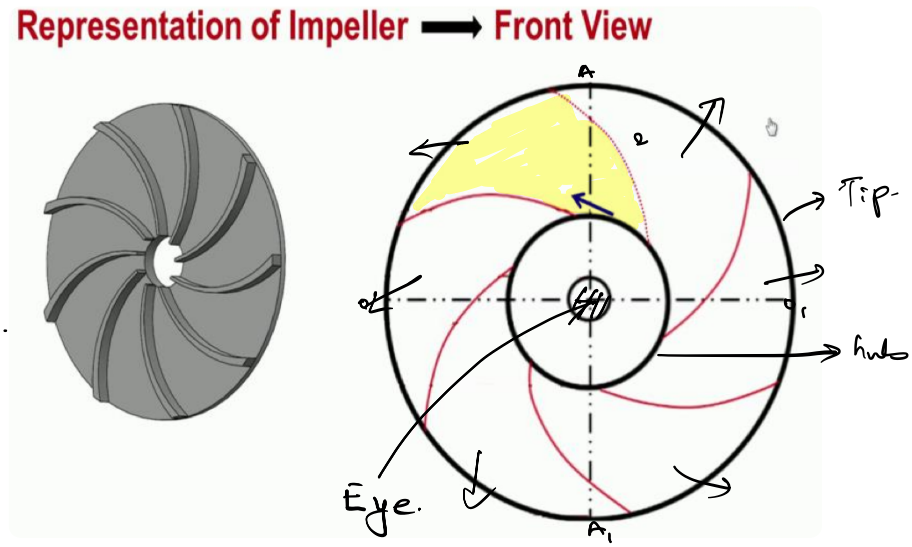
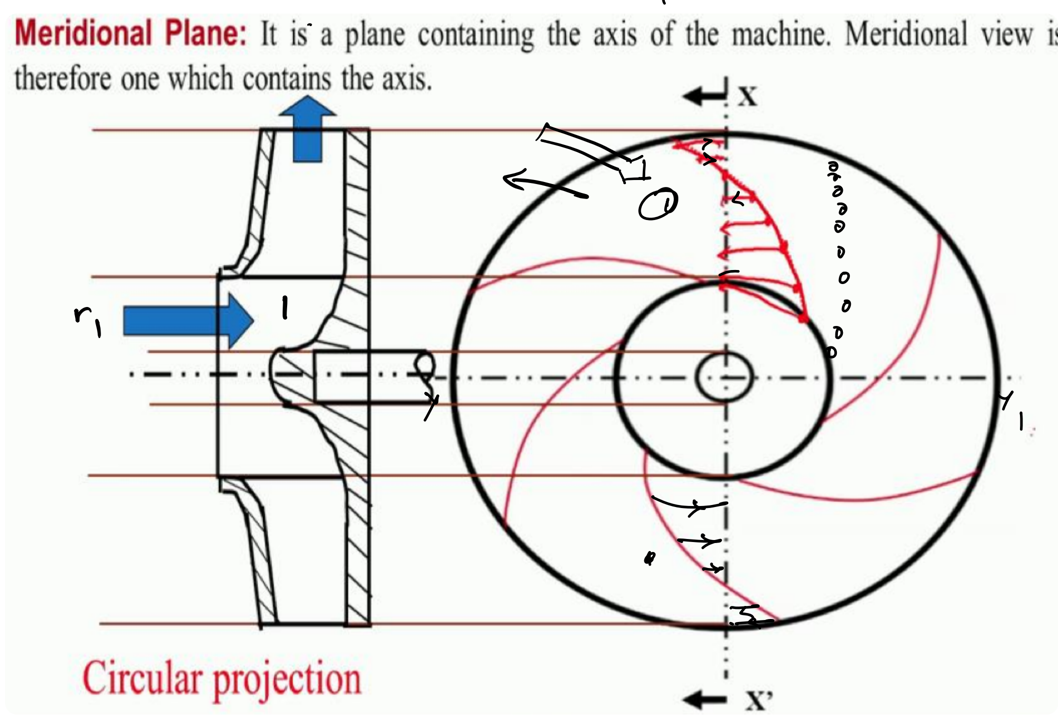
UNIT – 1

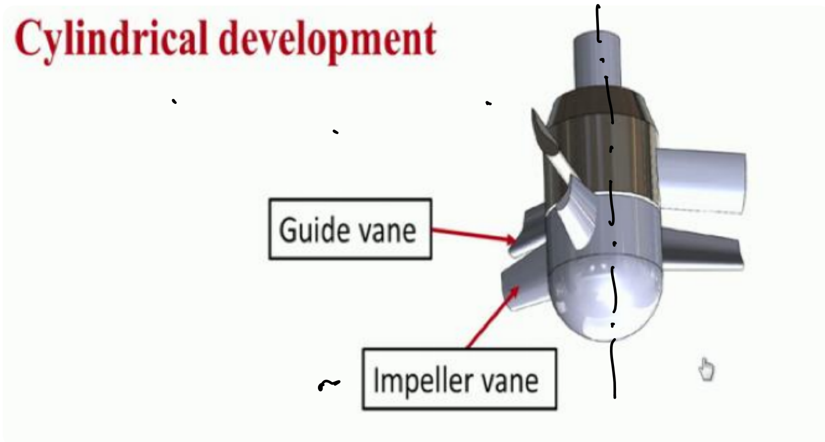
Introduction to Turbomachinery: Definition, Parts of a Turbomachine; Classification; Dimensionless parameters and their significance (No Derivation only Discussion); Specific speed, General Analysis of Turbomachines: Euler equation and its alternate forms-components of energy transfer **(05+1T) Hours**

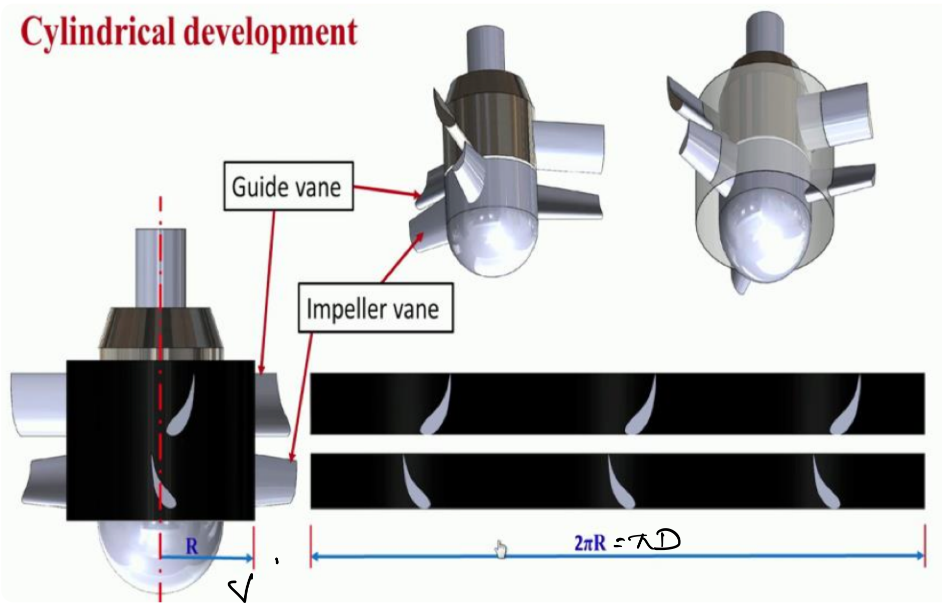


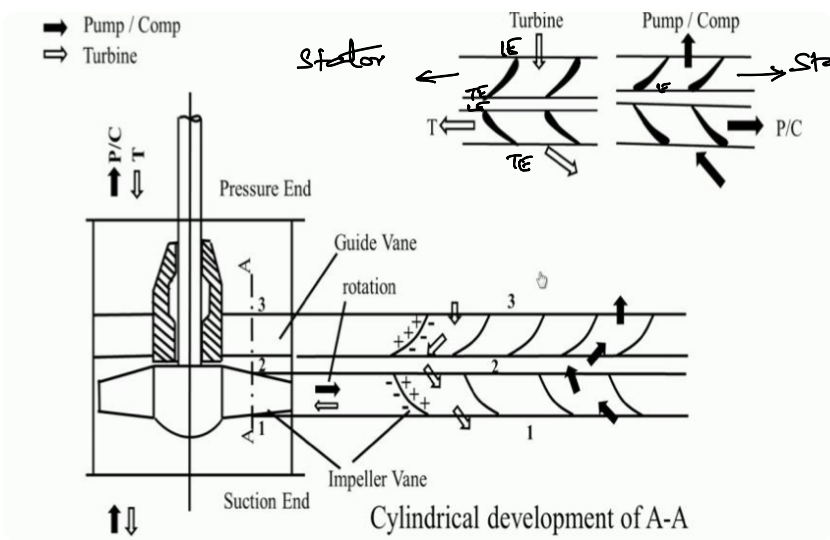


r_1 = hub radius. r_2 = tip radius.

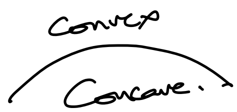
d_1 = hub diameter. d_2 = tip diameter.

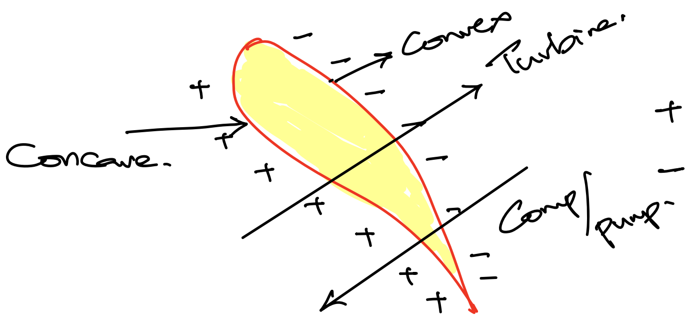

Cylindrical development
Guide vane
Impeller vane


Cylindrical development
Guide vane
Impeller vane
R
2πR = πD


Pump / Comp
Turbine
Turbine
Pump / Comp
Stator
LE
TE
T
TE
Stator
P/C
P/C
T
Pressure End
Guide Vane
rotation
A
3
2
1
A
3
2
1
Impeller Vane
Suction End
Cylindrical development of A-A

Blade profile of a turbomachine.





\+ pressure side.

\- Suction side.

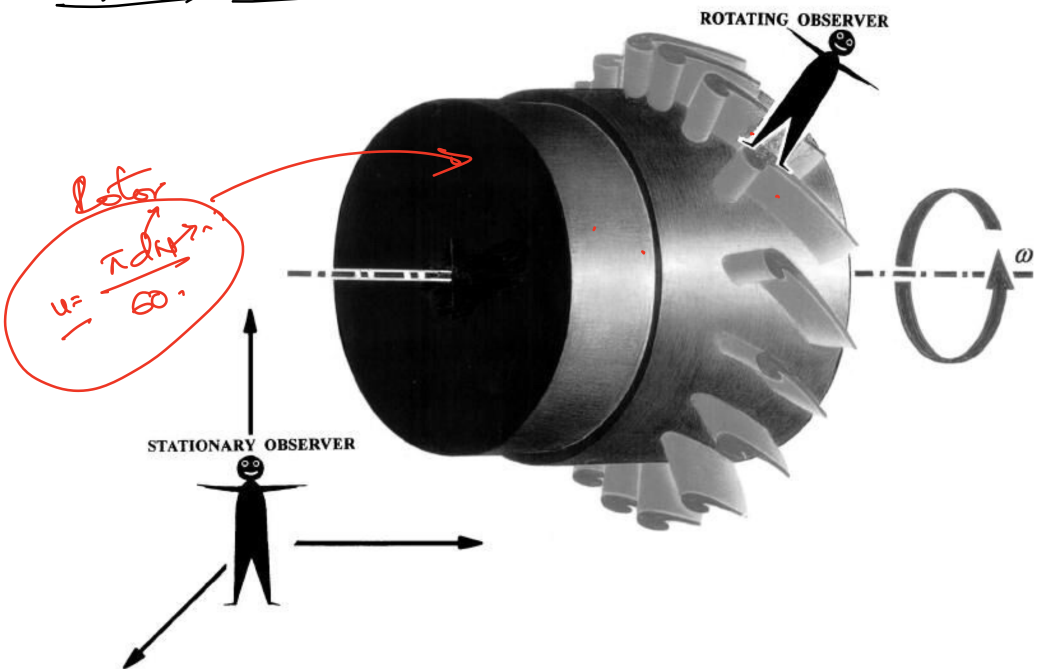
Turbine: Fluid on the pressure side will push the blade. → Expansion

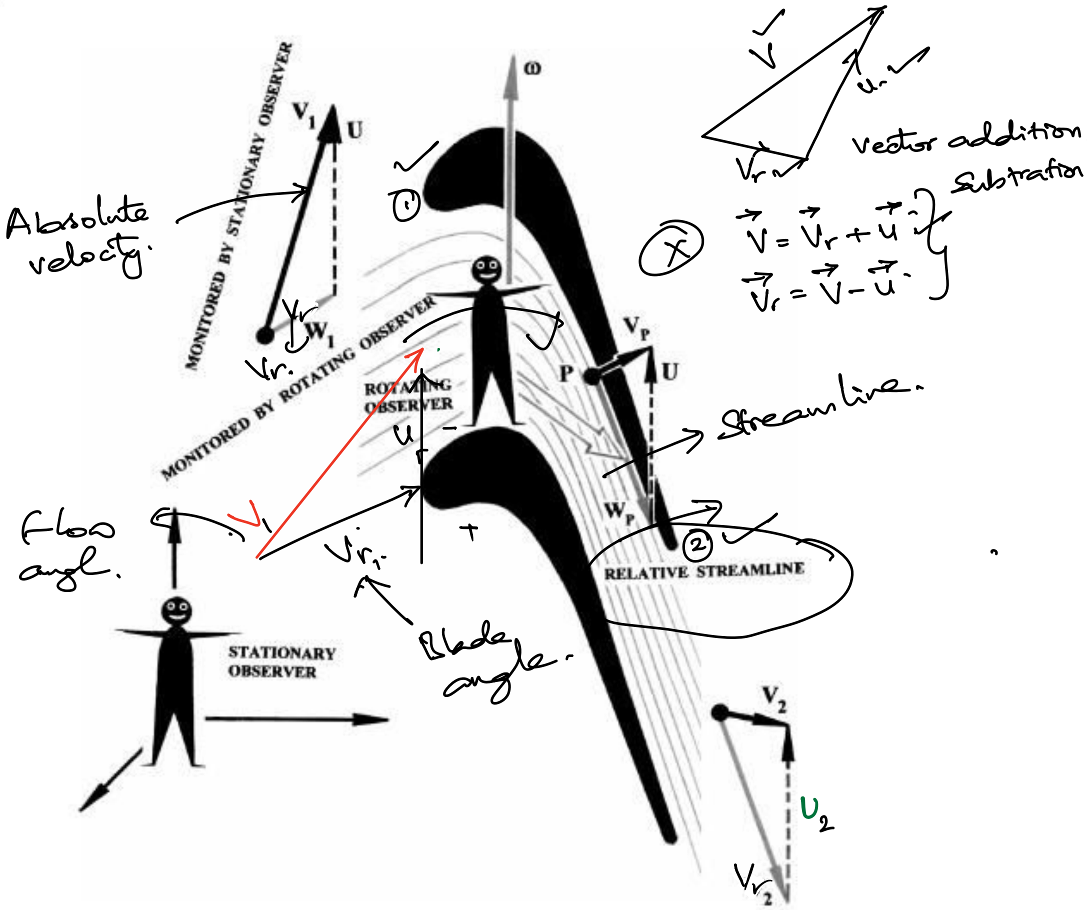
Compressor/Pump: Blades drive the fluid. → Compression

ⓧ **Turbine:** Pressure at the inlet is more than the outlet → Expansion device.

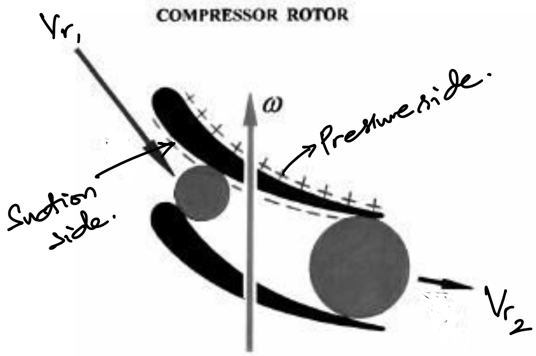
Comp/pump: Pressure at the inlet is less than the outlet → Compressive device.

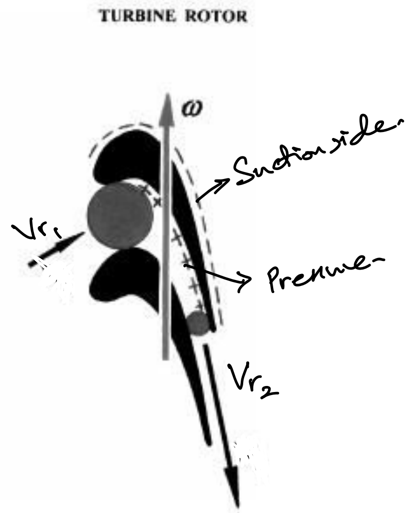
Reference planes.



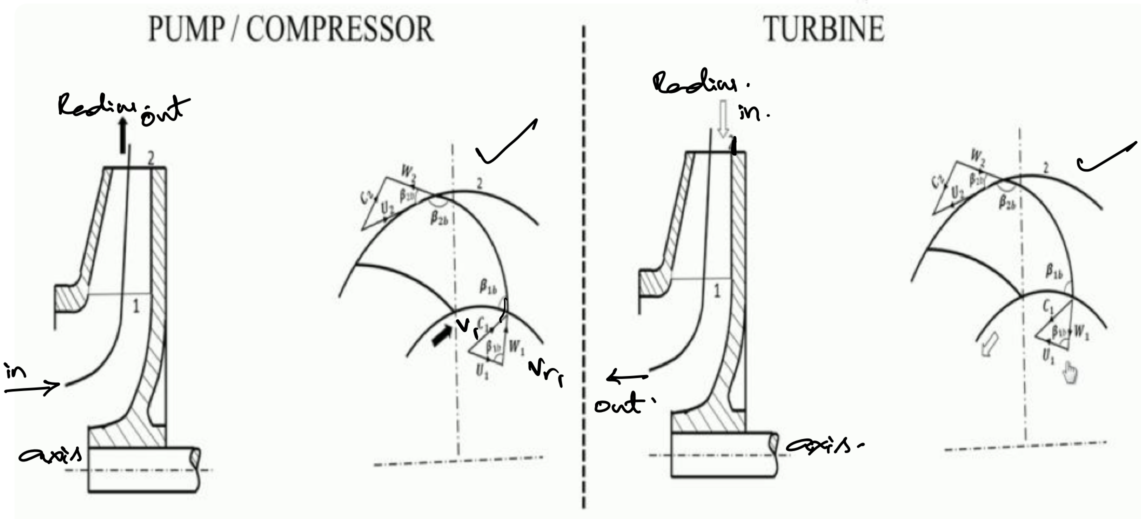


$$\vec{V} = \vec{V}_r + \vec{u}$$
$$\vec{V}_r = \vec{V} - \vec{u}$$

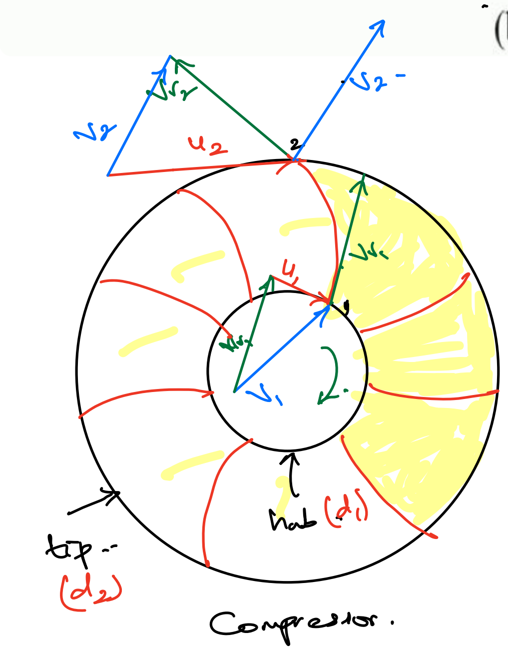


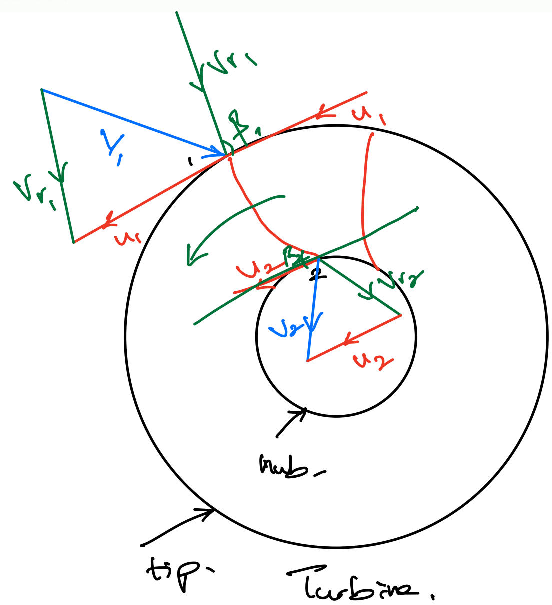


Radial flow machines.

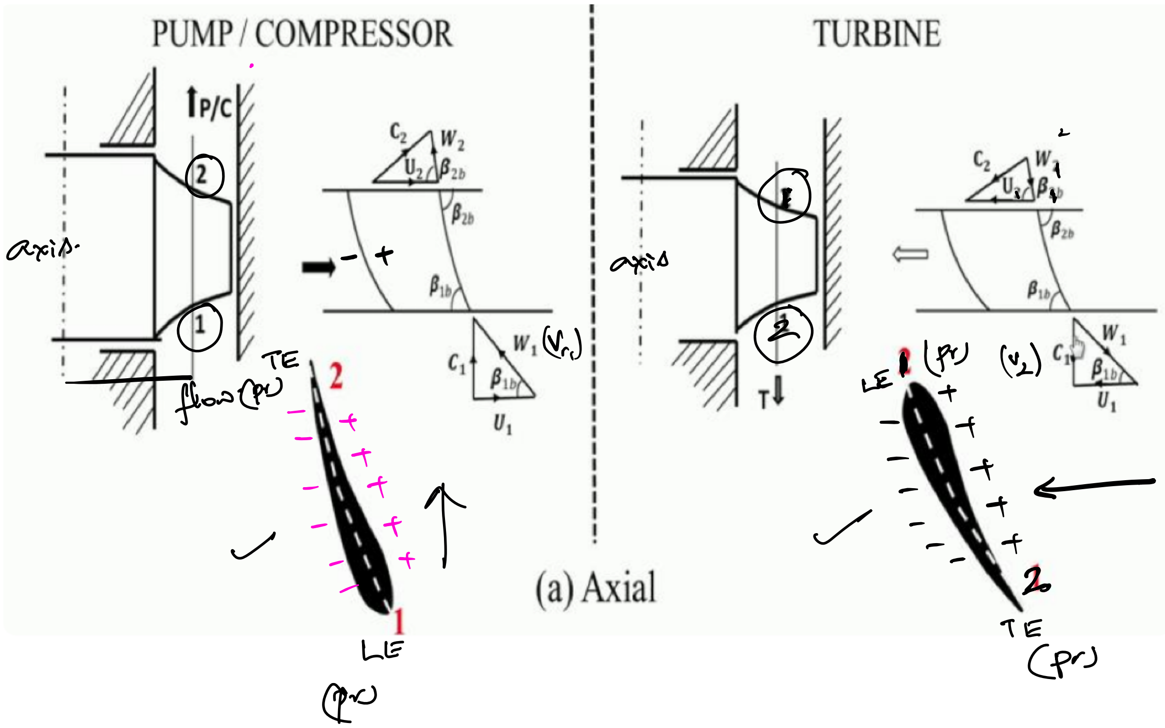


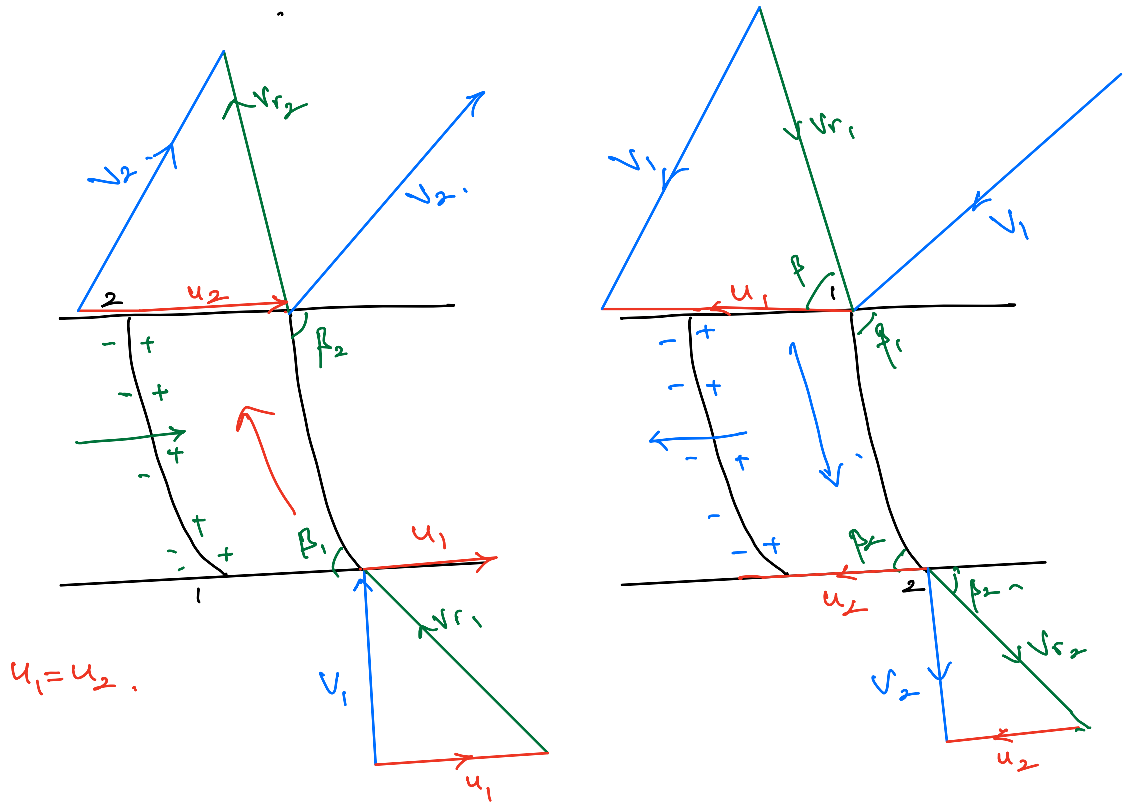
(b) Radial





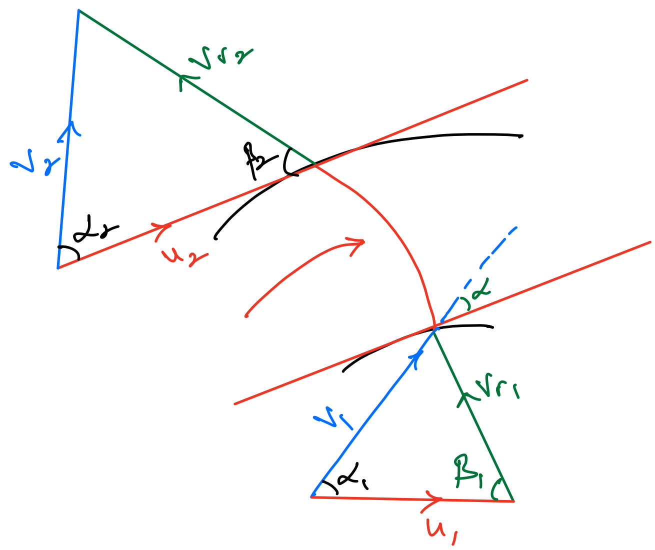
Axial flow machine.





Compressor

Turbine.

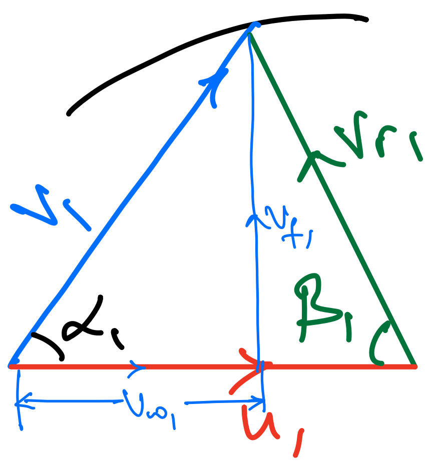


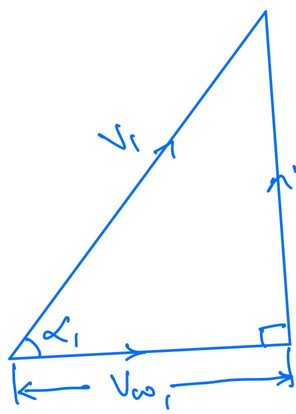
V_1 = Absolute velocity (m/s)
u_1 = Blade velocity (m/s)
V_{r_1} = Relative velocity (m/s)

} Inlet.

V_2 = Absolute velocity (m/s)
u_2 = Blade velocity (m/s)
V_{r_2} = Relative velocity (m/s)

} Outlet.





$V_{f_1} = V_1 \sin\alpha_1$

$\sin\alpha_1 = \dfrac{V_{f_1}}{V_1}$

$\cos\alpha_1 = \dfrac{V_{w_1}}{V_1} \rightarrow V_{w_1} = V_1 \cos\alpha_1$

V_w = Tangential (or) Peripheral (or) Whirl Component

V_f = Flow (or) Meridonial component.

α = Flow angle. (degrees).

β = Blade (or) Vane angle (degrees).

$V_f \rightarrow$ (a) Axial flow machines it will be its axial component.

(b) Radial flow machine it will be its radial Component.

Energy Transfer/ Euler's Energy equation.

1) Control volume includes all the blade passages.

2) Mass flow $\dot{m} = \rho Q = \rho A \vec{V}$.

3) Absolute velocity [V] almost remains uniform

4) For analysis $\rightarrow$ Angular momentum.

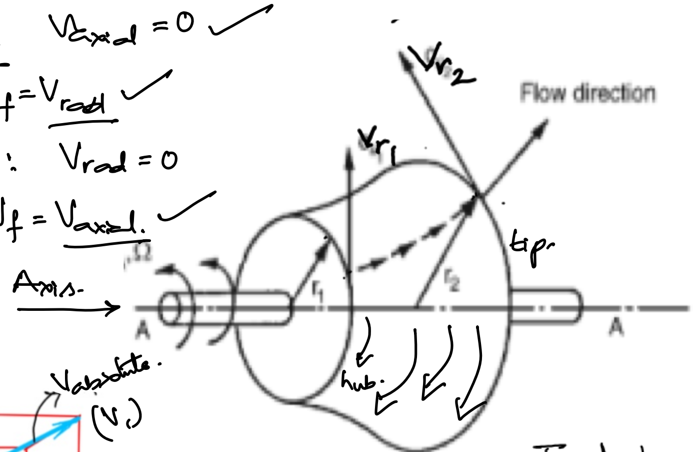
Rate of angular momentum = Applied Torque

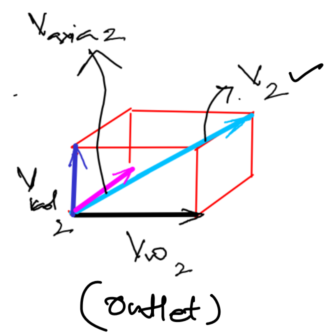
Radial m/c $V_{axial} = 0$

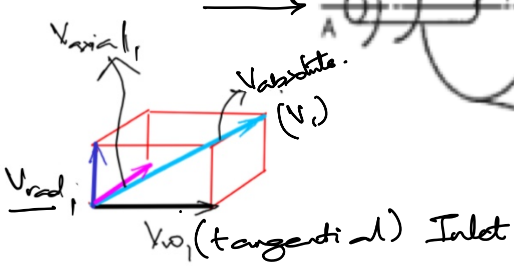
$V_w, u, V_f = V_{rad}$

Axial m/c: $V_{rad} = 0$

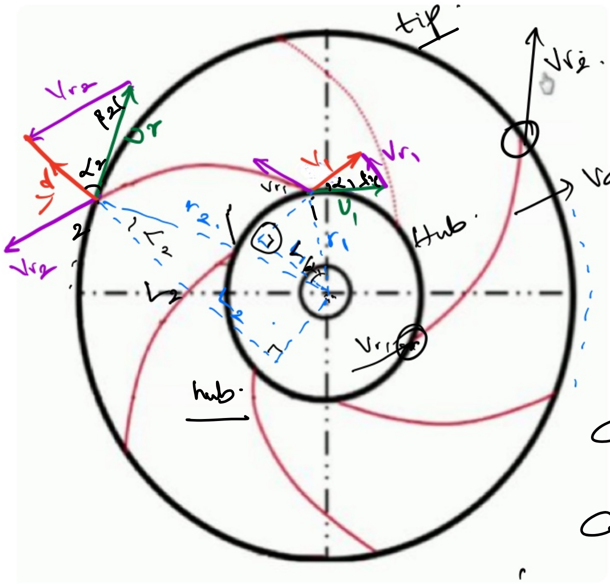
$V_w, u, V_f = V_{axial}$.

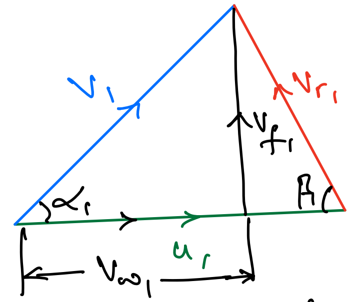






Inlet Vel triangle.





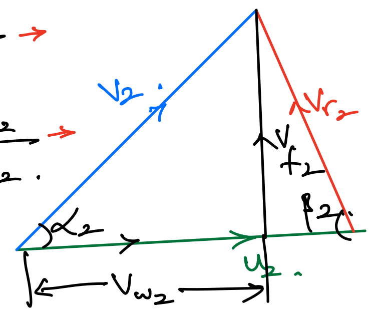
$\sin\alpha_1 = \frac{V_{f_1}}{V_1}$

$\sin\alpha_2 = \frac{V_{f_2}}{V_2}$

Outlet Velocity Δle.

$\cos\alpha_1 = \frac{V_{w_1}}{V_1}$

$\cos\alpha_2 = \frac{V_{w_2}}{V_2}$.



From the newton's II law.

$$F = m.a = m.\frac{d\vec{V}}{dt} = \frac{dm}{dt}.d\vec{V} = \dot{m}\,d\vec{V} \longrightarrow (1)$$

Torque $T = F \times \text{distance.} = F_2L_2 - F_1L_1$

$$T = \dot{m}(\vec{V}_2L_2 - \vec{V}_1L_1) \longrightarrow (2)$$

From the right angled triangle.

$$\left.\begin{aligned} \cos\alpha_1 &= \frac{L_1}{r_1} \rightarrow L_1 = r_1\cos\alpha_1 \\ \cos\alpha_2 &= \frac{L_2}{r_2} \rightarrow L_2 = r_2\cos\alpha_2 \end{aligned}\right\} (3)$$

∴ Substituting for L_1 & L_2 in (2)

$$T = \dot{m}(\vec{V}_2 r_2\cos\alpha_2 - \vec{V}_1 r_1\cos\alpha_1)$$

$$T = \dot{m}(\vec{V}_{w_2}r_2 - \vec{V}_{w_1}r_1) \longrightarrow (4)$$

Workdone per sec [Power]

$\omega = \dfrac{2\pi N}{60}$

$\dot{W} = T\omega$

$u = \omega r$

$\dot{m} = \rho A \vec{V}$

Compressor (Fan)

$$\dot{W} = \dot{m}\left(\vec{V}_{w_2} r_2 - \vec{V}_{w_1} r_1\right)\omega$$

$u_1 = \omega r_1$

$u_2 = \omega r_2$

$$\dot{W} = \dot{m}\left(\vec{V}_{w_2}\omega r_2 - \vec{V}_{w_1}\omega r_1\right)$$

$$\dot{W} = \dot{m}\left(\vec{V}_{w_2}u_2 - \vec{V}_{w_1}u_1\right) \longrightarrow (5)$$

Specific work → (Remain same)

$$W_{sp} = \frac{\dot{W}}{\dot{m}} = \left(\vec{V}_{w_2}u_2 - \vec{V}_{w_1}u_1\right) \longrightarrow (6) \Rightarrow \text{unit mass flow rate}$$

$$W_{sp.}\,(wf) = \frac{\dot{W}}{\dot{m}g} = \frac{\left(\vec{V}_{w_2}u_2 - \vec{V}_{w_1}u_1\right)}{g} \longrightarrow (7) \rightarrow \text{unit weight}$$

(X) Specific work is independent of the density of the working fluid.

Special case:

1) Axial flow machine. $d_1 = d_2 = d$, $r_1 = r_2 = r$; $u_1 = u_2 = u$.

$$W = \underbrace{(V_{w_2} - V_{w_1})}_{\Delta V_w}u$$

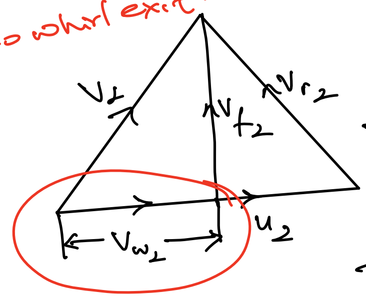
ΔV_w → Difference in whirl velocity.

2) $V_{w_2} = 0$, $\alpha_2 = 90^\circ$

$$W = V_{w_1}u_1 - \underbrace{\cancel{V_{w_2}u_2}}_{\to 0} = V_{w_1}u_1$$

Turbine.

No whirl exit.



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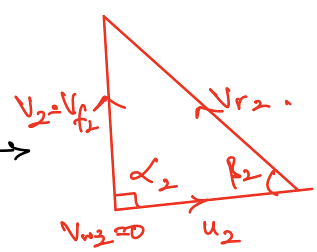
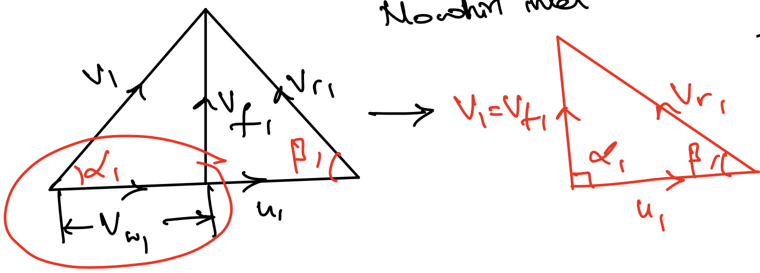


Turbine.

Compressor / Fan. $V_{w_1} = 0$.

No whirl inlet.

$$W = V_{w_2} u_2 - \cancel{V_{w_1}} u_1 \quad (\to 0)$$
$$= V_{w_2} u_2 .$$



Euler equation (Specific Work)

1) Compressor / Fan.

$W = V_{w_2} u_2 - V_{w_1} u_1 \rightarrow$ Radial flow machine $u_2 > u_1$

$W = (V_{w_2} - V_{w_1}) u \rightarrow$ Axial flow machine $u_1 = u_2 = u$.

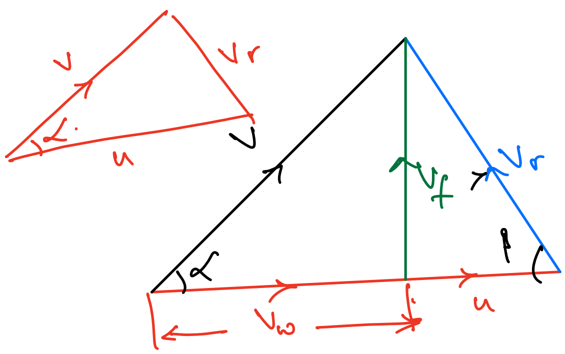
2) Turbine.

$W = V_{w_1} u_1 - V_{w_2} u_2 \rightarrow$ Radial flow machine $u_1 > u_1$

$W = (V_{w_1} - V_{w_2}) u \rightarrow$ Axial flow machine. $u_1 = u_2 = u$.

Alternate form of Euler equation.

$$W = V_{w_2} u_2 - V_{w_1} u_1$$



Applying the cosine rule for the velocity triangle.

$$V_r^2 = V^2 + u^2 - 2Vu \cos\alpha \rightarrow (1)$$

Inlet velocity triangle $\rightarrow V_{r_1}^2 = V_1^2 + u_1^2 - 2V_1 u_1 \cos\alpha_1$

Outlet velocity triangle $\rightarrow V_{r_2}^2 = V_2^2 + u_2^2 - 2V_2 u_2 \cos\alpha_2$ $\Big\}$ (2)

$$V_{r_1}^2 - V_{r_2}^2 = V_1^2 + u_1^2 - 2V_1 u_1 \cos\alpha_1 - (V_2^2 + u_2^2 - 2V_2 u_2 \cos\alpha_2)$$

$$V_{r_1}^2 - V_{r_2}^2 = V_1^2 - V_2^2 + u_1^2 - u_2^2 - 2V_1u_1\cos\alpha_1 + 2V_2u_2\cos\alpha_2$$

$$V_2u_2\cos\alpha_2 - V_1u_1\cos\alpha_1 = \frac{(V_2^2-V_1^2)+(u_2^2-u_1^2)+(V_{r_1}^2-V_{r_2}^2)}{2}$$

$$V_{w_2}u_2 - V_{w_1}u_1 = \frac{(V_2^2-V_1^2)+(u_2^2-u_1^2)+(V_{r_1}^2-V_{r_2}^2)}{2}.$$

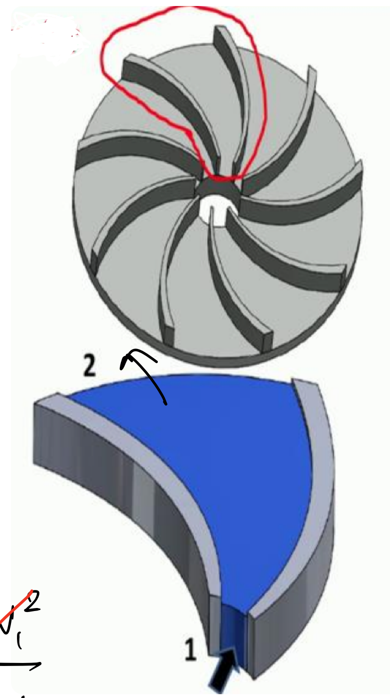
$$\text{Compressor (Fan)} \leftarrow W = \underbrace{\frac{V_2^2-V_1^2}{2}}_{\text{Change in KE due to absolute velocity}} + \underbrace{\frac{u_2^2-u_1^2}{2}}_{\text{Change due to centrifugal effects.}} + \underbrace{\frac{V_{r_1}^2-V_{r_2}^2}{2}}_{\text{Change due relative velocity}}. \rightarrow \text{Pump/Compressor}$$

Leads to change in static head for an incompressible fluid.

— x — x — x — x —

Extra reading to understand static head.

Component of energy transfer.



$$W' = \dot{m}(h_{o_2} - h_{o_1}) = \frac{W}{\dot{m}} = h_{o_2} - h_{o_1}$$

$$\frac{W}{\dot{m}} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} \qquad \because h_o = h + \frac{V^2}{2}$$

In the absence of any loses.

$$W = W_{bl} = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}.$$

$$\cancel{\frac{V_2^2-V_1^2}{2}} + \frac{U_2^2-U_1^2}{2} + \frac{V_{r_1}^2-V_{r_2}^2}{2} = h_2 - h_1 + \cancel{\frac{V_2^2-V_1^2}{2}}.$$

$$\therefore\ h_2 - h_1 = \frac{U_2^2 - U_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2} \Bigg\}$$ Steam turbine / Gas turbine

Also for an isentropic process.

$$dh = \frac{dP}{\rho} \quad \left(\because\ Tds = dh - v\,dP;\ \ Tds = 0\right). \qquad v = \frac{1}{\rho} \rightarrow \rho = \frac{1}{v}.$$

$$Tds = du + p\,dv.$$

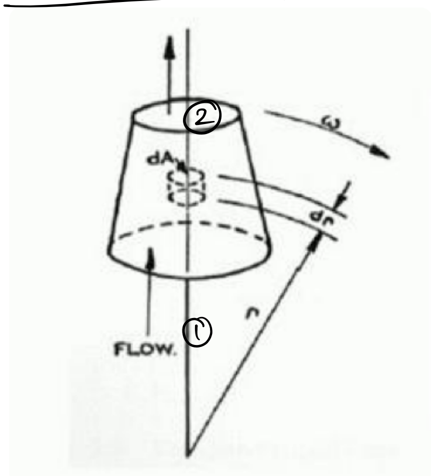
For a turbomachine handling incompressible flow;

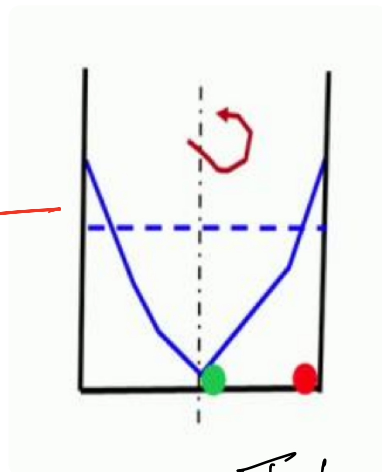
$$\frac{P_2 - P_1}{\rho} = h_2 - h_1 = \underbrace{\frac{U_2^2 - U_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}_{\text{Rise of static pressure inside the impeller}}$$

② ③

Centrifugal component

$A\,dr.\,\omega^2 r.$





Force balance:

$$(p+dp)\,dA - p\,dA = dm\,\omega^2 r = \rho d$$

$$dP\,dA = \rho\,dA\,dr\,\omega^2 r.$$

$$\int_1^2 \frac{dP}{\rho} = \int_1^2 \omega^2 r\,dr = \frac{1}{2}\left(U_2^2 - U_1^2\right)$$

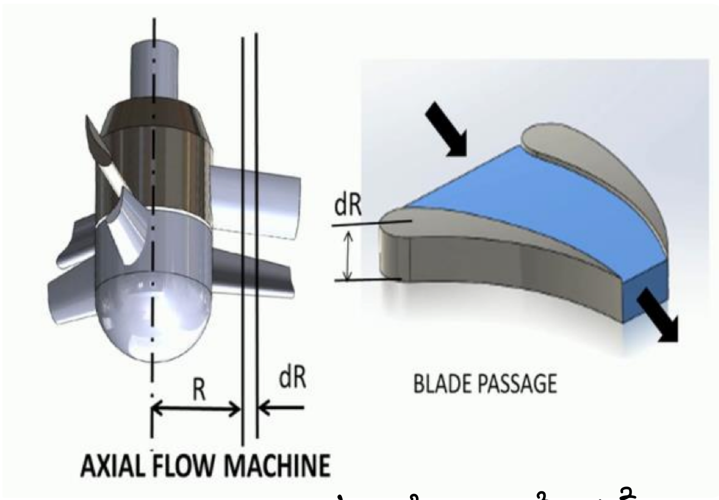
Turbine:

$U_1 > U_2$

Pump/Compressor

$U_2 > U_1$

Axial flow machine.



$$\left.\begin{aligned} u_1 = u_2 = u \\ d_1 = d_2 = d_m \end{aligned}\right\} \quad u = \frac{\pi d_m N}{60}$$

$$\therefore \quad W_{bl} = \underbrace{\frac{V_2^2 - V_1^2}{2}}_{\Delta KE} + \underbrace{\frac{V_{r_1}^2 - V_{r_2}^2}{2}}_{\Delta \text{ relative velocity} \to \text{Static head.}} \qquad \left[\because \cancelto{0}{\frac{u_2^2 - u_1^2}{2}}\right]$$

Euler equation for a turbine.

$W = V_{w_1} u_1 - V_{w_2} u_2 \quad \rightarrow$ Energy/unit mass.

$W = \dfrac{V_{w_1} u_1 - V_{w_2} u_2}{g} \quad \rightarrow$ Energy/unit weight

$W = \dfrac{V_1^2 - V_2^2}{2} + \dfrac{u_1^2 - u_2^2}{2} + \dfrac{V_{r_2}^2 - V_{r_1}^2}{2}. \quad \rightarrow$ Radial flow turbine.

$W = \dfrac{V_1^2 - V_2^2}{2} + \dfrac{V_{r_2}^2 - V_{r_1}^2}{2} \quad \rightarrow$ Axial flow turbine.

Euler equation for a Compressor/Pump.

$W = V_{w_2} u_2 - V_{w_1} u_1$

$W = \dfrac{V_{w_2} u_2 - V_{w_1} u_1}{g}.$

$W = \dfrac{V_2^2 - V_1^2}{2} + \dfrac{V_{r_1}^2 - V_{r_2}^2}{2} \quad \rightarrow$ Axial flow.

$W = \dfrac{V_2^2 - V_1^2}{2} + \dfrac{u_2^2 - u_1^2}{2} + \dfrac{V_{r_1}^2 - V_{r_2}^2}{2}. \quad \rightarrow$ Radial.

Component of Energy transfer:

1) Potential head (Elevation) $\rightarrow (Z_2 - Z_1)$
2) Pressure (or) flow Component $\rightarrow \dfrac{P_2 - P_1}{\rho}$
3) Enthalpy Component $\rightarrow \Delta h = h_2 - h_1$
4) Absolute velocity component $\rightarrow \dfrac{V_2^2 - V_1^2}{2}$
5) Centrifugal velocity component $\rightarrow \dfrac{u_2^2 - u_1^2}{2}$
6) Relative velocity component $\rightarrow \dfrac{V_{r_1}^2 - V_{r_2}^2}{2}$

ⓧ static head

ⓧ Kinetic head.

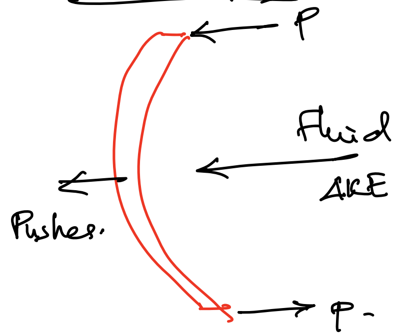
Impulse machine:

Across the rotor/impeller. there is no change in static pressure/head, but only there is change in kinetic energy ΔKE

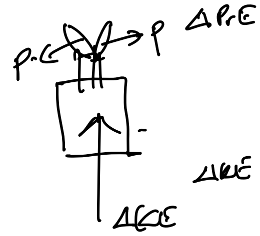
Reaction machine:

Across the impeller/rotor there is a change in static pressure/head. Also there is change in kinetic energy. (Both pr. E & ΔKE)

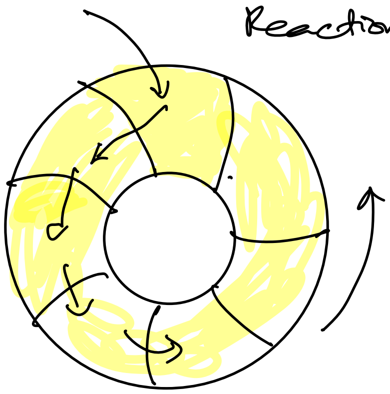
Curved plate:



Sprinklers



Reaction m/c.



Degree of reaction 'R' $0 < R < 1$

$$R = \frac{\text{Static Energy/head.}}{\text{Total Energy/head.}}$$

Total Energy = Static Energy + Dynamic Energy.

Steam Turbines & Gas turbines. } $R = \frac{\text{Static Enthalpy.}}{\text{Total Enthalpy}} = \frac{\Delta h}{\Delta h_0}.$

Radial flow machines.

$$R = \frac{\frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{\frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}} \rightarrow \text{Turbine.}$$

$$R = \frac{\frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}{\frac{V_2^2 - V_1^2}{2} + \frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}} \rightarrow \text{Comp/Pump.}$$

$$R = \frac{\frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{V_{w_1}u_1 - V_{w_2}u_2} \rightarrow \text{Turbine.}$$

$$R = \frac{\frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}{V_{w_2}u_2 - V_{w_1}u_1} \rightarrow \text{Comp/pump.}$$

$$0 < R < 1$$

No. Static Energy.

$$\underbrace{\frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}_{\text{Zero.}}$$

No. Kinetic energy

$$\underbrace{\frac{V_2^2 - V_1^2}{2}}_{\text{Zero.}}$$

Cases:

1) $R = 0 \rightarrow$ Pure impulse machine.
   eg: Pelton wheel, Delaval steam turbine.

2) $R = 1 \rightarrow$ Pure reaction machine. eg: Lawn sprinkler
   Not a turbomachine

3) $R < 1 \rightarrow$ Reaction machine.

Utilization factor (ϵ) [Diagram factor]

$$\epsilon = \frac{\text{Energy utilized/converted}}{\text{Energy available}}$$

$$\epsilon = \frac{E_u}{E_i}$$

$E_u \to$ Euler energy/ Actual energy.

$E_i \to$ Ideal energy.

$$\epsilon = \frac{\frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{\frac{V_1^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}} \longrightarrow V_2 = 0$$

$$\epsilon = \frac{\frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{\frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2} + \frac{V_2^2}{2}} \longrightarrow \epsilon = \underbrace{\frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}}_{\text{Home work.}}$$

$$\epsilon = \frac{\text{Euler equation}}{\text{Euler equation} + \text{losses}}$$

$$\epsilon = \frac{V_{w_1}u_1 - V_{w_2}u_2}{V_{w_1}u_1 - V_{w_2}u_2 + \frac{V_2^2}{2}} \to \text{Loss or Energy not utilized.}$$

$$0 < \epsilon < 1$$

It indicates how well the turbine perform/can convert the energy. While designing the turbine, this quantity has to be maximized.

Tutorial-2

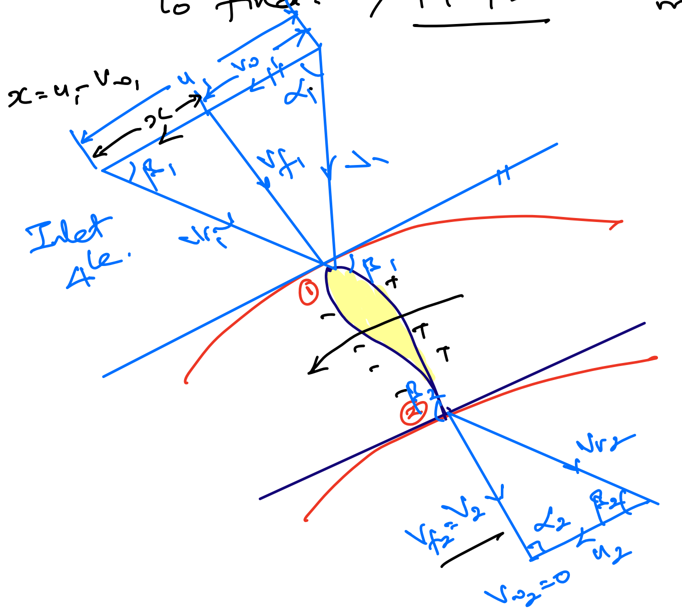
1. In a radially inward flow turbine, the diameter of the runner at the inlet is 50 cm and the diameter at the outlet is 15 cm. The speed of the machine is 1500 rpm. The fluid at a velocity of 35 m/s enters the runner at 20^o to the tangent and leaves the runner without any whirl component. The flow component of the fluid velocity remains constant in the runner. Calculate the blade angles at the inlet and outlet, the specific work and the degree of reaction.

Sol: Given: $d_1 = 0.5\,m$; $d_2 = 0.15\,m$, $N = 1500$ rpm, $\vec{V}_1 = 35\,m/s$; $\alpha_1 = 20^\circ$

$V_{w_2} = 0$; $V_{f_1} = V_{f_2} = V_f$. Assume $\dot{m}$ kg/s.

To find: i) β_1 & β_2 ii) $\frac{W}{\dot{m}}$ iii) R.

① → ②



Blade velocities

$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.5 \times 1500}{60}$

$u_1 = 39.27\,m/s$.

$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.15 \times 1500}{60}$

$u_2 = 11.78\,m/s$.

Consider the velocity triangle @ inlet ①

$\cos\alpha_1 = \frac{V_{w_1}}{V_1}$

$\vec{V}_{w_1} = \vec{V}_1 \cos\alpha_1$

$= 35 \times \cos 20$

$\vec{V}_{w_1} = 32.89\,m/s$.

$\sin\alpha_1 = \frac{V_{f_1}}{V_1}$

$V_{f_1} = V_1 \sin\alpha_1$

$= 35 \times \sin 20$

$V_{f_1} = 11.97\,m/s = V_{f_2}$

i) Blade angles β_1 & β_2.

$\tan\beta_1 = \frac{V_{f_1}}{x} \rightarrow \tan\beta_1 = \frac{V_{f_1}}{u_1 - V_{w_1}} = \frac{11.97}{39.27 - 32.89}$.

$\therefore \beta_1 = \tan^{-1}\left[\frac{8.97}{39.27 - 32.89}\right] = 61.94^\circ$

From the outlet velocity triangle.

$$\tan\beta_2 = \frac{V_{f2}}{u_2} = \frac{11.97}{11.78}$$

$$\beta_2 = \tan^{-1}\left[\frac{11.97}{11.78}\right] = 45.45^\circ$$

ii) Specific Work $\left[\frac{W}{\dot m}\right]$

$$\frac{W}{\dot m} = V_{w_1}u_1 - \cancel{V_{w_2}}^{0}u_2 \rightarrow \frac{W}{\dot m} = V_{w_1}u_1$$

$$= 32.87 * 39.27$$

$$\frac{W}{\dot m} = 1.291 \text{ KJ/kg}.$$

iii) Degree of reaction 'R'.

$$R = \frac{\text{Static energy}}{\text{Total energy}}$$

$$\frac{W}{\dot m} = \frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}$$

$$\downarrow$$

$$\frac{W}{\dot m} - \frac{V_1^2 - V_2^2}{2} = \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}$$

$$R = \frac{\frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{\frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}$$

$$R = \frac{\frac{W}{\dot m} - \frac{V_1^2 - V_2^2}{2}}{\frac{W}{\dot m}}$$

$$R = \frac{1291 - \left(\frac{35^2 - 11.97^2}{2}\right)}{1291}$$

$$R = 0.58$$

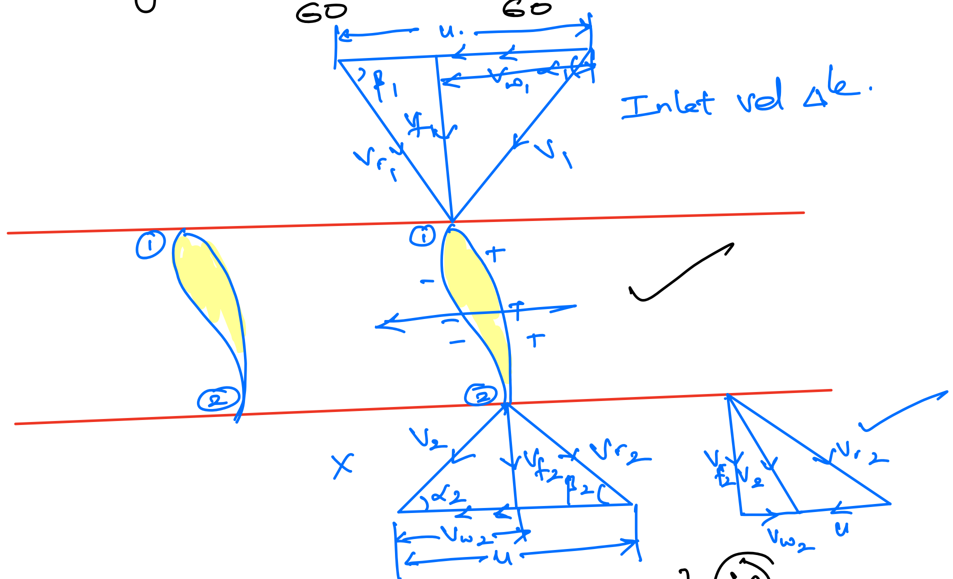
2. At a stage of impulse turbine the mean blade diameter is 0.75 m, its rotational speed being 3000 rpm. The absolute velocity of fluid discharging from a nozzle inclined at 18° to the plane of wheel is 275 m/sec. If the utilization factor is 0.9. The relative velocity at the rotor exit is 0.9 times that at the inlet. Find the inlet and exit rotor angles. Also find the power output from the stage for a mass flow rate of 2 kg/s and axial thrust on the shaft.

Given: $d = 0.75$ m; $N = 3000$ rpm; $\alpha_1 = 18°$; $\vec{V}_1 = 275$ m/s; $\epsilon = 0.9$,

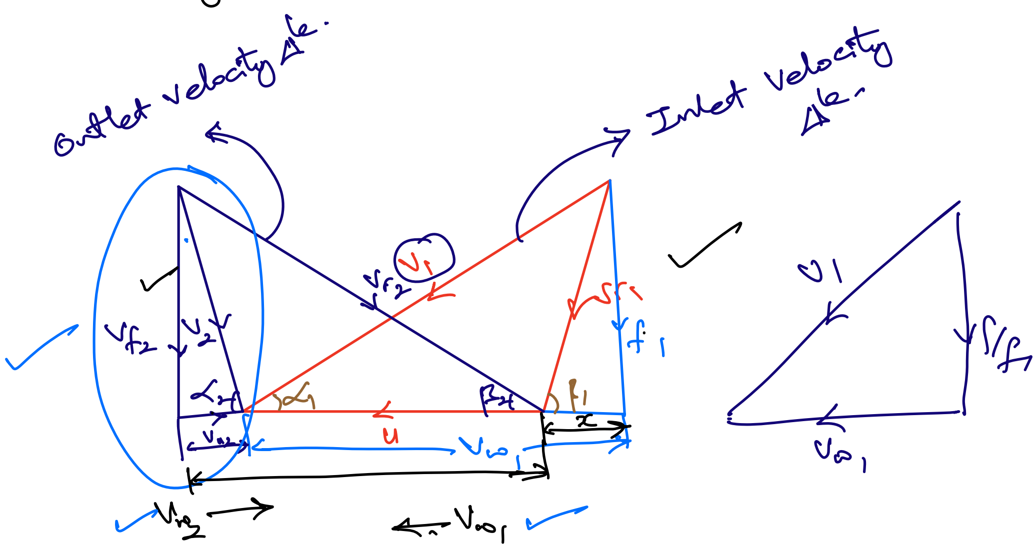
$V_{r_2} = 0.9\, V_{r_1}$, $\dot{m} = 2$ kg/s.

To find: i) β_1 & β_2 ii) P iii) F_{axial}.

Blade velocity $u = \frac{\pi d N}{60} = \frac{\pi \times 0.75 \times 3000}{60} = 117.81$ m/s.



Combined Velocity Δle → [when $u_1 = u_2 = u$]



Consider the inlet velocity Δ^{le} @ ①

$$\cos\alpha_1 = \frac{V_{w_1}}{V_1} \qquad \sin\alpha_1 = \frac{V_{f_1}}{V_1}$$

$$V_{w_1} = V_1 \cos\alpha_1 \qquad V_{f_1} = V_1 \sin\alpha_1$$

$$= 275 \times \cos 18 \qquad = 275 \times \sin 18$$

$$V_{w_1} = 261.54 \text{ m/s} . \qquad V_{f_1} = 84.98 \text{ m/s} .$$

$$\tan\beta_1 = \frac{V_{f_1}}{V_{w_1} - u} \rightarrow \beta_1 = \tan^{-1}\left[\frac{V_{f_1}}{V_{w_1} - u}\right]$$

$$= \tan^{-1}\left[\frac{84.98}{261.84 - 117.81}\right]$$

$$\beta_1 = 30.59^\circ$$

$$\sin\beta_1 = \frac{V_{f_1}}{V_{r_1}} \text{ (or) } \cos\beta_1 = \frac{V_{w_1} - u}{V_{r_1}}$$

$$V_{r_1} = \frac{V_{f_1}}{\sin\beta_1} = \frac{84.98}{\sin 30.59}$$

$$V_{r_1} = 167 \text{ m/s.}$$

$$V_{r_2} = 0.9\, V_{r_1} = 0.9 \times 167 = 150.27 \text{ m/s.}$$

Utilization factor $\epsilon = \dfrac{W}{W + \frac{V_2^2}{2}} = \dfrac{(V_{w_1} - V_{w_2})u}{(V_{w_1} - V_{w_2})u + \frac{V_2^2}{2}}$

Also $\epsilon = \dfrac{V_1^2 - V_2^2}{V_1^2 - R V_2^2}$ $\quad R = 0 \rightarrow$ Impulse.

$$\epsilon = \frac{V_1^2 - V_2^2}{V_1^2} \rightarrow 0.9 = \frac{275^2 - V_2^2}{275^2}$$

$$V_2 = 86.96 \text{ m/s} .$$

$$\epsilon = \frac{(V_{w_1} - V_{w_2})u}{(V_{w_1} - V_{w_2})u + \frac{V_2^2}{2}}$$

$$0.9 = \frac{(261.54 - V_{w_2}) \times 117.81}{(261.54 - V_{w_2}) \times 117.81 + \frac{86.96^2}{2}}$$

$V_{w_2} = -27.3$ m/s $\longrightarrow$ Opposite direction

From the outlet velocity triangle.

$$V_2^2 = V_{f_2}^2 + V_{w_2}^2$$

$$86.96^2 = V_{f_2}^2 + (-27.3)^2.$$

$$V_{f_2} = 82.54 \text{ m/s}.$$

$$\tan\beta_2 = \frac{V_{f_2}}{V_{w_2} + u}.$$

$$\beta_2 = \tan^{-1}\left[\frac{V_{f_2}}{V_{w_2} + u}\right]$$

$$= \tan^{-1}\left[\frac{82.54}{-27.3 + 117.81}\right]$$

$$\beta_2 = 28^\circ.$$

ii) $\frac{W}{m} = (V_{w_1} - V_{w_2})\, u$

$$= [261.84 - (-27.3)] * 117.81$$

$$\frac{W}{\dot m} = 34.03 \text{ KJ/kg}$$

iii) $P = \dot m * \frac{W}{\dot m} = 2 * 34.03 = 68.06 \text{ KW}.$

iv) $F_{axial} = \dot m (V_{f_1} - V_{f_2})$

$$= 2(84.98 - 82.54)$$

$F_{axial} = 4.88 \text{ N} \longrightarrow$ Axial direction.

3. In a radial flow pump, the impeller has its smaller diameter of 5 cm and bigger diameter of 12.5 cm. Its speed is 1500 rpm. The inlet blade angle is 50^o. The fluid enters the impeller without any whirl component. The flow component remains constant. $\rightarrow V_{f_1} = V_{f_2}$

- Find the specific work and degree of reaction at a blade outlet angle of 70^o.
- Also find at what outlet angle the impeller becomes a zero-work impeller.

Sol: **Given:** $d_1 = 0.05\,m$; $d_2 = 0.125\,m$; $\alpha_1 = 90^\circ$; $N = 1500$ rpm; $\beta_1 = 50^\circ$

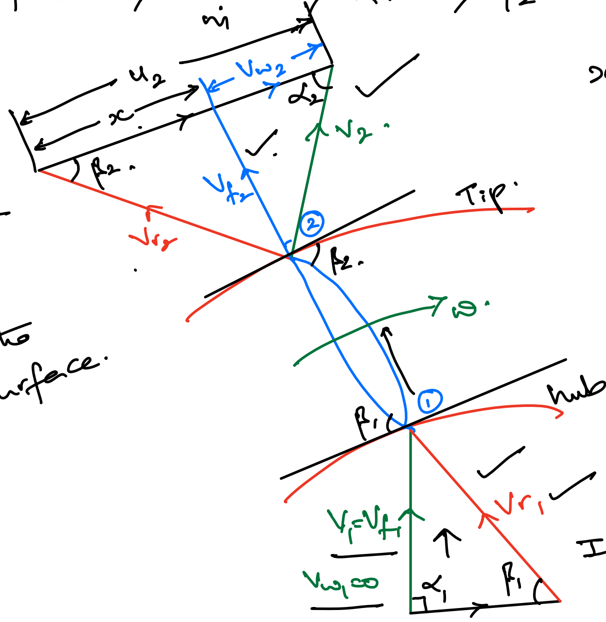
$\beta_2 = 70^\circ$ $\quad V_{w_1} = 0$, $\quad V_{f_1} = V_{f_2} = V_f$.

To find: i) $\frac{W}{m}$ ii) R. iii) β_2 @ $W = 0$

$x = u_2 - V_{w_2}$.

Exit velocity Δ^{le}.

$V_r \rightarrow$ Along the blade surface.



$d_2 > d_1$

$u_2 > u_1$

① $\rightarrow$ Inlet

② $\rightarrow$ Outlet.

Inlet Velocity Δ^{le}.
(Pump/Compressor)

No inlet whirl.

Blade velocity.

$$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.05 \times 1500}{60} = 3.9\,m/s.$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.125 \times 1500}{60} = 9.82\,m/s.$$

$u_2 > u_1$ Radial flow pump.

Let us Consider the inlet velocity triangle.

$$\tan\beta_1 = \frac{V_1}{u_1} \text{ (or) } \frac{V_{f_1}}{u_1} \rightarrow V_1 = u_1 \tan\beta_1$$

$$= 3.9 \times \tan 50$$

$$V_1 = 4.68\,m/s.$$

$$V_{f_1} = V_{f_2} = V_1 = 4.68\,m/s.$$

Similarly, consider the outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f2}}{x} \rightarrow \tan\beta_2 = \frac{V_{f2}}{u_2 - V_{w_2}}.$$

$$u_2 - V_{w_2} = \frac{V_{f2}}{\tan\beta_2}.$$

$$\left[\cot\theta = \frac{1}{\tan\theta}\right]$$

$$V_{w_2} = u_2 - V_{f_2}\cot\beta_2 \leftarrow \boxed{V_{w_2} = u_2 - \frac{V_{f2}}{\tan\beta_2}.}$$

$$= 9.82 - \frac{4.68}{\tan 70}.$$

$$V_{w_2} = 8.1\ m/s$$

i) Specific work $\frac{W}{\dot{m}}$ $(\dot{m} = 1\ kg/s)$

$$\frac{W}{\dot{m}} = V_{w_2}u_2 - \cancel{V_{w_1}u_1}^{0} \rightarrow \text{Pump. } V_{w_2} > V_{w_1} \text{ [Euler equation]} \rightarrow V_{w_2}u_2.$$

$$= 8.1 \times 9.82.$$

$$\frac{W}{\dot{m}} = 79.54\ J/kg.$$

$$R = \frac{\text{Static energy}}{\text{Total energy}}.$$

ii) Degree of reaction

$$R = \frac{\frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}{\frac{W}{\dot{m}}} = \frac{\frac{W}{\dot{m}} - \left(\frac{V_2^2 - V_1^2}{2}\right)}{\frac{W}{\dot{m}}}$$

From the inlet velocity triangle,

$$\sin\beta_1 = \frac{V_{f1}}{V_{r_1}} \quad \text{OR} \quad \left[\cos\beta_1 = \frac{u_1}{V_{r_1}}\right] \times$$

$$V_{r_1} = \frac{V_{f1}}{\sin\beta_1} = \frac{4.68}{\sin 50} = 6.11\ m/s.$$

$$\left[K = \frac{V_{r_2}}{V_{r_1}}\right] \times$$

Ily from the outlet velocity triangle.

$$\sin\beta_2 = \frac{V_{f2}}{V_{r_2}} \quad \text{OR} \quad \left[\cos\beta_2 = \frac{u_2 - V_{w_2}}{V_{r_2}}\right] \times$$

$$V_{r_2} = \frac{V_{f_2}}{\sin\beta_2} = \frac{4.68}{\sin 70} = 4.98 \text{ m/s}$$

$$\therefore \quad R = \frac{\frac{9.82^2 - 3.9^2}{2} + \frac{6.11^2 - 4.98^2}{2}}{79.54}$$

$R = 0.59. \ \underline{< 1} \rightarrow$ [$0 < R < 1$ → Check the calculated value]

iii) $\underline{\beta_2}$ @ $W = 0$.

$$\frac{W}{\dot{m}} = V_{\omega_2} u_2 - \cancel{V_{\omega_1}}^{0} u_1 \rightarrow \text{Euler equation.}$$

$$= \underline{(u_2 - V_{f_2} \cot\beta_2)} u_2.$$

$$\frac{\cancel{W}^{0}}{\dot{m}} = \underline{u_2^2 - V_{f_2} \cot\beta_2 u_2}.$$

$$0 = u_2^2 - V_{f_2} \cot\beta_2 u_2.$$

$$u_2^{\cancel{2}} = V_{f_2} \cot\beta_2 \cancel{u_2}$$

$$\cot\beta_2 = \frac{u_2}{V_{f_1}} \qquad V_{f_1} = V_{f_2} = V_1$$

$$\cot\beta_2 = \frac{u_2}{V_1}$$

$$\cot\beta_2 = \frac{9.82}{4.68}.$$

$$\underline{\beta_2 = 25.48^\circ \simeq 25.5^\circ}.$$

11. In a certain turbo machines, the inlet whirl velocity is 15 m/s, inlet flow velocity is 10 m/s, blade speeds are 30 m/s and 8 m/s at inlet and outlet respectively. Discharge is radial with absolute velocity of 15 m/s. If water is the working fluid flowing at a rate of $1.5m^3/s$, Calculate

(a) Power in KW

(b) Change in total pressure in bar

(c) Degree of reaction

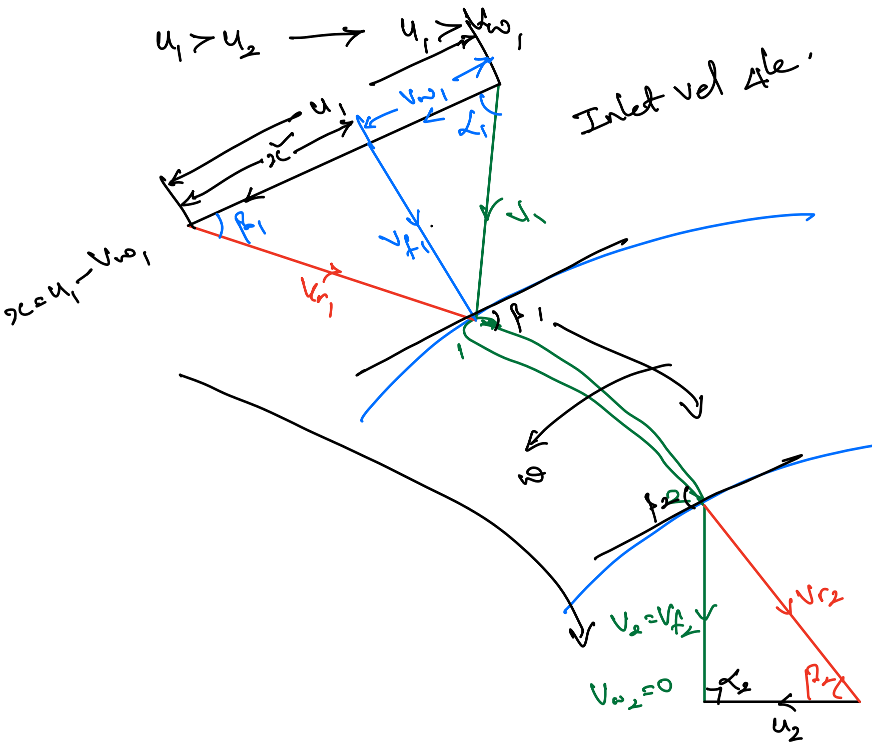
(d) Utilization factor

→ Turbine → $u_1 > u_2$.

Sol: Given: $V_{w_1} = 15$ m/s; $V_{f_1} = 10$ m/s; $u_1 = 30$ m/s; $u_2 = 8$ m/s; $Q = 1.5\, m^3/s$

$V_2 = 15$ m/s.

To find: i) P ii) ΔP iii) R iv) ϵ



$x = u_1 - V_{w_1}$

$= 30 - 15$

$x = 15$ m/s

Consider the inlet velocity triangle

$$\tan\beta_1 = \frac{V_{f_1}}{u_1 - V_{w_1}} \rightarrow \tan\beta_1 = \frac{10}{15} \rightarrow \beta_1 = \tan^{-1}\left[\frac{10}{15}\right]$$

$$\underline{\beta_1 = 33.69^\circ}.$$

$$\text{lly} \quad \sin\beta_1 = \frac{V_{f_1}}{V_{r_1}} \rightarrow V_{r_1} = \frac{V_{f_1}}{\sin\beta_1} = \frac{10}{\sin 33.69}$$

$$\underline{V_{r_1} = 18.03 \text{ m/s}}.$$

Consider the outlet velocity triangle

$$\tan\beta_2 = \frac{V_2}{u_2} \quad (\text{or}) \quad \frac{V_{f_2}}{u_2}. \qquad \sin\beta_2 = \frac{V_2}{V_{r_2}}$$

$$\tan\beta_2 = \frac{15}{8}. \rightarrow \beta_2 = \tan^{-1}\left[\frac{15}{8}\right] \qquad V_{r_2} = \frac{V_2}{\sin\beta_2}$$

$$\beta_2 = 61.93^\circ. \qquad V_{r_2} = \frac{15}{\sin 61.93}.$$

ii) $W = \frac{\Delta P}{\rho} = (V_{w_1}u_1 - \cancel{V_{w_2}u_2}^{\,0}) \rightarrow$ Incompressible fluid.

$V_{r_2} = 17\ m/s$.

$\Delta P = V_{w_1}u_1 * \rho$.

OR $\left\{ \Delta P = \left[\frac{V_1^2 - V_2^2}{2} + \frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}\right] * \rho. \right\}$

$\rho = 1000\ kg/m^3$ for water.

$\Delta P = 15 \times 30 \times 1000$

$\Delta P = 4.5$ bar.

iii) Power $= \dot{m} \times \frac{W}{m} = \dot{m}\left[V_{w_1}u_1 - \cancel{V_{w_2}u_2}^{\,0}\right]$ $\qquad \dot{m} = \rho A V = \rho Q$

$= \dot{m} \times V_{w_1}u_1$ $\qquad \therefore \dot{m} = \rho Q.$

$= 1500 \times 15 \times 30$ $\qquad = 1000 \times 1.5$

$= 675$ KW. $\qquad \dot{m} = 1500\ kg/s$

iv) $$R = \frac{\frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{V_{w_1}u_1 - \cancel{V_{w_2}u_2}^{\,0}} = \frac{\frac{u_1^2 - u_2^2}{2} + \frac{V_{r_2}^2 - V_{r_1}^2}{2}}{V_{w_1}u_1}$$

$$R = \frac{\frac{30^2 - 8^2}{2} + \frac{17^2 - 18.03^2}{2}}{15 \times 30}$$

$R = 0.89. < 1$

iv) Utilization factor 'ϵ'

$$\epsilon = \frac{W}{W + \frac{V_2^2}{2}} = \frac{V_{w_1} u_1}{V_{w_1} u_1 + \frac{V_2^2}{2}}$$

$$\epsilon = \frac{15 \times 30}{15 \times 30 + \frac{15^2}{2}}$$

$$\epsilon = 0.8 < 1$$

Problems:

1) Blade angles β_1, β_2.
2) Work (or) Specific work.
3) Power $\rightarrow \dot{m}$
4) $u_1, u_2, V_{r_1}, V_{r_2}, V_{w_1}, V_{w_2}, V_{f_1}, V_{f_2}$
5) R (or) ϵ.
6) Velocity triangles

Calculations.
Trignometric relations.

Velocity triangles → Radial flow → Velocity Δ^{le} @ hub & tip.
Velocity triangles → Axial flow → Combined Velocity Δ^{le}.

Velocity triangles ⇓ Vector addition/subtraction

2 (c) Absolute velocity of water at impeller exit is 14 m/s at an angle of 18°. The blade peripheral speed at the exit is 25 m/s, and the shaft speed is 1450 rpm. Whirl component of the absolute velocity at the inlet is zero. The flow rate is 0.018 m^3/s. Find

(i) The magnitude of the relative velocity and exit blade angle,

(ii) The power required,

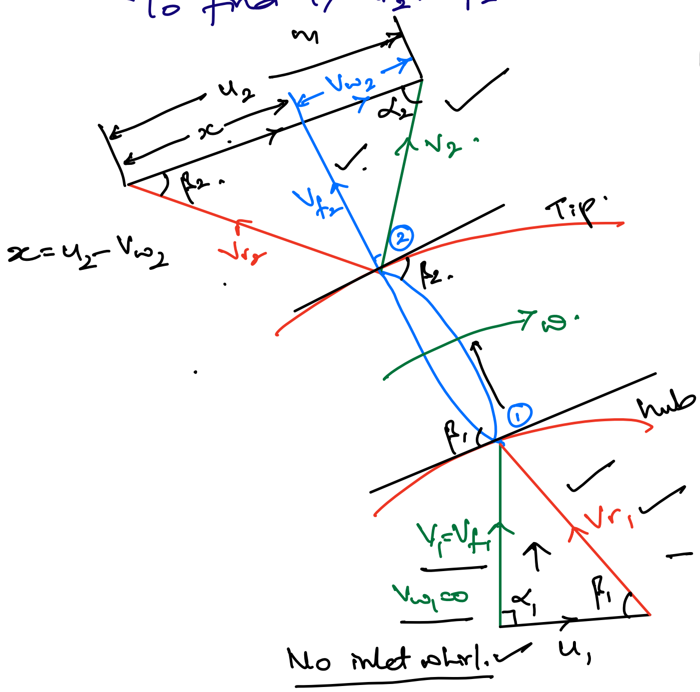
(iii) If inner diameter is 0.6 times outer diameter, determine inlet blade angle, if meridional velocity is constant.

(iv) Degree of Reaction. (CO3, PO2, L3)

Sol: Given: $V_2 = 14\ m/s$; $\alpha_2 = 18°$; $u_2 = 25\ m/s$; $N = 1450$ rpm; $V_{w1} = 0$

$Q = 0.018\ m^3/s$. $d_2 = 0.6 d_1$, V_f = constant

To find i) V_{r_2} & β_2 ii) P iii) β_1. iv) R.



From the exit velocity triangle

$$\cos\alpha_2 = \frac{V_{w_2}}{V_2}$$

$$V_{w_2} = V_2 \cos\alpha_2$$

$$= 14 \times \cos 18$$

$$V_{w_2} = 13.32\ m/s$$

$$\sin\alpha_2 = \frac{V_{f_2}}{V_2}$$

$$V_{f_2} = V_2 \sin\alpha_2$$

$$= 14 \times \sin 18$$

$$V_{f_2} = 4.33\ m/s.$$

i) Exit blade angle.

$$\tan\beta_2 = \frac{u_2 - V_{w_2}}{V_{f_2}} = \frac{25 - 13.32}{4.33}.$$

$$\therefore \beta_2 = \tan^{-1}\left[\frac{25 - 13.32}{4.33}\right]$$

$$\beta_2 = 69.67°$$

$$\cos\beta_2 = \frac{u_2 - V_{w_2}}{V_{r_2}} \rightarrow V_{r_2} = \frac{u_2 - V_{w_2}}{\cos\beta_2} = \frac{25 - 13.32}{\cos 69.67} = 33.62\ m/s$$

ii) Power $P = \dot{m}\left[V_{w_2}u_2 - \cancel{V_{w_1}} \times u_1\right]$ $V_{w_1} = 0$

$= \dot{m}\left[V_{w_2}u_2\right]$ $\dot{m} = \rho Q$

$= 1000 \times 0.018$

$P = 18 \times 13.32 \times 25$

$\dot{m} = 18\ kg/s$

$P = 5.99\ kW \simeq 6\ kW.$

iii) $u_2 = \frac{\pi d_2 N}{60} \rightarrow d_2 = \frac{60 u_2}{\pi N} = \frac{60 \times 25}{\pi \times 1450} = 0.33\ m.$

$\therefore d_1 = 0.6 \times d_1 = 0.198\ m.$

$\therefore u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.198 \times 1450}{60} = 15.03\ m/s$

From the inlet velocity triangle.

$\tan\beta_1 = \frac{V_{f1}}{u_1}$ $V_{f1} = V_{f2} = V_f = 4.33\ m/s.$

$\beta_1 = \tan^{-1}\left[\frac{4.33}{15.03}\right]$

$\beta_1 = 16.07°.$

$\cos\beta_1 = \frac{u_1}{V_{r_1}} \rightarrow V_{r_1} = \frac{u_1}{\cos\beta_1} = \frac{15.03}{\cos 16.07}$

$V_{r_1} = 15.642$

iv) Degree of reaction 'R'

$$R = \frac{\frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}{V_{w_2}u_2 - \cancel{V_{w_1}u_1}} = \frac{\frac{u_2^2 - u_1^2}{2} + \frac{V_{r_1}^2 - V_{r_2}^2}{2}}{V_{w_1}u_1}$$

$$R = \frac{\frac{25^2 - 15.03^2}{2} + \frac{15.64^2 - 33.62^2}{2}}{13.32 \times 25}$$

$$R = \frac{199.55 - 443.16}{333} = -0.73$$ as it is a pump.