$\therefore$ Number of turbines required

$$n_t = \frac{P_a}{P}$$

$$= \frac{3640}{909.5}$$

$$n_t = 4\ \text{turbines}.$$