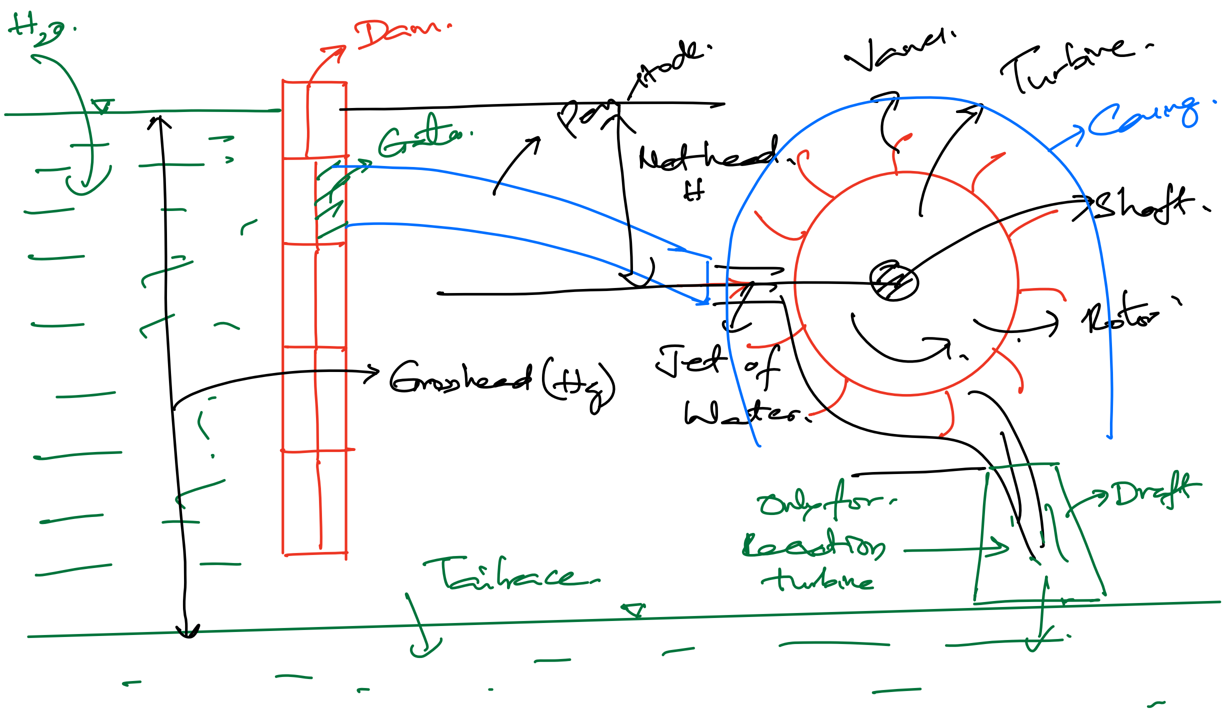
UNIT – 5

Hydraulic Turbines: Classification; Efficiencies; Pelton wheel-velocity triangles, Design parameters; Francis turbine–velocity triangles and design parameters; Analysis of Draft tube, Kaplan turbine– Velocity triangles and design parameters. **07 Hours**

What is a hydraulic turbine?

Hydraulic turbine are the machines which convert the hydro energy into mechanical work. The hydro energy is the stored energy (potential) in reservoirs by constructing the dams.

Layout of a hydro-electric power plant.



Classification:

1) Based on the energy available at the vanes.

a) Impulse type → Pelton turbine.

b) Reaction type → Francis & Kaplan.

2) Based on the head of water available.

a) Low head (5 to 75 m) → Kaplan.

b) Medium head (50 to 250 m) → Francis, Modern Francis.

c) High head (150 to 350 m) → Pelton turbine.

3) Based on the direction of fluid flow.

a) Radial flow → Francis

b) Axial flow → Kaplan, Propeller.

c) Tangential flow → Pelton turbine.

d) Mixed flow → Modern Francis turbine.

4) Based on specific speed.

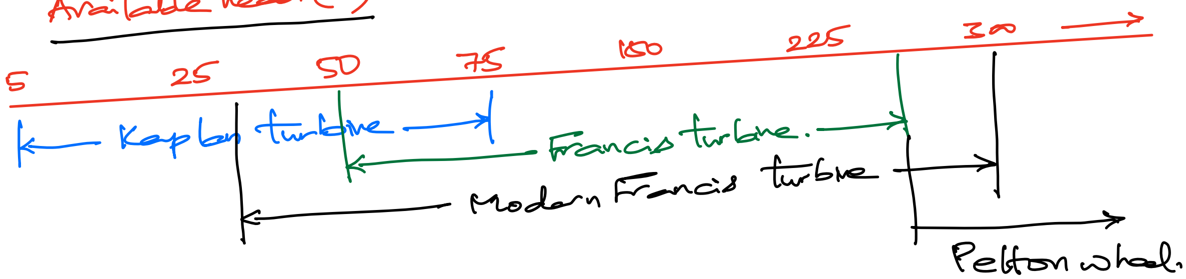
a) 5 – 30 → Single jet impulse turbine.
b) 30 – 65 → Multi-jet impulse turbine
} Pelton

c) 65 – 180 → Francis

d) 150 – 300 → Modern Francis.

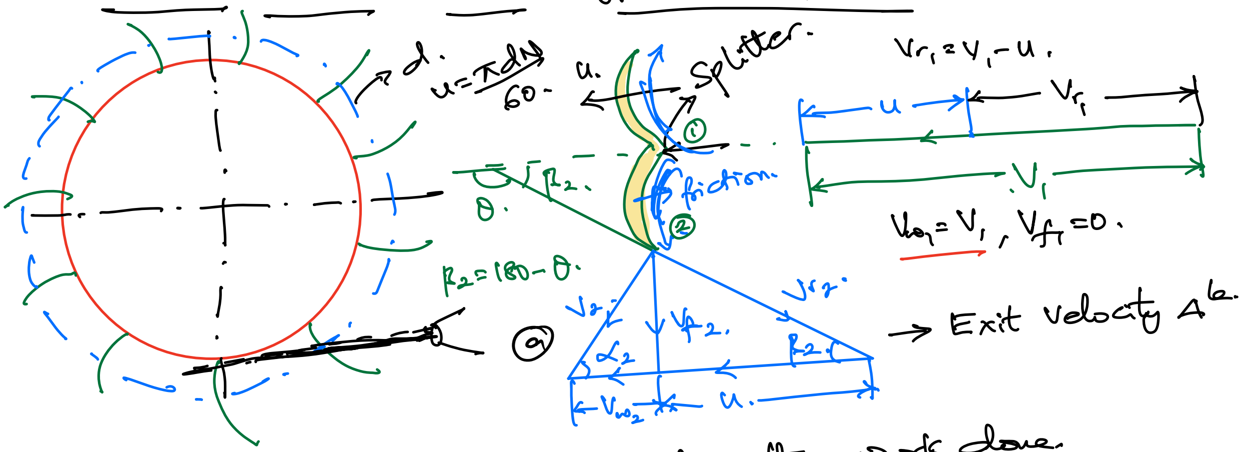
e) 300 – 1000 → Kaplan, Propeller.

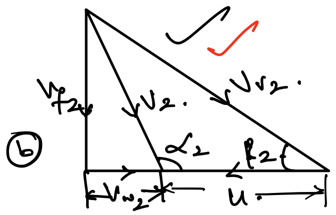
Available head (m)



Pelton wheel [Impulse type, high speed]

u, V_r.





Expression for the work done.

From the Euler's equation.

$$\frac{W}{m} = (V_{w_1} - V_{w_2})\, u \rightarrow \text{Specific work.}$$

a → V_{w_2} is in same direction.

b → V_{w_2} is in opposite direction. ✓

$$\frac{W}{m} = (V_{w_1} + V_{w_2})\, u \quad [\because -V_{w_2}] \rightarrow ①$$

From the exit velocity triangle.

$$\cos\beta_2 = \frac{V_{w_2} + u}{V_{r_2}} \rightarrow V_{w_2} = V_{r_2}\cos\beta_2 - u. \rightarrow ②.$$

Friction →

Let $C_b = \frac{V_{r_2}}{V_{r_1}} \rightarrow V_{r_2} = C_b . V_{r_1} \rightarrow ③$.

From the inlet velocity triangle. $V_{r_1} = V_1 - u \rightarrow ④$

Substitute for V_{r_1} in ③

$$\therefore V_{r_2} = C_b(V_1 - u) \rightarrow ⑤$$

Substitute V_{r_2} in ②

$$\therefore V_{w_2} = C_b(V_1 - u)\cos\beta_2 - u. \rightarrow ⑥$$

If V_{w_2} is substituted in ① for workdone

$$\frac{W}{m} = [V_{w_1} + C_b(V_1 - u)\cos\beta_2 - u]\,u.$$

From the inlet velocity triangle $V_{w_1} = V_1$

$$\therefore \frac{W}{m} = [(V_1 - u) + C_b(V_1 - u)\cos\beta_2]\,u.$$

$$\frac{W}{m} = (V_1 - u)\,u\,[1 + C_b\cos\beta_2] \longrightarrow ⑦$$

To obtain the maximum workdone. u, V_1, C_b, β_2.

$$\frac{dW}{du} = 0. \rightarrow \frac{d}{du}\left((V_1 - u)\,u\,[1 + C_b\cos\beta_2]\right) = 0.$$

$$\frac{d}{du}\left(V_1 u - u^2\right) = 0$$

$$V_1 - 2u = 0.$$

$$u = \frac{V_1}{2} \longrightarrow ⑧$$

Substitute the value of 'u' in ⑦.

$$W_{max} = \left(V_1 - \tfrac{1}{2}V_1\right)\tfrac{1}{2}V_1\left[1 + C_b.\cos\beta_2\right].$$

$$W_{max.} = \frac{V_1^2}{4}\left(1 + C_b.\cos\beta_2\right) \longrightarrow ⑨.$$

Speed ratio $\phi = \frac{u}{V_1} \rightarrow u = \phi V_1 \quad \therefore$ ⑦ becomes.

$$W = (V_1 - \phi V_1)\,\phi V_1\left(1 + C_b.\cos\beta_2\right)$$

For design $\phi = 0.46$.

$$W = \phi V_1^2(1 - \phi)\left(1 + C_b\cos\beta_2\right) \longrightarrow ⑩ \rightarrow \frac{dW}{d\phi} = 0$$

& $\phi = 0.5$

Workdone in terms of Speed ratio.

Efficiencies associated with Pelton wheel.

1) Hydraulic (OR) Blade (OR) Vane Efficiency.

$$\eta_{hyd} = \frac{\text{Energy converted by the fluid (Workdone)}}{\text{Energy available with the water.}}$$

$$\eta_{hyd} = \frac{(V_{w_1} - V_{w_2})u}{wQH}$$

(a) $(V_{w_1} - V_{w_2})u$ (x) (b) $(V_{w_1} + V_{w_2})u$. if $-V_{w_2}$.

H → Net head.

$$\eta_{hyd} = \frac{(V_{w_1} - V_{w_2})u}{\frac{V_1^2}{2}}$$

$\frac{V_1^2}{2}$ → Pure Kinetic energy.

2) Volumetric Efficiency:

$$\eta_{vol} = \frac{\text{Quantity of water striking the blade}}{\text{Quantity of water entering the turbine.}}$$

$$\eta_{vol} = \frac{Q - \Delta Q}{Q} \quad (OR) \quad \frac{\dot{m} - \Delta\dot{m}}{\dot{m}}$$

3) Mechanical Efficiency:

$$\eta_{mech} = \frac{\text{Energy available at the shaft.}}{\text{Energy available at the rotor (Workdone)}}$$

$$\eta_{mech} = \frac{P_{shaft}}{(V_{w_1} - V_{w_2})u}$$

$\eta_{overall} = \eta_{hyd} \times \eta_{vol} * \eta_{mech}$.

4) Overall Efficiency:

$$\eta_{overall} = \frac{\text{Energy available at the shaft}}{\text{Energy available with the water.}} = \frac{P_{shaft}}{wQH} = \frac{P_{shaft}}{\frac{V_1^2}{2}}$$

Design parameters of the Pelton wheel.

$V^2 = 2gH$

1) Jet velocity $\rightarrow V_1 = C_V\sqrt{2gH}$

H = Net head.

2) Discharge. $\rightarrow Q = n\,\underbrace{A V_1}_{Q_{nozzle}}$

A = C/s area of the jet

V_1 = Jet velocity.

n = Number of jets. [nozzles]

3) Blade speed $\rightarrow u = \dfrac{\pi D N}{60}$

D = Mean diameter of the wheel.

N = Speed in rpm.

4) Jet ratio $m = \dfrac{D}{d}$

$D \rightarrow$ Mean dia of the wheel.

$d \rightarrow$ Jet diameter.

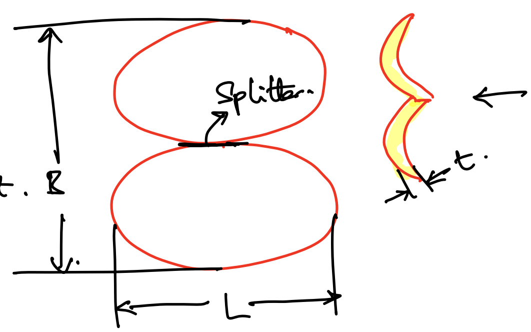
5) Number of buckets (OR) Vanes.

$$Z = \frac{m}{2} + 15 = \frac{D}{2d} + 15$$

6) L = Length of the bucket.

B = Width of the bucket.

t = Thickness of the bucket.



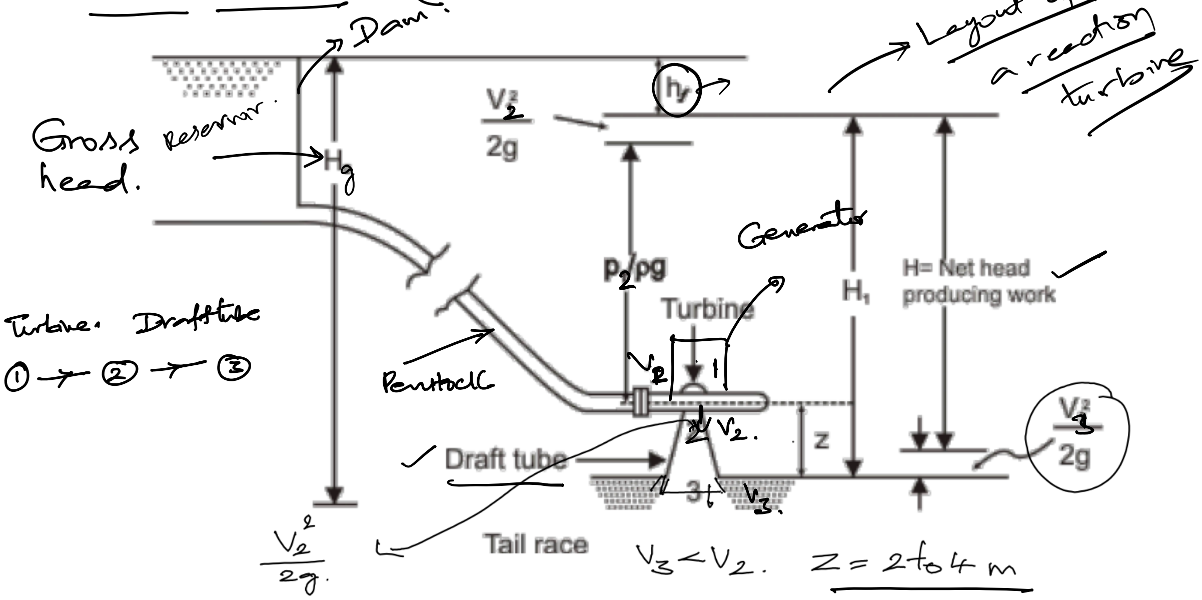
$L = 2.3d$ to $2.8d$.

$B = 2.8d$ to $3.2d$.

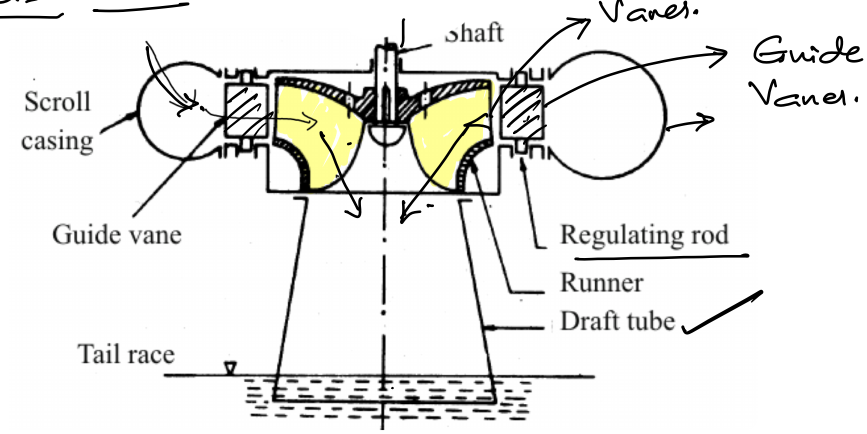
$t = 0.6d$ to $0.9d$.

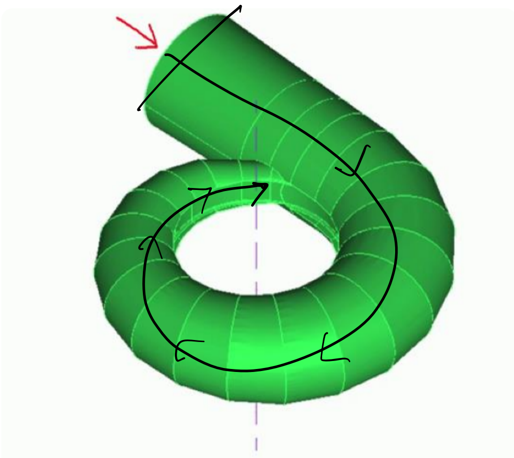
d = Diameter of the jet.

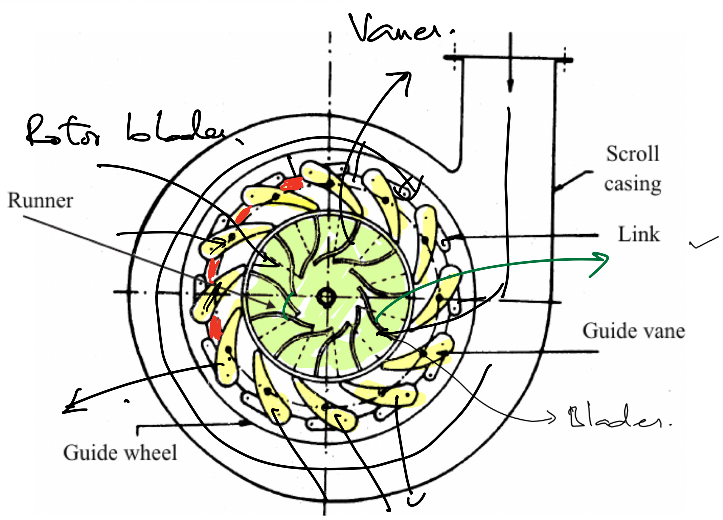
Reaction turbine. (Francis (OR) Kaplan).

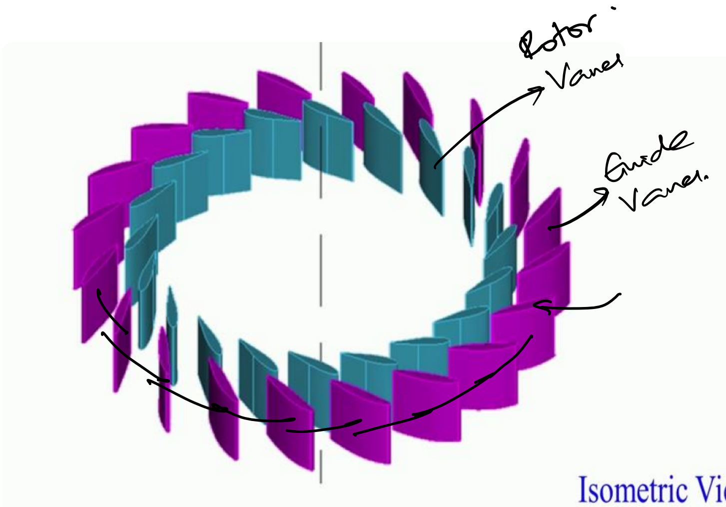


Francis turbine. Generator.

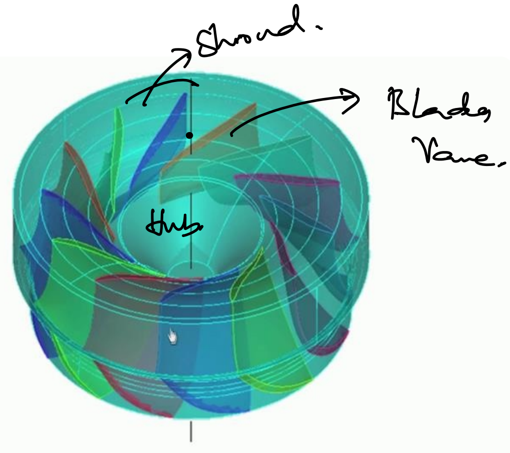



Isometric View of Spiral Casing

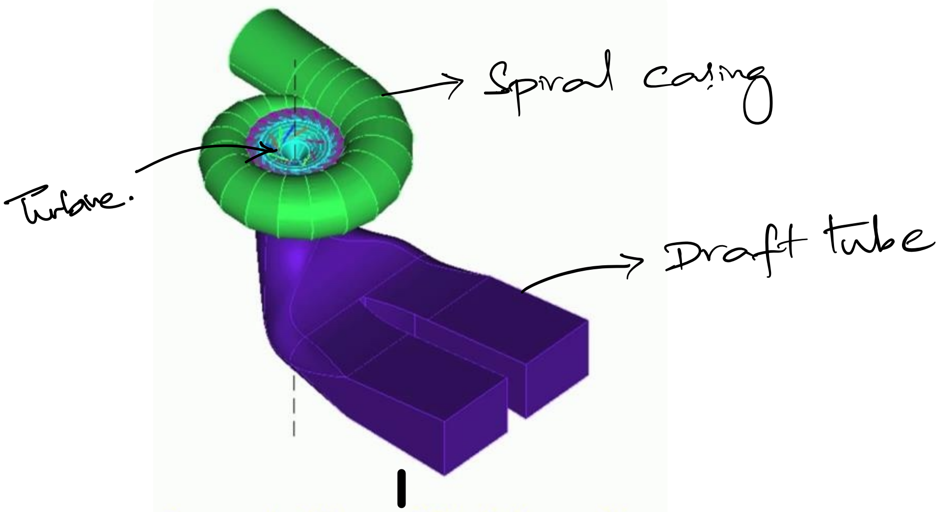




Isometric View of Guide and Stay Vane



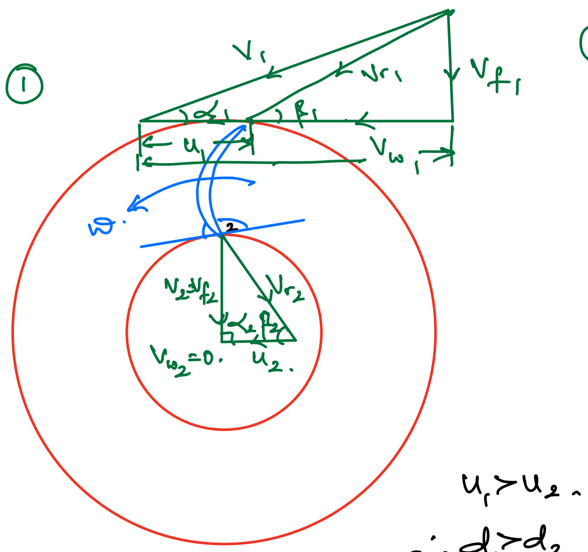
Isometric View of the Hub & Shroud with Runner Blades



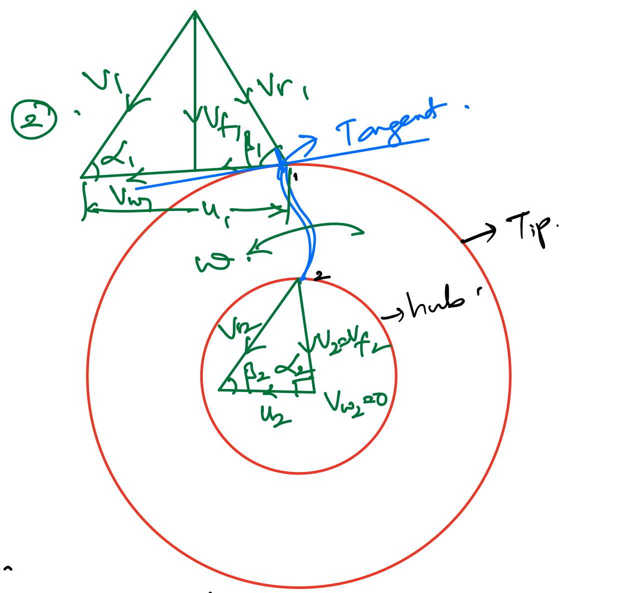
Isometric View of Total Assembly

Analysis of Francis turbine:

①



②



$u_1 > u_2.$

$\therefore d_1 > d_2.$

$u = \frac{\pi d N}{60}.$

1- Inlet

2- Outlet/Exit.

Specific work:

① $\frac{W}{m} = V_{w_1}u_1 - V_{w_2}u_2$ $\quad \because V_{w_2} = 0.$ → Euler equation.

$\frac{W}{m} = V_{w_1}u_1$ → ① N-m (or J.

In terms of pressure head.

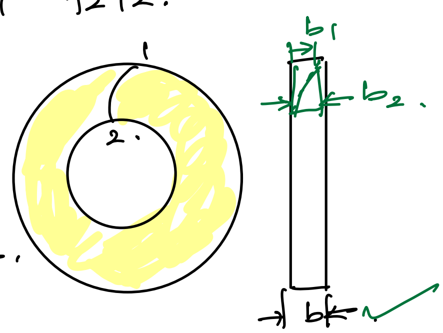
$H = \frac{W}{mg} = \frac{V_{w_1}u_1 - V_{w_2}u_2}{g}.$ $\quad \because V_{w_2} = 0$ → $H = \frac{V_{w_1}u_1}{g}$ → ② metres of liquid column

② Discharge $Q = A_f V_f.$ → $Q = A_{f_1}V_{f_1} = A_{f_2}V_{f_2}.$

d_1 & d_2 → Tip & hub diameter

b_1 & b_2 → Width of the blade @ Tip & hub

V_{f_1} & V_{f_2} → Flow velocities @ Tip & hub.



$Q = \pi d_1 b_1 V_{f_1} = \pi d_2 b_2 V_{f_2}$ → ③

$A_{f_1} = \pi d_1 b_1$

$A_{f_2} = \pi d_2 b_2.$

$\frac{u_2}{u_1} = \frac{d_2}{d_1} \simeq 0.6$ to 0.7

(3) Hydraulic efficiency.

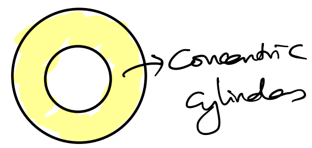
$$\eta_{hyd} = \frac{\text{Available head.}}{\text{Net head.}} \rightarrow \text{Euler head.}$$

$$\eta_{hyd} = \frac{\dfrac{V_{w_1} u_1}{g}}{H}$$

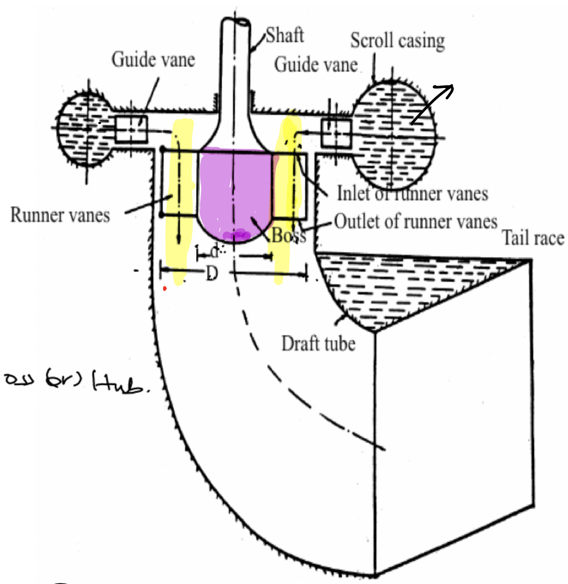
$H \longrightarrow$ Head under which turbine is operating

$$\eta_{hyd} = \frac{V_{w_1} u_1}{gH} \longrightarrow (4)$$

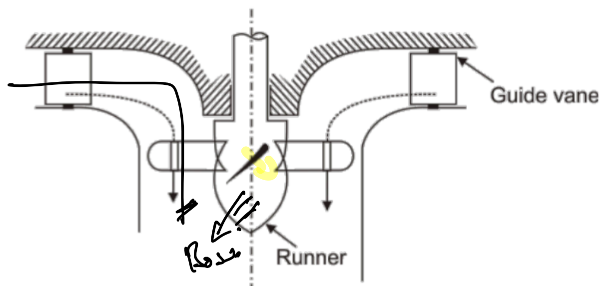
(4) η_{vol}, η_{mech} & $\eta_{overall}$ $\rightarrow$ (*) Same as discussed in pelton wheel.

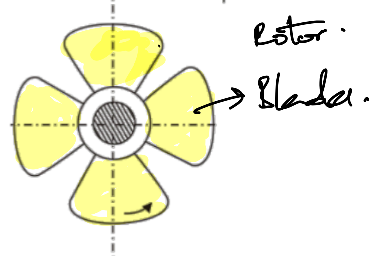


Kaplan (or) Propeller turbine:



Boss (or) Hub.



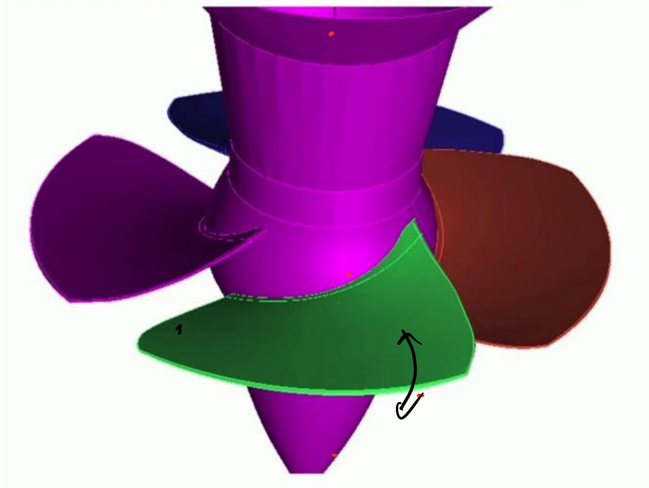


(*) Flow is similar to flow in between concentric-cylinders (flow area).

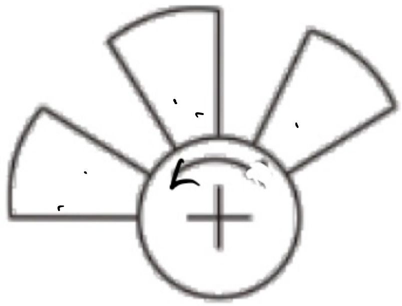
Flow area $A_f = \frac{\pi (D^2 - d^2)}{4}$.

D = Tip diameter.

d = Boss/hub diameter.

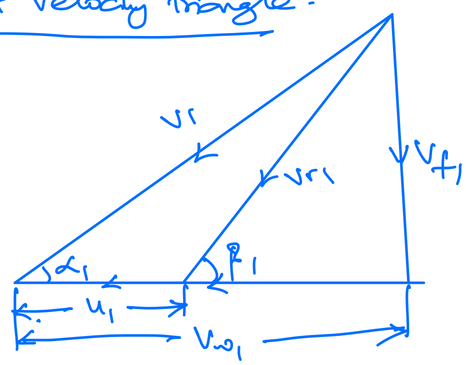


Zoomed View of the Runner Blades with Hub

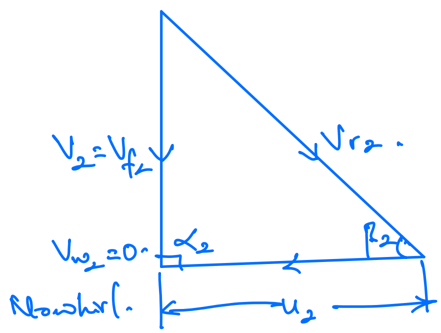


Analysis of Kaplan turbine.

Inlet velocity triangle.



Outlet velocity triangle $V_{w_2}=0$.



① Specific work.

$$\frac{W}{m} = V_{w_1}u_1 - V_{w_2}u_2 \,. \quad \because V_{w_2}=0\,; \quad \therefore \frac{W}{m} = V_{w_1}u_1$$

Euler equation derivation.

② Discharge $Q = A_f V_f = \dfrac{\pi\left(D^2-d^2\right)}{4} V_f$.

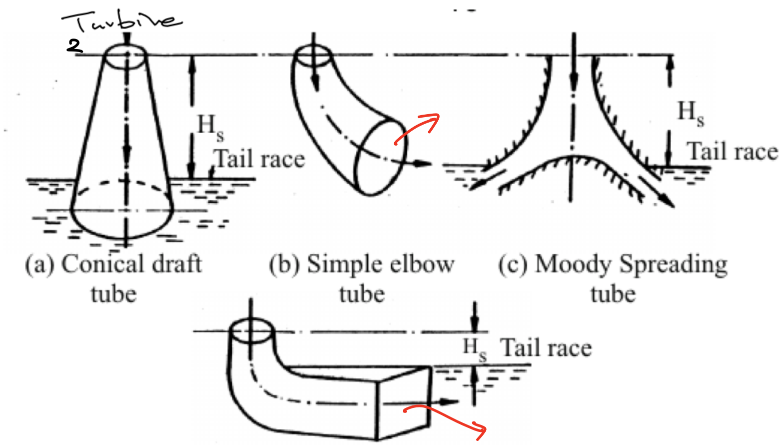
③ Speed ratio $\phi = \dfrac{u}{V_1} \rightarrow \phi = 1.5$ to 2.2.

$V_1 = \sqrt{2gH} \rightarrow$ Net head.

④ Flow ratio $\psi = \dfrac{V_f}{V_1} \rightarrow \psi = 0.55$ to 0.75.

⑤ $\dfrac{d}{D} = 0.35$ to 0.6.

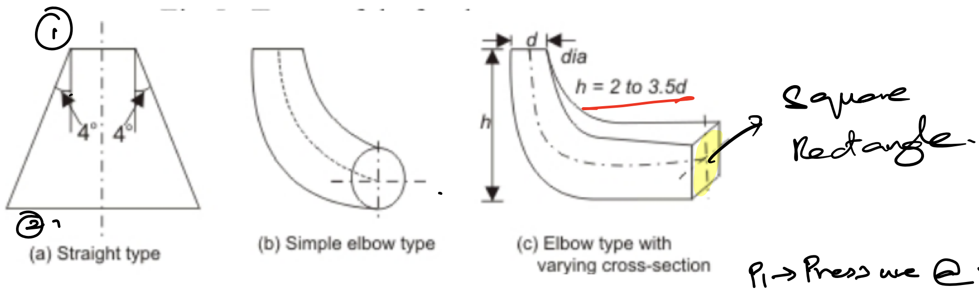
Draft tube: It is the device used to recover the kinetic energy of the fluid exiting the reaction turbine. It also allows the reaction turbine to be installed above the tail race



(a) Conical draft tube (b) Simple elbow tube (c) Moody Spreading tube

(d) Draft tube with circular inlet and rectangular outlet

Note: If angle > 5° the losses will increase drastically.



(a) Straight type (b) Simple elbow type (c) Elbow type with varying cross-section

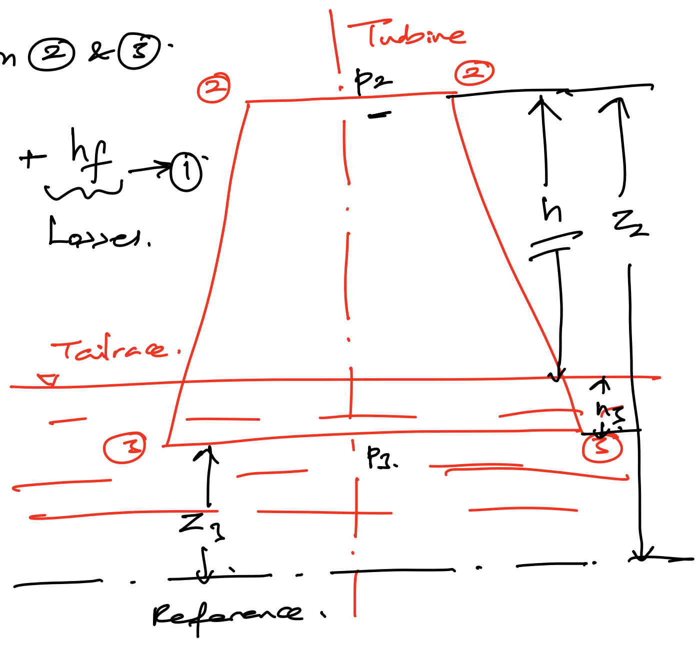
$p_1 \to$ Pressure @ inlet to turbine

Analysis of Draft tube

Apply the Bernoulli's equation (2) & (3).

$$\underbrace{\frac{p_2}{\rho g} + \frac{\vec{V}_2^2}{2g} + z_2}_{\text{Entry}} = \underbrace{\frac{p_3}{\rho g} + \frac{\vec{V}_3^2}{2g} + z_3}_{\text{Exit}} + \underbrace{h_f}_{\text{Losses}} \to (1)$$

$$\frac{p_3}{\rho g} = \frac{p_a}{\rho g} + h_3 \to (2)$$



Substituting for $\frac{p_3}{\rho g}$ in ①.

$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 = \frac{p_a}{\rho g} + h_3 + \frac{V_3^2}{2g} + Z_3 + h_f.$$

$$\frac{p_2}{\rho g} = \frac{p_a}{\rho g} - \left(\frac{V_2^2 - V_3^2}{2g}\right) - \underbrace{(Z_2 - Z_3 - h_3)}_{h} + h_f.$$

$\because h = Z_2 - Z_3 - h_3.$

$$\frac{p_2}{\rho g} = \frac{p_a}{\rho g} - \frac{(V_2^2 - V_3^2)}{2g} - h + h_f. \rightarrow ③$$

If 'h_f' is expressed in terms of Kinetic energy losses.

$$h_f \simeq \frac{V_2^2 - V_3^2}{2g} \cdot K. \rightarrow \text{Less.}$$

$$\frac{p_2}{\rho g} = \frac{p_a}{\rho g} - h - \underbrace{(1-K)}_{\downarrow}\left(\frac{V_2^2 - V_3^2}{2g}\right) \uparrow$$

$(1-K) \rightarrow$ Draft tube efficiency.

$\rightarrow$ Lower the value of 'K', higher the efficiency.

Cavitation: It is the phenomenon of formation of bubbles at the exit of the turbine, when the pressure falls below the vapour pressure of the water (fluid). These bubbles will attach to the surface of the casing, blade, draft tube and are collapsed when the stream of fluid strikes them.

Difference between impulse and reaction turbines.

Impulse turbine	Reaction Turbine
The entire available energy of the water is first converted into kinetic energy.	The available energy of the water is not converted from one form to another.
The water flows through the nozzles and impings on the buckets, which are fixed to the outer periphery of the wheel.	The water is guided by the glide blades to flow over the moving vane.
The water impings on the buckets with KE	The water glides over the moving vanes with PE.
The pressure of the flowing water remains unchanged and is equal to the atmospheric pressure.	The pressure of the flowing water is reduced after gliding over the vane.
It is not essential that the wheel should run full.	It is essential that the wheel should always run full and kept full of water.
It is possible to regulate the flow without loss.	It is not possible to regulate the flow without loss.
Impulse Turbine has more hydraulic efficiency.	Reaction Turbine has relatively less efficiency.
Impulse Turbine operates at high water heads.	Reaction turbine operates at low and medium heads.
Example of Impulse turbine is Pelton wheel.	Examples of Reaction Turbine are Francis turbine, Kaplan and Propeller Turbine, Deriaz Turbine, Tubuler Turbine, etc.

1. A single jet impulse turbine of 10 MW capacity is to work under a head of 500 m. If the specific speed = 10, over all efficiency = 0.8 and the coefficient of velocity = 0.98, find the diameter of the jet and bucket wheel. Assume $\phi = 0.46$.

Sol: Data: $P = 10\,MW$; $n = 1$; $H = 500\,m$; $N_{s_T} = 10$; $\eta_o = 0.8$; $C_v = 0.98$.

$\phi = 0.46 = \frac{u}{V_1}$, $w = 9810\ N/m^3$. [water], $w = \rho g$

To find ① d. ② D.

i) $\eta_o = \frac{P_{shaft}}{wQH} \Rightarrow 0.8 = \frac{10 \times 10^6}{9810 \times Q \times 500}$

$Q = 2.54\ m^3/s$.

ii) Specific speed $N_{s_T} = \frac{N\sqrt{P \rightarrow kW}}{H^{5/4}} \rightarrow 10 = \frac{N\sqrt{10 \times 10^3}}{500^{5/4}} \rightarrow N = 236.14\,rpm$

iii) Velocity of the jet.

$C_v = \frac{V_{act}}{V_{theo}} = \frac{V_{act}}{\sqrt{2gH}} \rightarrow V_{act} = C_v\sqrt{2gH}$ (Fluid mechanics).

$V_1 = C_v\sqrt{2gH} = 0.98\sqrt{2 \times 9.81 \times 500} = 97.06\ m/s$.

iv) Speed ratio $\phi = \frac{u}{V_1} \rightarrow 0.46 = \frac{u}{97.06} \rightarrow u = 44.6\ m/s$.

v) $Q = n \times \frac{\pi d^2}{4} \times V_1 \rightarrow 2.54 = 1 \times \frac{\pi \times d^2}{4} \times 97.06$.

$Q = nAV_j$

$d = 0.18\,m \simeq 180\,mm$.

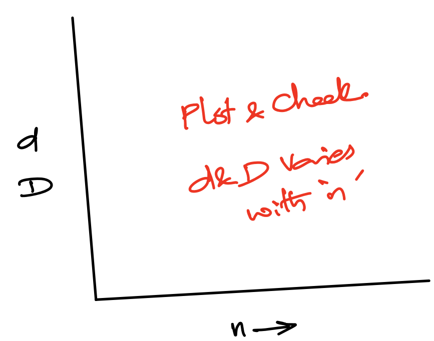
vi) $u = \frac{\pi DN}{60} \rightarrow$ $\cancel{X}$ D = Mean diameter of rotor.

$D = \frac{60u}{\pi N}$

$D = \frac{60 \times 44.6}{\pi \times 236.4}$

$D = 3.6\,m$.

H.W: n = 2, n = 4. & find d & D.



Plot & Check.

d & D varies with 'n'

3. The gross head available at a project site is 350 m of water. The penstock pipe is estimated to be 60 m long. The pipe friction factor is *f=0.0007*. The total pipe losses have to be limited to 4% of gross head. The expected power from the project is 2600 kW. The turbine speed is 600 rpm. Calculate (a) the required flow rate Q in m^3/s, (b)the pipe line diameter (c) the jet diameter d, and (d) the rotor diameter D. The speed ratio is 0.46, the nozzle velocity coefficient is 0.985 and the overall efficiency is 0.92.

Sol: Data: $H_g = 350\,m$; $L_p = 60\,m$; $f = 0.0007$; $h_f = 4\%\, H_g$, $P = 2600\,kW$.

$N = 600\,rpm$, $\phi = 0.46$, $C_V = 0.985$, $\eta_o = 0.92$.

To find i) Q. 2) d_p 3) d 4) D.

d_p = Pipe diameter

d = Jet diameter

D = Wheel diameter.

Net head $H = H_g - \text{losses} \to h_f$.

$$= H_g - 4\%\, H_g$$

$$= H_g - \frac{4}{100} H_g$$

$$H = \left(1 - \frac{4}{100}\right) H_g = 0.96\, H_g$$

$$H = 0.96 \times 350$$

$$H = 336\,m \to \text{Net head.}$$

$$h_f = \frac{4}{100} H_g$$

$$= \frac{4}{100} \times 350$$

$$h_f = 14\,m$$

i) $\eta_o = \dfrac{P_{shaft}}{w Q H} \to 0.92 = \dfrac{2600 \times 10^3}{9810 \times Q \times 336} \to Q = 0.86\, m^3/s$.

$$Q = A_1 V_1 = A_2 V_2.$$

ii) $Q = A_p V_p = A_j V_j$

$$V_p = \frac{Q}{A_p} = \frac{Q}{\frac{\pi d_p^2}{4}} \to V_p = \frac{4Q}{\pi d_p^2}.$$

$h_f = \dfrac{4 f L_p V_p^2}{2 g d_p}$ [frictional loss through pipe], $h_f = \dfrac{4fLV^2}{2gd}$.

$$h_f = \frac{4 f L_p}{2 g d_p} \cdot \left(\frac{4Q}{\pi d_p^2}\right)^2.$$

$$14 = \frac{4 \times 0.0007 \times 60}{2 \times 9.81 \times d_p} \times \left(\frac{4 \times 0.86}{\pi d_p^2}\right)^2.$$

$$d_p = 0.59\,m.$$

iii) Velocity of the jet.

$$C_v = \frac{V_{act}}{V_{theo}} \rightarrow V_j$$

$$V_j = C_v\sqrt{2gH}$$

$V_j = C_v\sqrt{2gH}$.

$= 0.985\sqrt{2\times9.81\times336}$

$V_j = 79.97$ m/s.

$$Q = n\times\frac{\pi d^2}{4}\times V_j$$

$$0.86 = 1\times\frac{\pi d^2}{4}\times 79.97$$

$d = 0.117$ m $\simeq 117$ mm.

$n = 1, 2, 4, 6 \rightarrow$ plot.

iii) $\phi = \frac{u}{V_j} \rightarrow u = \phi V_j$

$= 0.46\times 79.97$

$u = 36.79$ m/s.

$$u = \frac{\pi D N}{60} \rightarrow D = \frac{60u}{\pi N}$$

$$D = \frac{60\times 36.79}{\pi\times 600}$$

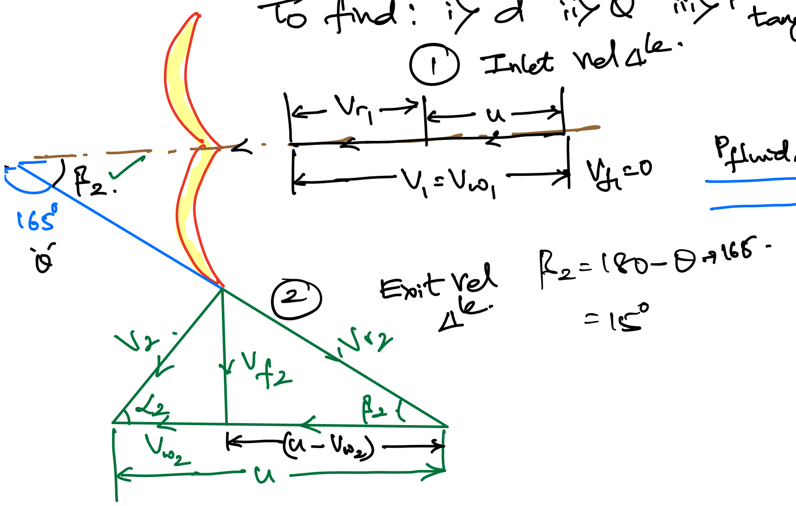
$D = 1.17$ m.

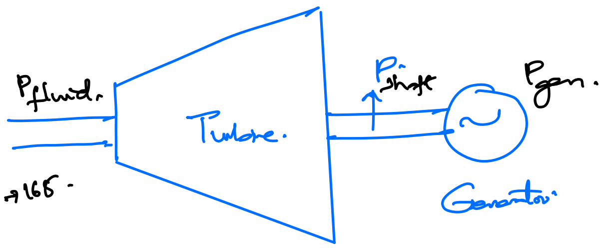
6. A double jet pelton -wheel is required to generate 7500 kW when the available head at the base of the nozzle is 400 m. The jet is deflected through 165°and the relative velocity of the jet is reduced by 15% in passing over the buckets. Determine (i) the diameter of each jet (ii) Total flow (iii) Force exerted by the jets in the tangential direction. Assume generator efficiency is 95%, η_o = 80%, speed ratio = 0.47.

Sol: Given: $n=2$; $P_{gen} = 7500$ kW ; $H = 400$ m; $\beta_2 = 180-\theta = 180-165 = 15^\circ$.

$V_{r_2} = (1-0.15)V_{r_1} = 0.85 V_{r_1}$ $\eta_{gen} = 95\%$. $\eta_o = 80\%$. $\phi = 0.47$.

To find: i) d ii) Q iii) F_{tang}





$$\eta_{gen} = \frac{P_{generator}}{P_{shaft}}.$$

$$P_{shaft} = \frac{P_{gen}}{\eta_{gen}} = \frac{7500}{0.95}$$

$P_{shaft} = 7895$ kW.

i) $$\eta_o = \frac{P_{shaft}}{w Q H}.$$

$$0.8 = \frac{7895 \times 10^3}{9810 \times Q \times 400}$$

$Q = 2.52\ m^3/s$.

ii) $V_1 = C_v\sqrt{2gH}$ Assume $C_v = 0.98$ to 0.985

$V_1 = 0.98\sqrt{2 \times 9.81 \times 400}$

$V_1 = 86.83$ m/s $= V_{w_1}$

$$Q = n \times \frac{\pi d^2}{4} \times V_1$$

$$\left\{ u = \frac{\pi D N}{60} \rightarrow D = \frac{60u}{\pi N} \right\}.$$

$$2.52 = 2 \times \frac{\pi d^2}{4} \times 86.83.$$

$d = 0.136$ m.

iii) $\phi = \dfrac{u}{V_1} \rightarrow u = \phi V_1 = 0.47 \times 86.83$.

$u = 40.81$ m/s.

From the inlet velocity triangle.

$V_{r_1} = V_1 - u = 86.83 - 40.81 = 46.02$ m/s.

$V_{r_2} = 0.85\, V_{r_1} = 0.85 \times 46.02 = 39.12$ m/s.

From the exit velocity triangle

$\sin\beta_2 = \dfrac{V_{f_2}}{V_{r_2}}$

$\sin 15 = \dfrac{V_{f_2}}{39.12}$

$V_{f_2} = 10.12$ m/s.

$\cos\beta_2 = \dfrac{(u - V_{w_2})}{V_{r_2}}$

$\cos 15 = \dfrac{(40.81 - V_{w_2})}{39.12}$

$V_{w_2} = 3.03$ m/s.

Tangential force.

$F_t = \dot{m}(V_{w_1} - V_{w_2})$ $\qquad \dot{m} = \rho Q.$

$= 1000 \times 2.52 (86.83 - 3.03)$

$F_t = 211.18$ kN. → Force applied tangentially on the turbine.

8. A Francis turbine is to be designed for the flow rate of $2m^3/s$ available at a project site at a net head of 10 m of water. The expected overall efficiency is 80%. The speed coefficient and the flow coefficient can be assumed as 0.8 and 0.6 respectively. The hydraulic losses in the turbine are 15% of avaialble energy. Design the turbine rotor, with the salient dimensions and angles, to run at 300 rpm. The water leaves the rotor without any whirl component.

Sol: Given: $Q = 2 m^3/s$; $H = 10m$; $\eta_o = 0.8$; $\phi = 0.8$ $\varphi = 0.6$ $\eta_{hyd} = 0.85$

$N = 300$ rpm; $\alpha_2 = 90°$; $V_{w_2} = 0$.

To find: i) u_1 & u_2 ii) V_f; iii) d_1 & d_2 iv) β_1 & β_2. v) P.

i) Speed ratio $\phi = \frac{u_1}{V_1} \rightarrow u_1 = \phi\sqrt{2gH}$ $[\because C_v = 1]$

$$u_1 = 0.8\sqrt{2\times9.81\times10}$$ $V = C_v\sqrt{2gH}$

$$u_1 = 11.2 \ m/s.$$

ii) Flow coefficient $\varphi = \frac{V_{f_1}}{V_1} \rightarrow V_{f_1} = \varphi\sqrt{2gH}$

$$V_{f_1} = 0.6\sqrt{2\times9.81\times10}$$

$$V_{f_1} = 8.4 \ m/s.$$

iii) Overall efficiency $\eta_o = \frac{P_{shaft}}{wQH} \rightarrow P = \eta_o wQH$ P = Shaft power.

$$P = 0.8\times9810\times2\times10$$

$$P = 157 \ KW.$$

iv) Specific speed $N_{S_T} = \frac{N\sqrt{P}}{H^{5/4}} = \frac{300\sqrt{157}}{10^{5/4}}$ KW.

$$N_{S_T} = 211.4 \ rpm.$$

v) $u_1 = \frac{\pi d_1 N}{60} \rightarrow d_1 = \frac{60 u_1}{\pi N}$

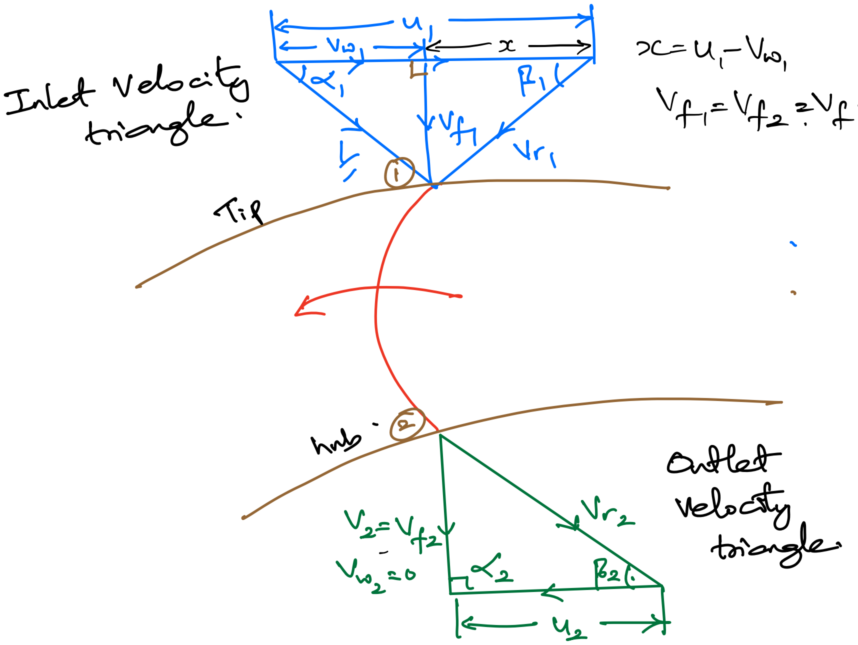
$$d_1 = \frac{60\times11.2}{\pi\times300}$$

$$d_1 = 0.7 m.$$

$$V_1 = \sqrt{2gH} = \sqrt{2\times9.81\times10} = 14 \ m/s.$$

As per design $\frac{d_2}{d_1} = 0.6$ [Assume].

$\therefore d_2 = 0.6 d_1 = 0.6 \times 0.7 = 0.42\,m.$



$x = u_1 - V_{w_1}$

$V_{f_1} = V_{f_2} = V_f = 8.4\,m/s.$

$V_{w_1} = V_1 \cos\alpha_1$

$V_{f_1} = V_1 \sin\alpha_1$

$u_1 = \frac{\pi d_1 N}{60}$

$u_2 = \frac{\pi d_2 N}{60}$

$u_1 > u_2$

$d_1 > d_2$

vi) Hydraulic efficiency.

$$\eta_{hyd} = \frac{V_{w_1}u_1 - \cancel{V_{w_2}u_2}^{\,0}}{gH} = \frac{V_{w_1}u_1}{gH}$$

$$0.85 = \frac{V_{w_1} \times 11.2}{9.81 \times 10}$$

$$V_{w_1} = 7.45\,m/s.$$

From the inlet velocity triangle.

$\tan\alpha_1 = \frac{V_{f_1}}{V_{w_1}}$

$\tan\alpha_1 = \frac{8.4}{7.45}$

$\alpha_1 = \tan^{-1}\left[\frac{8.4}{7.45}\right]$

$\alpha_1 = 48.4^\circ.$

$\tan\beta_1 = \frac{V_{f_1}}{u_1 - V_{w_1}}$

$\beta_1 = \tan^{-1}\left[\frac{V_{f_1}}{u_1 - V_{w_1}}\right]$

$\beta_1 = \tan^{-1}\left[\frac{8.4}{11.2 - 7.45}\right]$

$\beta_1 = 65.86^\circ.$

From the exit velocity triangle.

$$\tan\beta_2 = \frac{V_{f2}}{u_2}.$$

$$\beta_2 = \tan^{-1}\left[\frac{V_{f2}}{u_2}\right]$$

$$\beta_2 = \tan^{-1}\left[\frac{8.4}{6.72}\right]$$

$$\beta_2 = 51^\circ.$$

$\because u_2 = 0.6u_1 \quad \left(\because \frac{d_2}{d_1} = 0.6\right)$

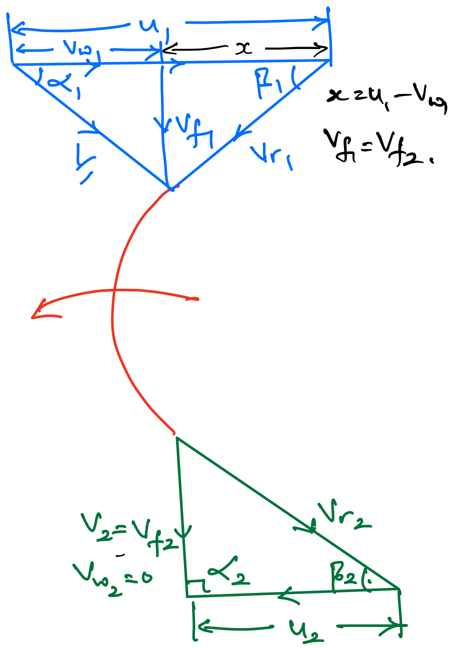
$= 0.6 \times 11.2$

$u_2 = 6.72\ m/s.$

$\because V_{f1} = V_{f2}.$

11. The head available at the inlet to a Francis turbine is 28 m. The output of the turbine is 1000 kW at an overall efficiency of 0.88, when there is no draft tube attached to the exit of the turbine. The level of the turbine is 2.5 m above the tailrace. The flow coefficient of the turbine is 0.3. Assess the effect of the installing (a) A straight cylindrical draft tube and (b) a draft tube with the half-cone angle of 5^o at the exit of the turbine. The efficiency of the draft tube may be taken as 95% in either case.

Sol: Given: $H = 28\,m$; $P = 1000\,kW$; $\eta_o = 0.88$; $\varphi = 0.3$;



i) $\eta_o = \dfrac{P}{wQH} \rightarrow Q = \dfrac{P}{\eta_o wH}$

$$Q = \frac{1000 \times 10^3}{0.88 \times 9810 \times 28}.$$

$$Q = 4.14\ m^3/s.$$

ii) $\varphi = \dfrac{V_f}{V_1} \rightarrow V_f = \varphi\sqrt{2gH}$

$$= 0.3\sqrt{2 \times 9.81 \times 28}.$$

$$V_f = 7.1\,m/s = V_2.$$

iii) $Q = A_1V_{f1} = A_2V_{f2}.$

$A_2 = \dfrac{\pi d_2^2}{4}.$

$V_{f1} = V_{f2} = V_f.$

$$Q = \frac{\pi d_2^2}{4} \times V_f$$

$$4.14 = \frac{\pi \times d_2^2}{4} \times 7.1$$

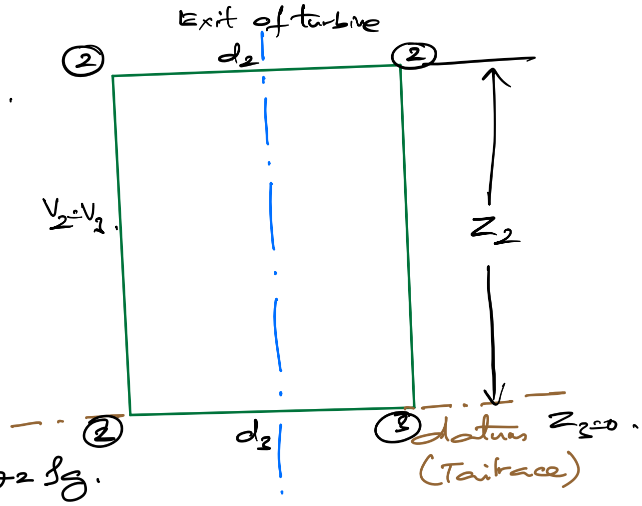
$$d_2 = 0.865\,m.$$

$\dfrac{P_2}{w}$ = Pressure head at the exit of the turbine.

a) Cylindrical draft tube

Applying the Bernoulli's equation.

$$\frac{P_2}{w} = \frac{P_a}{w} - \underbrace{h}_{z_2} - \underbrace{(1-k)}_{\eta_{draft\,tube}}\cancelto{0}{\left(\frac{V_2^2 - V_3^2}{2g}\right)}$$



$$\frac{P_2}{w} = \frac{P_a}{w} - Z_2.$$

$$= 10.3 - 2.5$$

$$\frac{P_2}{w} = 7.8 \text{ m} \quad [\text{Head}]$$

$w = \rho g.$

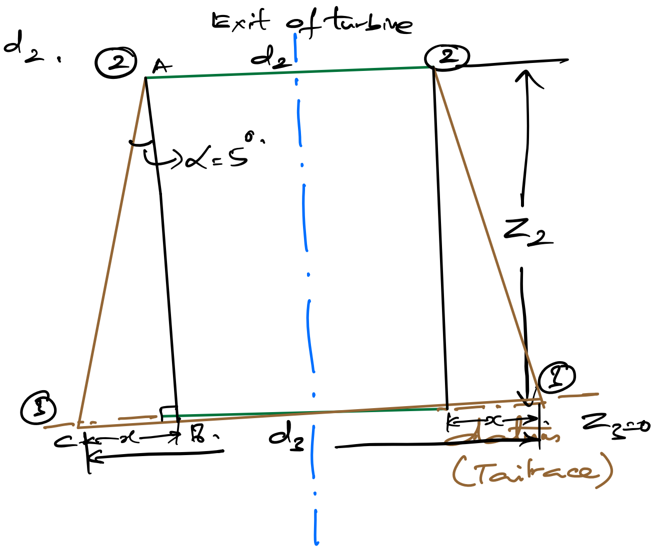
$w = 9810 \text{ N/m}^3.$

$\rho = 1000 \text{ kg/m}^3$

$g = 9.81 \text{ m/s}^2$

$$P_2 = 76.52 \text{ kpa.} \quad [\text{pressure}].$$

b) Conical draft tube: $d_3 > d_2$.



$$d_3 = d_2 + 2x$$

From the Δ^{le} ABC.

$$\tan\alpha = \frac{x}{Z_2}$$

$$x = Z_2 \tan\alpha.$$

$$x = 2.5 \times \tan 5$$

$$\therefore d_3 = d_2 + 2 \times 2.5 \times \tan 5$$

$$d_3 = 0.865 + 2 \times 2.5 \times \tan 5$$

$$d_3 = 1.3 \text{ m}.$$

$$Q = A_2 V_2 = A_3 V_3.$$

$$V_3 = \frac{Q}{\frac{\pi d_3^2}{4}} = \frac{4.14 \times 4}{\pi \times 1.3^2} = 3.1 \text{ m/s}.$$

Applying the Bernoulli's equation.

$$\frac{P_2}{w} = \frac{P_a}{w} - h - (1-k)\left(\frac{V_2^2 - V_3^2}{2g}\right) \qquad h = Z_2 = 2.5$$

$$= 10.3 - 2.5 - 0.95\left(\frac{7.1^2 - 3.1^2}{2 \times 9.81}\right)$$

saving.

$$\frac{P_2}{w} = 5.8\,m. \text{ [Head]}$$

$$P_2 = 56.9 \text{ Kpa [pressure]}.$$

Comment:

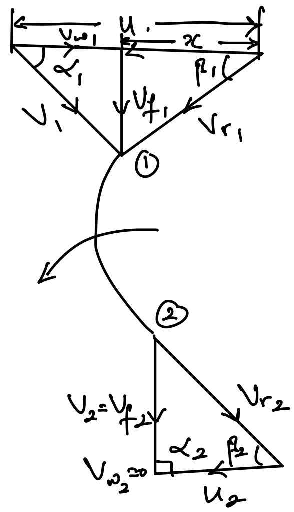
i) Cylindrical draftube: 2.5 m of head is saved.

ii) Conical draftube: 10.3 - 5.8 = 4.5 m of head is saved.

12. In a Francis turbine, the discharge is radial. The blade speed at inlet = 25 m/s. At the inlet tangential component of velocity = 18 m/s. The radial velocity of flow is constant and equal to 2.5 m/s. Water flows at the rate of $0.8m^3/s$. The utilization factor is 0.82. Find: i) Euler's head ii) Power developed iii) Inlet blade angle iv) Degree of reaction(R). Draw the velocity triangles.

Sol: Given: $u_1 = 25\,m/s$; $V_{w_1} = 18\,m/s$; $V_f = 2.5\,m/s$; $Q = 0.8\,m^3/s$; $\epsilon = 0.82$.

To find i) H_E ii) P iii) β_1 iv) R. [$\because V_{w_2} = 0$]



i) Euler head.

$\because$ Euler's eqn $= V_{w_1}u_1 - V_{w_2}u_2$

$$H_E = \frac{V_{w_1}u_1 - \cancel{V_{w_2}}^{\,0}u_2}{g} = \frac{V_{w_1}u_1}{g}.$$

$$H_E = \frac{18 \times 25}{9.81} = 45.87\,m.$$

iii) $P = wQH_E$ $\quad [\because \eta_o = 100\%]$ ✓

$= 9810 \times 0.8 \times 45.87$

$P = 359.98 \simeq 360 \text{ kW}$ — Theoritical power.

$$\eta_o = \frac{P}{wQH}$$

Note: $\eta_h = 0.9$. $\quad \eta_h = \frac{H}{H_E} \rightarrow H = \eta_h H_E = 0.9 \times 45.87 = 41.28 \text{ m}$.

$P = wQH$

$= 9810 \times 0.8 \times 41.28$

$P = 324 \text{ kW}$

From the outlet velocity triangle.

$V_{f_2} = V_2 = 2.5 \text{ m/s}$.

Consider the inlet velocity triangle.

$V_1^2 = V_{f_1}^2 + V_{w_1}^2 \rightarrow V_1 = \sqrt{(V_{f_1}^2 + V_{w_1}^2)}$

$V_1 = \sqrt{2.5^2 + 18^2}$

$V_1 = 18.17 \text{ m/s}$.

$\tan \beta_1 = \frac{V_{f_1}}{u - V_{w_1}}$

$\beta_1 = \tan^{-1}\left[\frac{2.5}{25 - 18}\right]$

$\beta_1 = 19.84°$

iv) Degree of reaction.

$$R = \frac{H_E - \frac{(V_1^2 - V_2^2)}{2g}}{H_E} = \frac{\text{Static Energy}}{\text{Total Energy}}$$

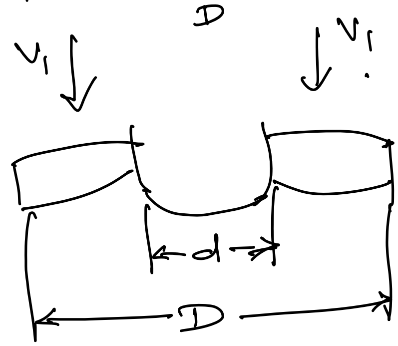
$$R = \frac{45.87 - \left(\frac{18.17^2 - 2.5^2}{2 \times 9.81}\right)}{45.87}$$

$R = 0.64$

14. A kaplan turbine produces 80,000 HP (58,800 kW) under a head of 25m which has an overall efficiency of 90%. Taking the value of speed ratio $\phi = 1.6$, flow ratio $\psi = 0.35$ and the hub diameter = 0.35 times the outer diameter. Find the diameter and the speed of the turbine.

<u>Sol:</u> Given: $P = 58{,}800\,kW$; $H = 25m$; $\eta_o = 0.9$; $\phi = 1.6$; $\psi = 0.35$; $\frac{d}{D} = 0.35$

To find: i) D ii) N.



Overall efficiency $\eta_o = \dfrac{P}{wQH}$

$$Q = \frac{P}{\eta_o w H}.$$

$$Q = \frac{58800}{0.9 \times 9810 \times 25}$$

$$Q = 266.4\ m^3/s$$

V_1 = absolute velocity

$$\psi = \frac{V_f}{V_1} \rightarrow V_f = \psi V_1 = \psi\sqrt{2gH}.$$

$V_1 = C_v\sqrt{2gH}$.

$\because C_v = 0.98 - 0.985$

$$V_f = 0.35\sqrt{2 \times 9.81 \times 25}$$

$$V_f = 7.75\ m/s.$$

Discharge; $Q = A_f V_f$.

$A_f = \frac{\pi}{4}(D^2 - d^2)$

$$Q = \frac{\pi}{4}(D^2 - d^2) \times V_f.$$

$$Q = \frac{\pi}{4}\left(D^2 - (0.35D)^2\right) \times V_f$$

$\because d = 0.35D$.

$$266.4 = \frac{\pi}{4} \times D^2(1 - 0.35^2) \times 7.75$$

$D = 5.91\ m.$ ✓

$$\phi = \frac{u}{V_1} \rightarrow u = \phi V_1 = \phi\sqrt{2gH}$$

$d = 0.35 \times 5.91$

$u = 1.6\sqrt{2 \times 9.81 \times 25}$

$d = 2.07\ m.$ ✓

$u = 35.44\ m/s$

$$u = \frac{\pi D N}{60} \rightarrow N = \frac{60u}{\pi D}$$

$$N = \frac{60 \times 35.44}{\pi \times 5.91} \rightarrow N = 114.5\ rpm.$$

(c) The mean diameter of a Pelton wheel is 2.6 m. The bucket outlet angle is 20°. The net head available at the nozzle is 500 m of water, if the available flow is 3.5 m^3/s. Calculate the following

(i) Speed of the rotor

(ii) Theoretical power

(iii) Specific speed. (CO3, PO2, L3)

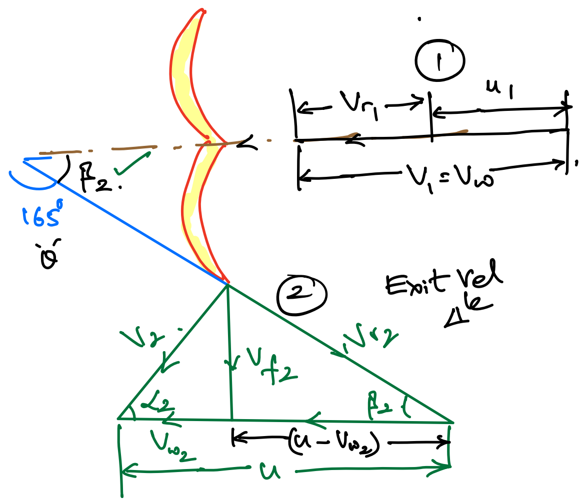
Sol: Given: $d = 2.6\,m$; $\beta_2 = 20°$; $H = 500\,m$; $Q = 3.5\,m^3/s$.

To find: i) N ii) P iii) N_{ST} Assume $\phi = 0.46$. $C_v = 0.98$

ii) Theoritical power.

$w = 9810\,N/m^3$ for water

$$P = wQH$$
$$= 9810 \times 3.5 \times 500$$
$$P = 17.16\,MW.$$



i) $$V_1 = C_v\sqrt{2gH}$$
$$= 0.98\sqrt{2 \times 9.81 \times 500}$$
$$V_1 = 97.06\,m/s.$$

$$0.46 = \frac{u}{V_1}$$
$$\therefore u = 0.46 \times V_1$$
$$= 0.46 \times 97.06$$
$$u = 44.65\,m/s.$$

$$\therefore u = \frac{\pi d N}{60} \rightarrow N = \frac{60u}{\pi d} = \frac{60 \times 44.65}{\pi \times 2.6}$$
$$N = 327.98\,rpm.$$

iii) $$N_{ST} = \frac{N\sqrt{P}}{H^{5/4}}$$
$$= \frac{327.98\sqrt{17.16 \times 10^3}}{500^{5/4}}$$ (has to be in kW)

$$N_{ST} = 18.17\,rpm.$$