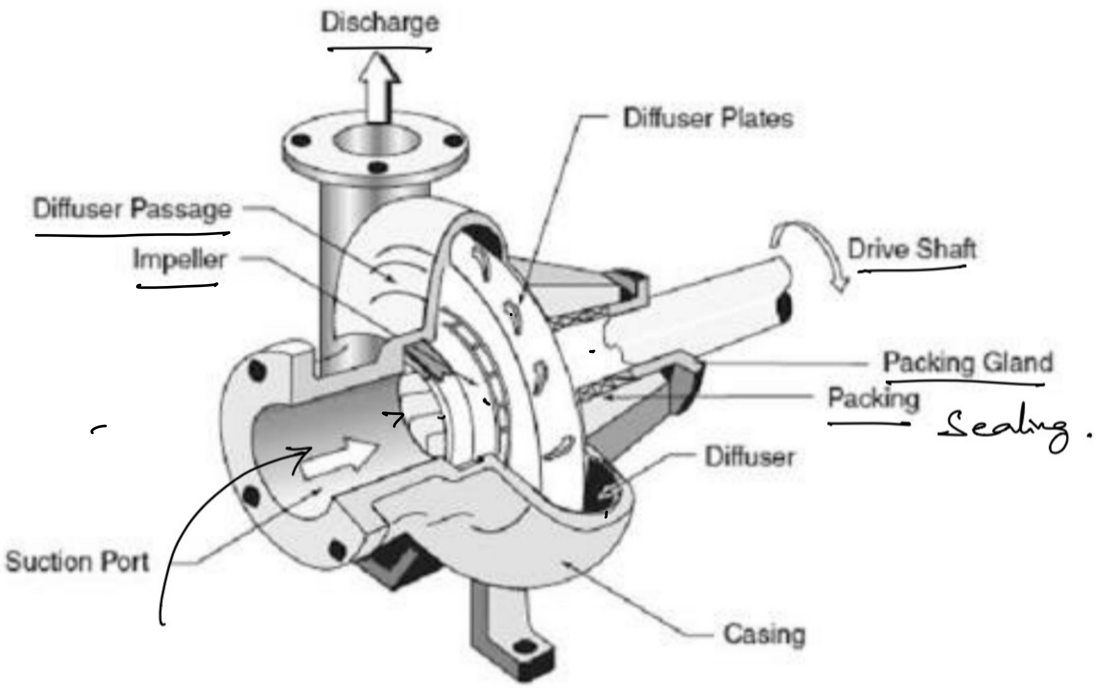
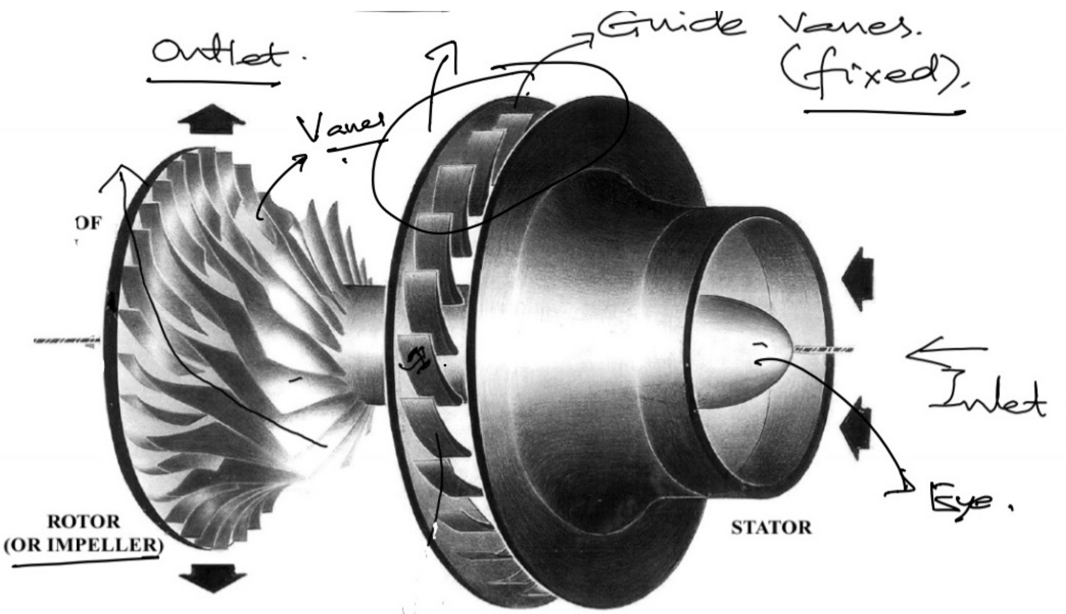
UNIT - 2

Centrifugal Compressors: Schematic diagram, Expression for overall pressure ratio; Velocity triangles; Slip factor, power input factor; Compressibility effect – need for pre-whirl vanes; Effect of Blade discharge angle on energy transfer and head coefficient, Surging and choking in centrifugal compressors.





Classification:

i) Compressible fluids $\rightarrow$ Fans, Blowers, Compressor.

2) Incompressible fluids $\rightarrow$ Water/oil. } Pump.

i) a) Fans $\rightarrow$ Pressure ratio (1.02 to 1.07) $\frac{P_2}{P_1}$

b) Blower $\rightarrow$ Pressure ratio (1.1 to 2)

c) Compressor $\rightarrow$ Pressure ratio (more than 2)

i) Single stage. $\rightarrow$ Pressure ration ($2 < Pr < 10$)

ii) Multi stage $\rightarrow$ Pressure ration (more than 10)

i) Radial flow Compressor/ Centrifugal Compressor.

$\rightarrow$ Flow is in radial direction.

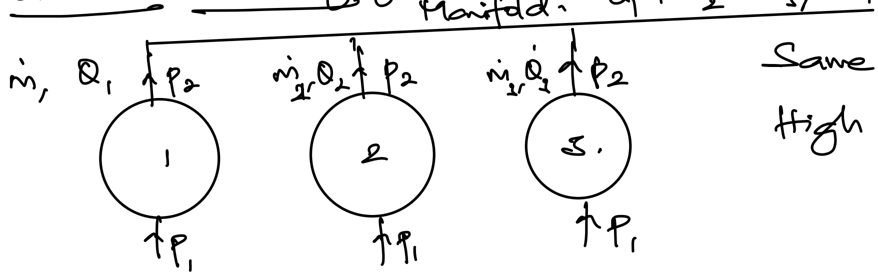
$\rightarrow$ Lesser mass flow rate.

ii) Axial flow compressor.

$\rightarrow$ Flow is in axial direction.

$\rightarrow$ More mass flowrate.

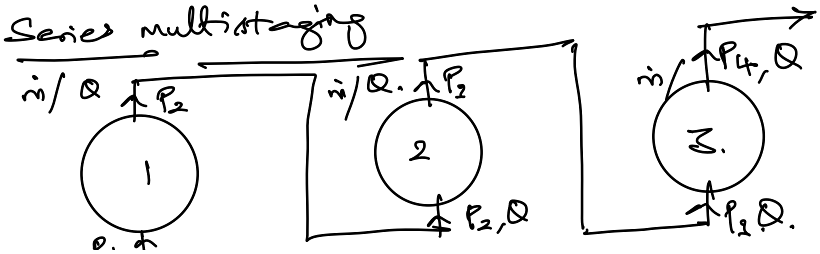
Parallel multistaging



Same exit pressure.

High discharge.

Series multistaging



High pressure

Same discharge.

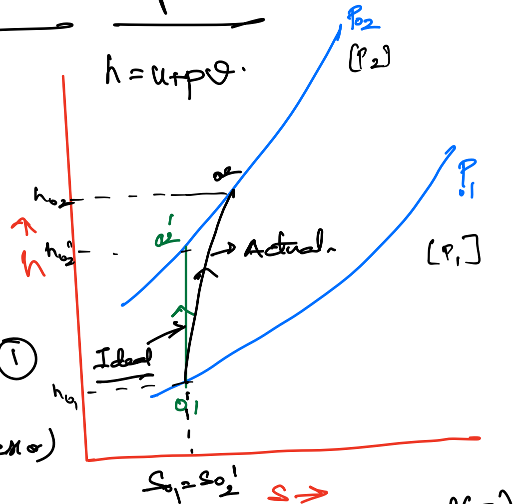
$P_4 > P_1$.

Expression for pressure ratio of a compressor.

Steady flow energy equation.

$h = u + p\vartheta$

$$\cancel{\dot{Q}} + \dot{m}\left[h_1 + \frac{\vec{V}_1^2}{2} + \cancel{gZ_1}\right] = \dot{m}\left[h_2 + \frac{\vec{V}_2^2}{2} + \cancel{gZ_2}\right] \pm \dot{W}$$



Let us assume $\dot{Q} = 0$ $\quad Z_1 = Z_2$.

$$\dot{m}\Big[\underbrace{h_1 + \frac{\vec{V}_1^2}{2}}_{h_{01}}\Big] = \dot{m}\Big[\underbrace{h_2 + \frac{\vec{V}_2^2}{2}}_{h_{02}}\Big] \pm \dot{W} \rightarrow (1)$$

$-W =$ Work absorbing m/c (Compressor)

$+W =$ Work producing m/c (Turbine).

$h = f(T)$

$h_0 = h + \frac{V^2}{2}$ $\rightarrow$ Specific stagnation enthalpy.

$P_{01} = P_1 + \frac{V_1^2}{2g}$

$\dot{m}\, h_{01} = \dot{m}\, h_{02} - W$.

$P_{02} = P_2 + \frac{V_2^2}{2g}$

$W = \dot{m}\left[h_{02} - h_{01}\right]$

$W = H_{02} - H_{01}$ $\rightarrow (2)$ Total enthalpy.

(X) $h = C_p T$ & $u = C_v T$ $\quad \because h = f(T)$ & $u = f(T)$

$W = \dot{m} C_p (T_{02} - T_{01})$ $\quad T_{02}$ & T_{01} $\rightarrow$ Absolute Temperatures.

Efficiency for a compressor:

$$\eta_c = \eta_{t-t} = \frac{h_{02}' - h_{01}}{h_{02} - h_{01}}$$

$$\eta_c = \frac{\text{Ideal work}}{\text{Actual work}}$$

$$= \frac{C_p(T_{02}' - T_{01})}{C_p(T_{02} - T_{01})}$$

$$\eta_c = \eta_{t-t} = \frac{T_{02}' - T_{01}}{T_{02} - T_{01}} \rightarrow (3)$$

$$T_{o_2} - T_{o_1} = \frac{T_{o_2}' - T_{o_1}}{\eta_{t-t}} \rightarrow (4)$$

Substitute for temperature change from (4) in 'W'

$$W = C_p \frac{(T_{o_2}' - T_{o_1})}{\eta_{t-t}}.$$

$$\frac{W}{C_p} = \frac{(T_{o_2}' - T_{o_1})}{\eta_{t-t}}. \rightarrow (5)$$

For an isentropic compression:

$$\frac{T_{o_2}'}{T_{o_1}} = \left(\frac{P_{o_2}}{P_{o_1}}\right)^{\frac{\gamma-1}{\gamma}}. \qquad \left[\because \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right]$$

$$\frac{P_{o_2}}{P_{o_1}} = \left(\frac{T_{o_2}'}{T_{o_1}}\right)^{\gamma/\gamma-1} \longrightarrow (6)$$

$$= \left(\frac{T_{o_2}' - T_{o_1} + T_{o_1}}{T_{o_1}}\right)^{\gamma/\gamma-1} \qquad \left[\because \text{Add \& subtract } 'T_{o_1}' \text{ in the numerator}\right]$$

$$\frac{P_{o_2}}{P_{o_1}} = \left(\frac{\frac{W}{C_p} \cdot \eta_{t-t} + T_{o_1}}{T_{o_1}}\right)^{\gamma/\gamma-1}$$

$$\left.\begin{aligned} \frac{P_{o_2}}{P_{o_1}} &= \left(\frac{W}{C_p} \cdot \frac{\eta_{t-t}}{T_{o_1}} + 1\right)^{\gamma/\gamma-1} \\ \text{(OR)} \quad \frac{P_{o_2}}{P_{o_1}} &= \left(1 + \frac{W}{C_p} \frac{\eta_{t-t}}{T_{o_1}}\right)^{\gamma/\gamma-1} \end{aligned}\right\} (7)$$

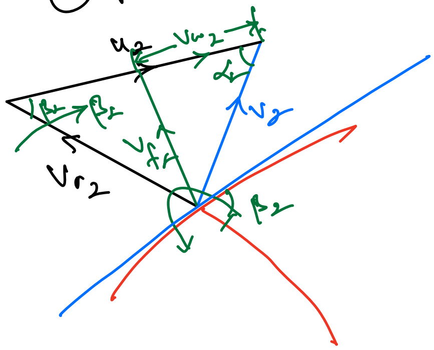
$$\frac{P_{o_2}}{P_{o_1}} = \left[1 + \frac{V_{w_2}u_2 - V_{w_1}u_1}{C_p} \frac{\eta_{t-t}}{T_{o_1}}\right]^{\gamma/\gamma-1}$$

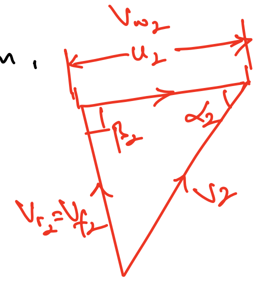
From the Euler's equation $W = V_{w_2}u_2 - V_{w_1}u_1$

Special cases: (a) For no whirl condition @ inlet $V_{w_1} = 0$. $W = V_{w_2}u_2$.

$$\frac{P_{o2}}{P_{o1}} = \left(\frac{V_{w_2} u_2}{C_p} \cdot \frac{\eta_{t-t}}{T_{o1}} + 1 \right)^{\gamma/\gamma-1} \longrightarrow (8)$$

(b) Radial exit condition.





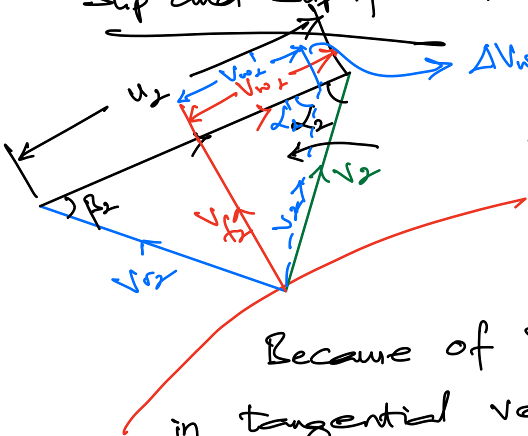
$\beta_2 = 90^\circ$

$V_{w_2} = u_2$.

$$W = V_{w_2} u_2 = V_{w_2}^2 \text{ (or) } u_2^2.$$

$$\frac{P_{o2}}{P_{o1}} = \left(\frac{u_2^2}{C_p} \cdot \frac{\eta_{t-t}}{T_{o1}} + 1 \right)^{\gamma/\gamma-1} \longrightarrow (9).$$

Slip and Slip factor:



ΔV_w. Slip.

a) $\text{Slip} = V_{w_2} - V_{w_2}' = \Delta V_{w_2}$.

$W = V_{w_2} u_2$, $\quad W' = V_{w_2}' u_2$.

Because of the slip there is a difference in tangential velocities of the ideal & actual at the exit [@ 2]

b) Slip factor 'σ' $= \dfrac{\text{Actual whirl velocity @ exit}}{\text{Ideal whirl velocity @ exit}}$

$$\sigma = \frac{V_{w_2}'}{V_{w_2}} < 1 \qquad \because V_{w_2}' < V_{w_2}$$

c) Slip coefficient 'ϕ' $= \dfrac{\text{Slip}}{V_{w_2}} = \dfrac{V_{w_2} - V_{w_2}'}{V_{w_2}} = \dfrac{\Delta V_{w_2}}{V_{w_2}}$.

If slip is considered, then.

$$\frac{P_{02}}{P_{01}} = \left(\frac{V_{w_2}' u_2}{C_p} \cdot \frac{\eta_{t-t}}{T_{01}} + 1\right)^{\gamma/\gamma-1} \qquad \because W' = V_{w_2}' u_2$$

$$\sigma = \frac{V_{w_2}'}{V_{w_2}} \rightarrow V_{w_2}' = \sigma V_{w_2}.$$

$$\therefore \quad \frac{P_{02}}{P_{01}} = \left(\frac{\sigma V_{w_2} u_2}{C_p} \cdot \frac{\eta_{t-t}}{T_{01}} + 1\right)^{\gamma/\gamma-1} \rightarrow (9)$$

For static condition.

$$\frac{P_2}{P_1} = \left(\frac{\sigma V_{w_2} u_2}{C_p} \cdot \frac{\eta_{s-s}}{T_1} + 1\right)^{\gamma/\gamma-1} \rightarrow (10)$$

Special case:

For a radial exit condition $\beta_2 = 90°$. $V_{w_2} = u_2$.

$$\left.\begin{aligned} \frac{P_{02}}{P_{01}} &= \left(\frac{\sigma u_2^2}{C_p} \cdot \frac{\eta_{t-t}}{T_{01}} + 1\right)^{\gamma/\gamma-1} \\ \frac{P_2}{P_1} &= \left(\frac{\sigma u_2^2}{C_p} \cdot \frac{\eta_{s-s}}{T_1} + 1\right)^{\gamma/\gamma-1} \end{aligned}\right\} (11)$$

Power input factor (OR) Work done factor: (Ω)

$$\Omega = \frac{\text{Actual work done}}{\text{Ideal workdone.}} = \frac{W_a}{W_i} \qquad W_i = V_{w_2}' u_2 \rightarrow \text{Ideal with slip.}$$

$$W_a = \Omega W_i \rightarrow W_i = V_{w_2} u_2 \text{ (Euler equation)} \rightarrow \text{Ideal.}$$

Both slip factor & power factor will reduce ideal work

$$V_{w_1} = 0 \qquad \frac{P_{02}}{P_{01}} = \left(\frac{\sigma \Omega V_{w_2} u_2}{C_p} \cdot \frac{\eta_{t-t}}{T_{01}} + 1\right)^{\gamma/\gamma-1} \rightarrow (12)$$

For a radial exit condition $V_{w2} = u_2$ & $\beta_2 = 90^\circ$

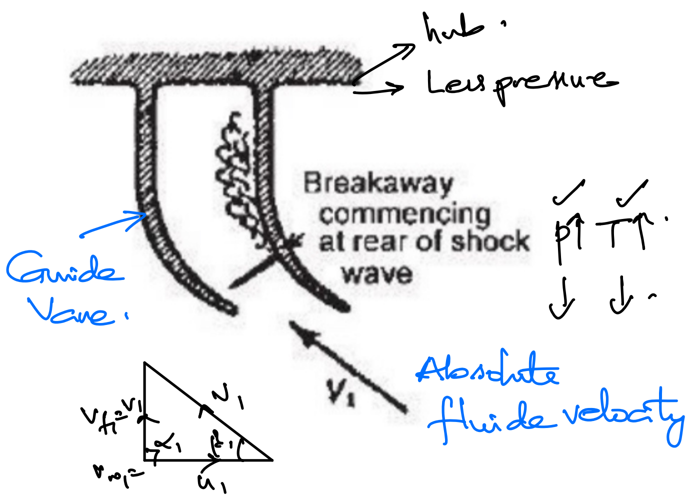
$$\frac{P_{o2}}{P_{o1}} = \left(\frac{\sigma \Omega u_2^2}{C_p} \frac{\eta_{t-t}}{T_{o1}} + 1\right)^{\gamma/\gamma-1} \rightarrow (13)$$

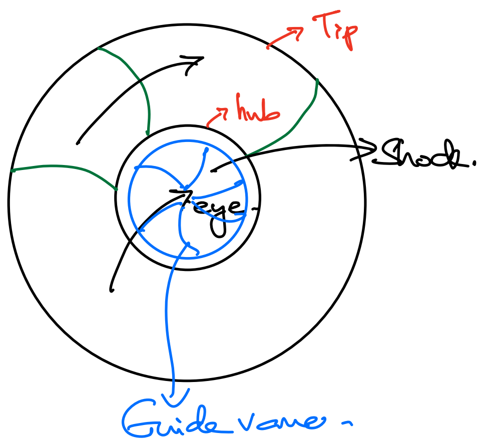
For static conditions.

$$\frac{p_2}{p_1} = \left(\frac{\sigma \Omega V_{w2} u_2}{C_p} \frac{\eta_{s-s}}{T_1} + 1\right)^{\gamma/\gamma-1} \rightarrow (14)$$

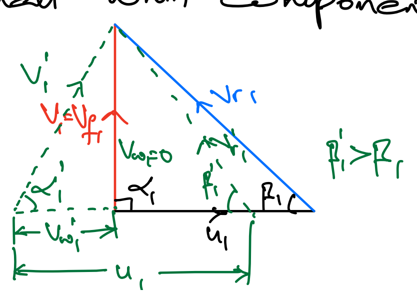
$$\frac{p_2}{p_1} = \left(\frac{\sigma \Omega u_2^2}{C_p} \frac{\eta_{s-s}}{T_1} + 1\right)^{\gamma/\gamma-1} \rightarrow (15)$$

Compressibility effect (or) Need for pre-whirling.





Prewhirling is produced by giving the incoming fluid.
a small whirl component using inlet guid vanes.



$u_1 = \frac{\pi d_1 N}{60}$ d_1 = hub diameter

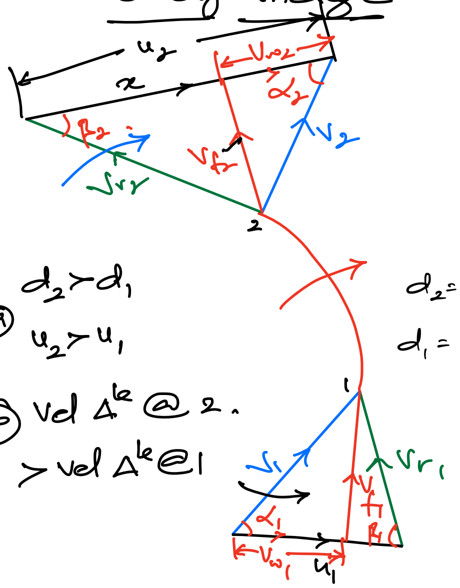
u_1 = hub speed.

$W = V_{w2} u_2 - V_{w1} u_1$

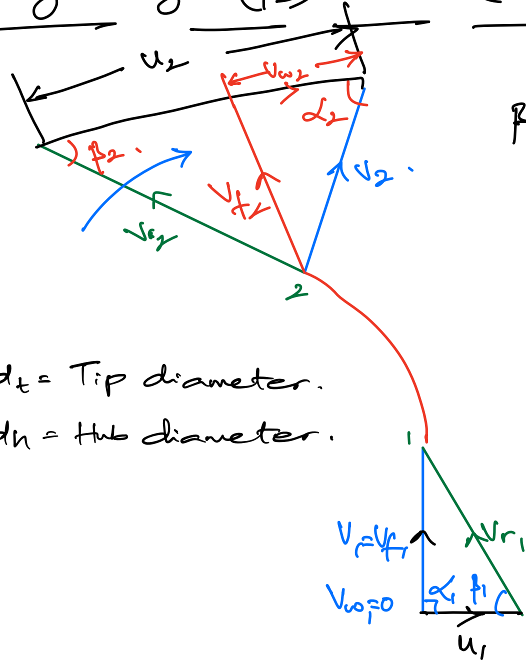
Fluid prewhirl in the direction of the impeller rotation will reduce the compressor work at a given rpm (speed).

Effect of blade discharge angle (β_2) on 'E' (or) 'H'

Velocity triangle



Pre-whirl condition.



No prewhirl condition

$\beta_2 < 90°$

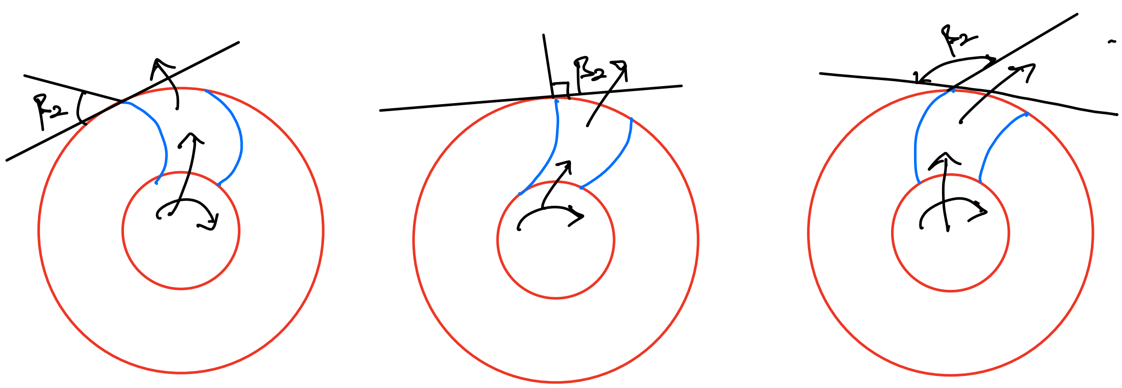
(a) $d_2 > d_1$
$u_2 > u_1$

(b) Vel Δ^{le} @ 2 $>$ vel Δ^{le} @ 1

$d_2 = d_t =$ Tip diameter.
$d_1 = d_h =$ Hub diameter.

Variation of blade angle 'β_2'

$A_f = \frac{\pi}{4}(d_t^2 - d_h^2)$



$\beta_2 < 90°$ Backward curved.

$\beta_2 = 90°$ Radial.

$\beta_2 > 90°$ Forward curved.

Workdone from the Euler equation is given by.

a) $W = V_{w_2}u_2 - V_{w_1}u_1$ $\quad (\because V_{w_1} = 0)$ & $V_{w_2} > V_{w_1}$

b) $W = V_{w_2}u_2$

c) $H_E = \dfrac{V_{w_2}u_2}{g}$ $\rightarrow$ In term of head developed.

From the velocity triangle at the exit. @ 2.

$$\tan\beta_2 = \frac{V_{f_2}}{(u_2 - V_{w_2})} \quad (OR) \quad \cot\beta_2 = \frac{u_2 - V_{w_2}}{V_{f_2}} \qquad \left[\because \cot\theta = \frac{1}{\tan\theta}\right]$$

$$V_{f_2}\cot\beta_2 = u_2 - V_{w_2}.$$

$$V_{w_2} = u_2 - V_{f_2}\cot\beta_2 \rightarrow (1)$$

Discharge $Q = A_1V_1 = A_2V_2$. $\rightarrow$ Velocities & Area.

V_1 & V_2 $\rightarrow$ Flow velocities. $(V_{f_1}$ & $V_{f_2})$

$$Q = A_1V_{f_1} = A_2V_{f_2}$$

$$V_{f_2} = \frac{Q}{A_2} \rightarrow (2)$$

Substitute for V_{w_2} and V_{f_2} in workdone equation

$$W = (u_2 - V_{f_2}\cot\beta_2)u_2. \qquad u_2 = \frac{\pi d_2 N}{60}$$

$$W = u_2^2 - u_2V_{f_2}\cot\beta_2.$$

$$W = u_2^2 - u_2 \cdot \frac{Q}{A_2}\cot\beta_2. \rightarrow (3).$$

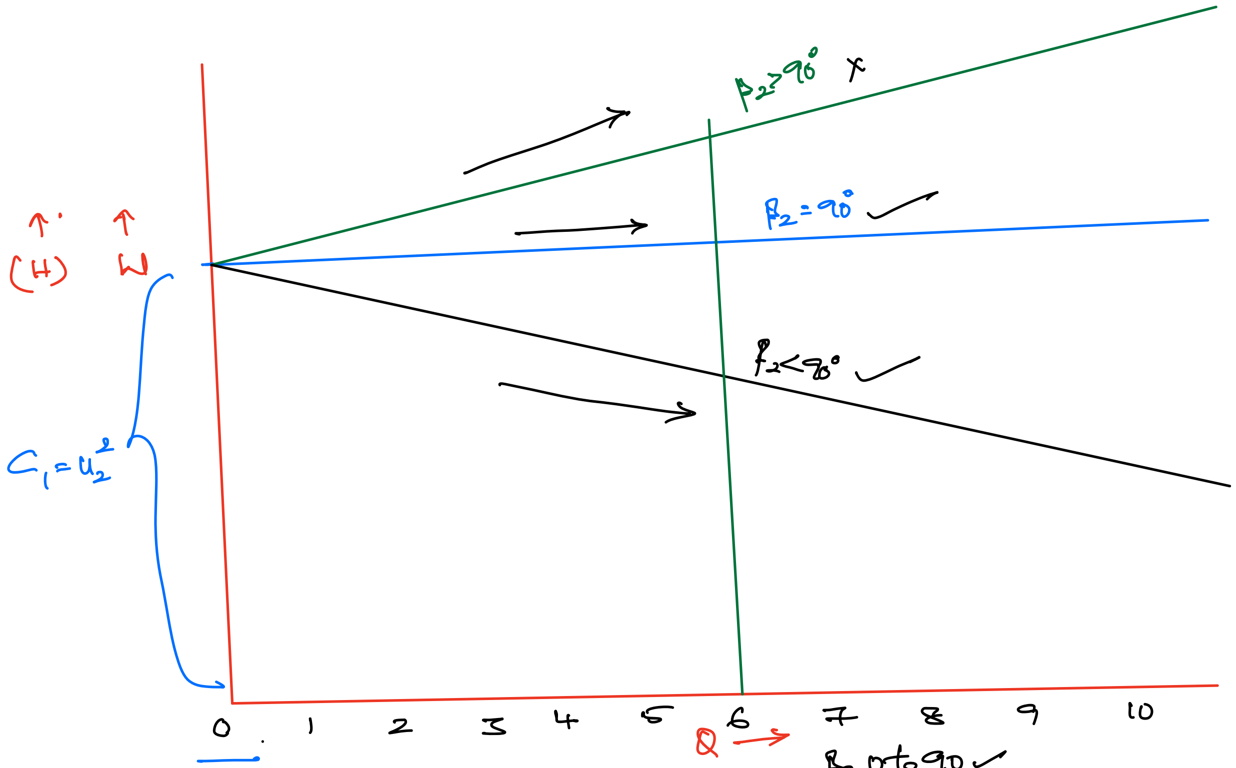
At any given diameter 'd_2', speed 'N' and 'β_2'

Let $C_1 = u_2^2$, $\quad C_2 = \dfrac{u_2\cot\beta_2}{A_2}$

$W = C_1 - C_2 Q$ → Equation of a straight line. $y = a + bx$.

$C_1 = u_2^2$, $C_3 = \frac{u_2 Q}{A_2}$.

$W = C_1 - C_2 \cot \beta_2$ → Equation of st line



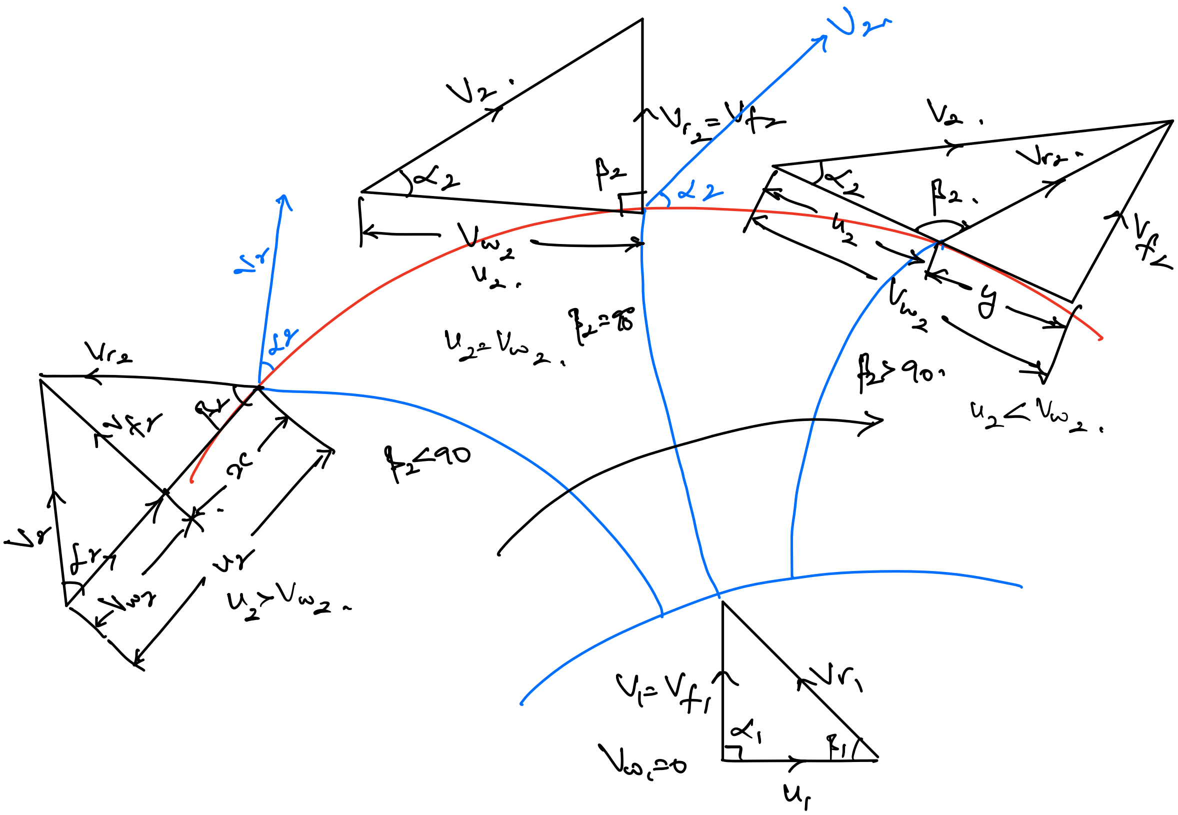
$d_2 = 0.25$ m; N = 3000 rpm. ; β_2 0 to 180

β_2 0 to 90

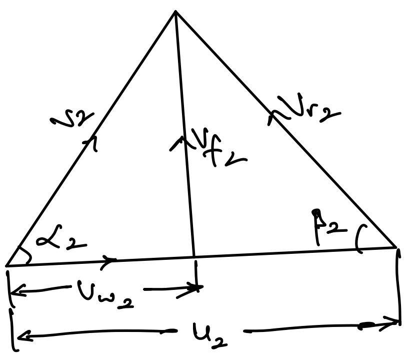
$\beta_2 = 90°$

β_2 90 to 180

Sl. No.	$u_2 = \frac{\pi d_2 N}{60}$	$C_1 = u_2^2$	$\beta_2 = 40°$	$\beta_2 = 90°$	$\beta_2 = 110°$	C_2	



$\beta_2 < 90°$

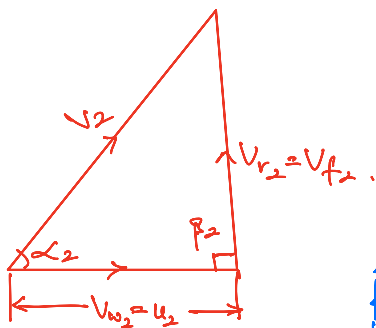


$V_{w_2} < u_2$

$W = V_{w_2} u_2$.

$$H = \frac{V_{w_2} u_2}{g}$$

$\beta_2 = 90°$



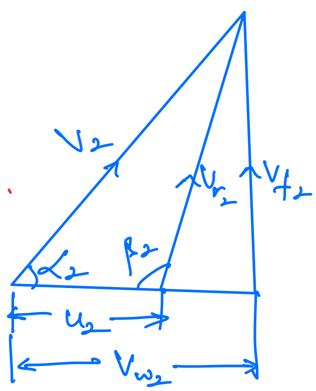
$V_{w_2} = u_2$

$W = V_{w_2} u_2$.

$W = V_{w_2}^2$ (or) u_2^2

$$H = \frac{u_2^2}{g}$$

$\beta_2 > 90°$

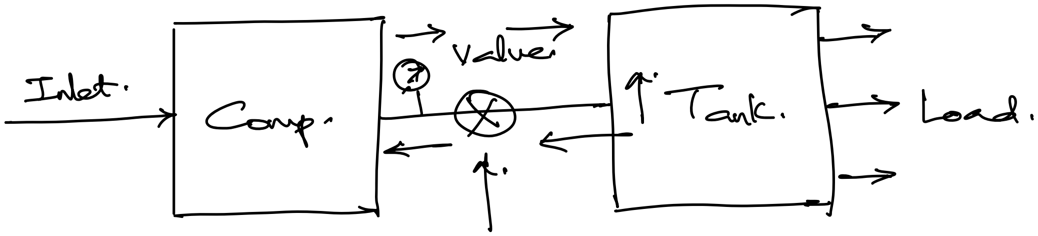


$V_{w_2} > u_2$.

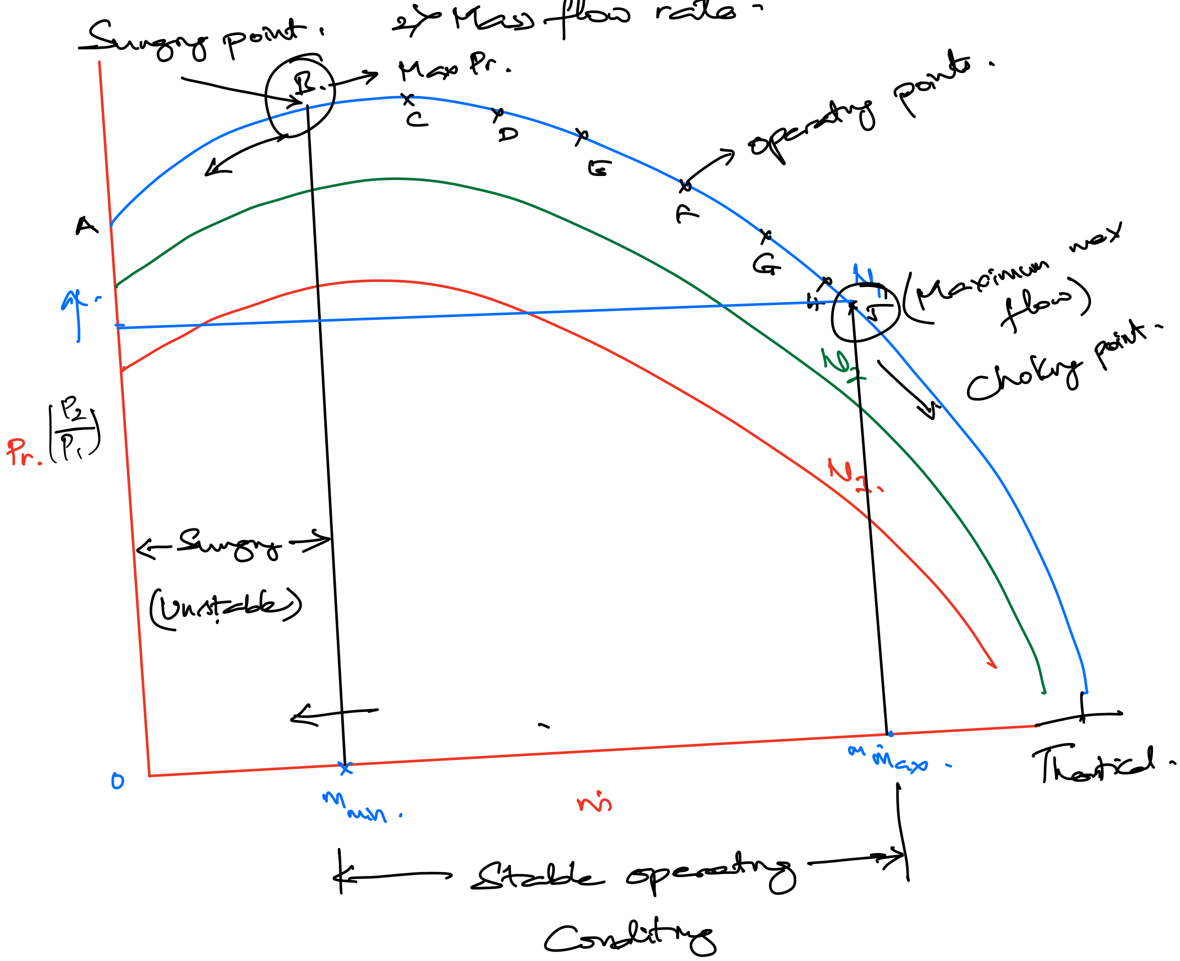
$W = V_{w_2} u_2$.

$$H = \frac{V_{w_2} u_2}{g}.$$

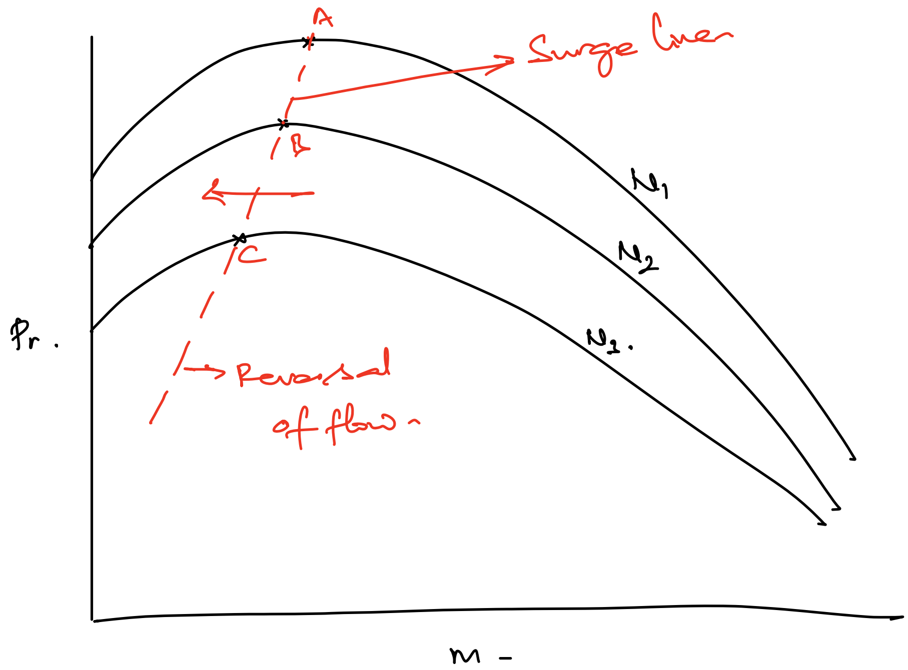
Surging and Choking



i) Controls the pressure
2) Mass flow rate.



Both Surging & Choking → Instability problems.



A
B
C
Surge line
Reversal of flow
N_1
N_2
N_3
Pr
m

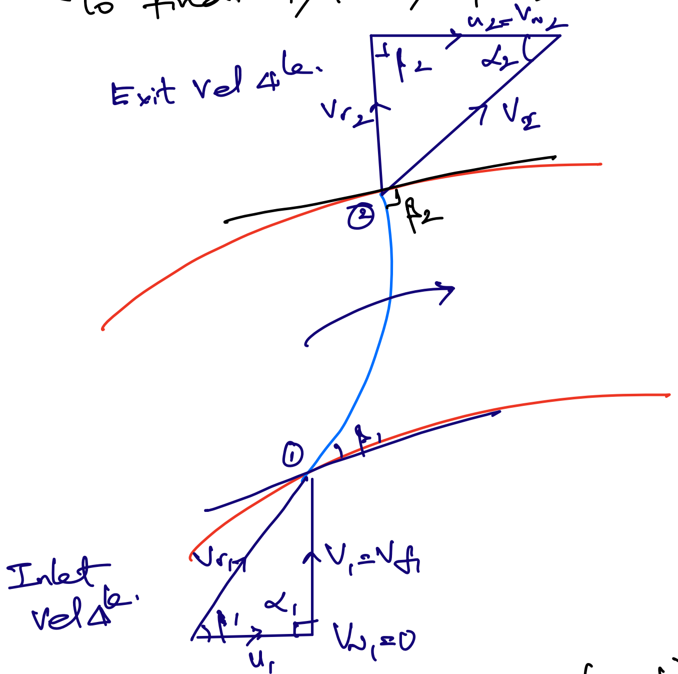
1. The diameter ratio of the impeller of a centrifugal compressor is 2 and the pressure ratio is 4. At a speed of 12000 rpm, the flow rate is 10 m^3/s of free air. The isentropic efficiency of the compressor is 84%. The blades are radial at the outlet and entry is radial at the inlet. The velocity of flow remains constant at 60 m/s through the impeller. Calculate (a) power input to the machine, (b)the impeller diameters at the inlet and outlet, and (c) the blade angle at the inlet. The suction is from the atmosphere at 100 kPa and 300 K.

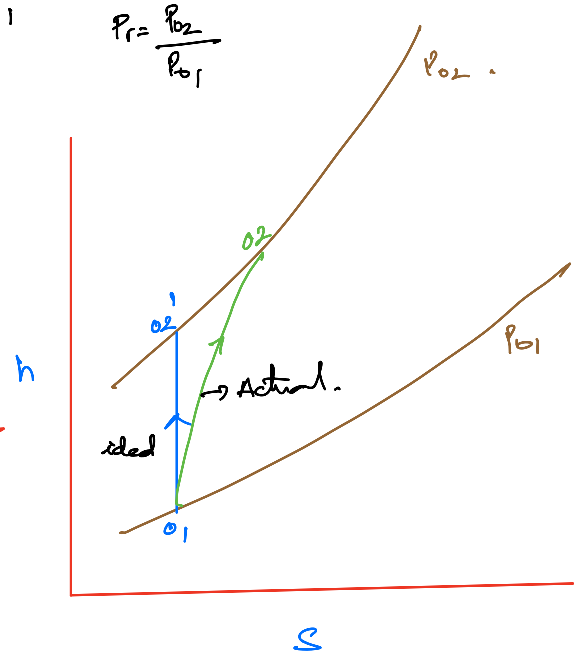
Sol: Given: $\frac{d_2}{d_1}=2$; $P_r = 4$; $N = 12000$ rpm; $Q = 10\ m^3/s$; $\eta_c = 84\%$; $\beta_2 = 90^\circ$

$\alpha_1 = 90^\circ$; $V_f = 60$ m/s; $P_{01} = 100$ kPa; $T_{01} = 300$ K; $\gamma = 1.4$, $\rho = 1.184\ kg/m^3$.

To find: i) P ii) d_1 & d_2 iii) β_1

$P_r = \frac{P_{02}}{P_{01}}$





Consider the isentropic process. $(0_1 - 0_2')$

$$\frac{T_{02}'}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_{02}' = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} = 300 * (4)^{\frac{1.4-1}{1.4}}$$

$$T_{02}' = 445.8\ K.$$

Compressor Efficiency is given by:

$$\eta_c = \frac{W_{isen}}{W_{act}} = \frac{C_p(T_{02}' - T_{01})}{C_p(T_{02} - T_{01})} \rightarrow \frac{T_{02}' - T_{01}}{T_{02} - T_{01}}$$

$$0.84 = \frac{445.8 - 300}{T_{02} - 300}$$

$$T_{02} = 473.6\ K > T_{02}'$$

Actual work done $W_{act} = C_p(T_{02} - T_{01})$ $\quad C_p = 1.005$ KJ/kg-K.

$= 1.005 \times 10^3 (473.6 - 300)$

$W_{act} = 174.5$ KJ/kg.

From the Euler eqn: $W_{act} = V_{w_2} u_2 - \cancel{V_{w_1} u_1} = V_{w_2}^2 \text{ (or) } u_2^2$

$\therefore W_{act} = u_2^2.$

$174.5 \times 10^3 = u_2^2.$

$u_2 = \sqrt{174.5 \times 10^3}$

$u_2 = 417.73$ m/s.

ii) $\therefore u_2 = \frac{\pi d_2 N}{60} \rightarrow d_2 = \frac{60 u_2}{\pi N} = \frac{60 \times 417.73}{\pi \times 12000}.$

$d_2 = 0.665$ m.

$\frac{d_2}{d_1} = 2 \quad \therefore d_1 = \frac{d_2}{d_2} = \frac{0.665}{2} = 0.3325$ m.

i) $P = \dot{m} W$ $\qquad \dot{m} = \rho A V = \rho Q, \rightarrow \dot{m} = 1.184 \times 10 = 11.84$ kg/s.

$= 11.84 \times 174.5 \times 10^3$

$P = 2066.1$ KW.

iii) From the inlet velocity triangle.

$\tan\beta_1 = \frac{V_f}{u_1}$

$\beta_1 = \tan^{-1}\left[\frac{V_f}{u_1}\right]$

$= \tan^{-1}\left[\frac{60}{208.91}\right]$

$\beta_1 = 16.02^\circ$

$u_1 = \frac{\pi d_1 N}{60}$

$= \frac{\pi \times 0.3325 \times 12000}{60}$

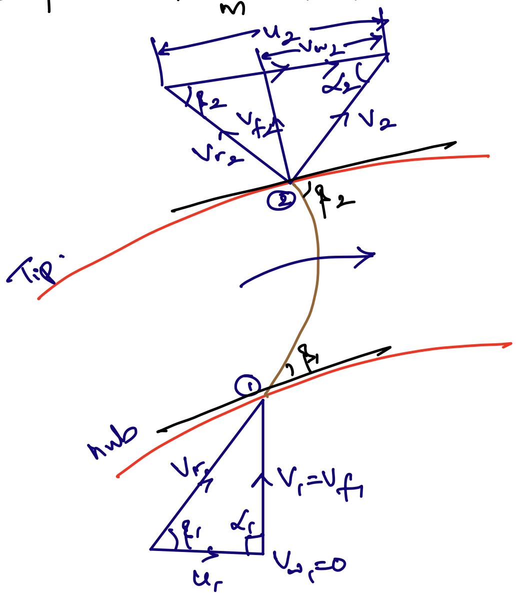
$u_1 = 208.91$ m/s.

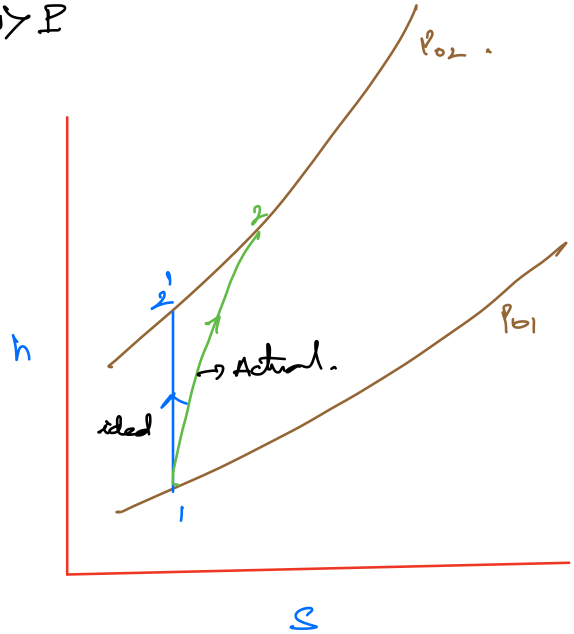
2. The impeller of a centrifugal compressor has the inlet and outlet diameters of 0.3 and 0.6 m, respectively. The intake is from the atmosphere at 100 kPa and 300 K, without any whirl component. The outlet blade angle is 75^o. The speed is 10000 rpm and the velocity of flow is 120 m/s. If the blade width at the intake is 60 mm, calculate (a) specific work (b) exit pressure (c)mass flow rate, and (d) power required to drive the compressor if the overall efficiency can be assumed as 0.7.

Sol: Given: $d_1 = 0.3m$; $d_2 = 0.6m$; $P_1 = 100kpa$; $T_1 = 300k$; $V_{w_1} = 0$; $\beta_2 = 75°$.

$N = 10000$ rpm; $V_f = 120$ m/s; $b_1 = 60mm$; $\eta = 0.7$.

To find: i) $\frac{W}{m}$ ii) P_2 iii) $\dot{m}$ iv) P





$$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.3 \times 10000}{60} = 157.07 \text{ m/s}$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.6 \times 10000}{60} = 314.15 \text{ m/s}.$$

From the inlet velocity triangle.

$$V_{r_1}^2 = V_{f_1}^2 + u_1^2.$$

$$= 120^2 + 157.07^2$$

$$V_{r_1} = \sqrt{120^2 + 157.07^2}$$

$$V_{r_1} = 197.66 \text{ m/s}.$$

$$\tan\beta_1 = \frac{V_{f_1}}{u_1} \rightarrow \beta_1 = \tan^{-1}\left[\frac{V_{f_1}}{u_1}\right] = \tan^{-1}\left[\frac{120}{157.07}\right]$$

$$\beta_1 = 37.37°$$

From the exit velocity Δ^{le}. $\therefore V_{f1} = V_{f2} = 120 m/s$

$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{w2}} \rightarrow \tan 75 = \frac{120}{314.16 - V_{w2}}.$$

$$\therefore V_{w2} = 282 \ m/s.$$

From the Euler equation:

$$\frac{W}{m} = V_{w2}u_2 - \cancel{V_{w1}}u_1 = V_{w2}u_2.$$

$$\frac{W}{m} = 282 * 314.15$$

$$\frac{W}{m} = 88.59 \ kJ/kg.$$

Also. $\frac{W}{m} = C_p(T_2 - T_1)$

$$88.59 \times 10^3 = 1.005 \times 10^3 (T_2 - 300)$$

$$T_2 = 388.15 \ K.$$

Efficiency is given by:

$$\eta = \frac{T_2' - T_1}{T_2 - T_1} \rightarrow 0.7 = \frac{T_2' - 300}{388.15 - 300}.$$

$$T_2' = 361.71 \ K.$$

For an isentropic compression process.

$$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow \frac{P_2}{P_1} = \left(\frac{T_2'}{T_1}\right)^{\gamma/\gamma-1}$$

$$P_2 = P_1 * \left(\frac{T_2'}{T_1}\right)^{\gamma/\gamma-1}$$

$$= 100 \times 10^3 * \left(\frac{361.71}{300}\right)^{1.4/1.4-1}$$

$$P_2 = 192.46 \ kPa.$$

$\dot{m} = \rho A_1 V_1 = \rho A_2 V_2$. $A_1 = \frac{\pi d_1^2}{4} = \frac{\pi \times 0.3^2}{4} = 0.071 \ m^2$.

$\dot{m} = 1.184 * 0.071 * 120 = 10.08 \ kg/s$.

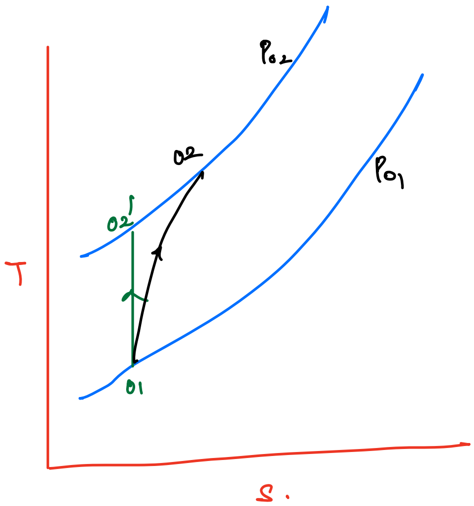
$P = \dot{m} * \frac{W}{m} = 10.08 * 88.591 = 893 \ kW.$

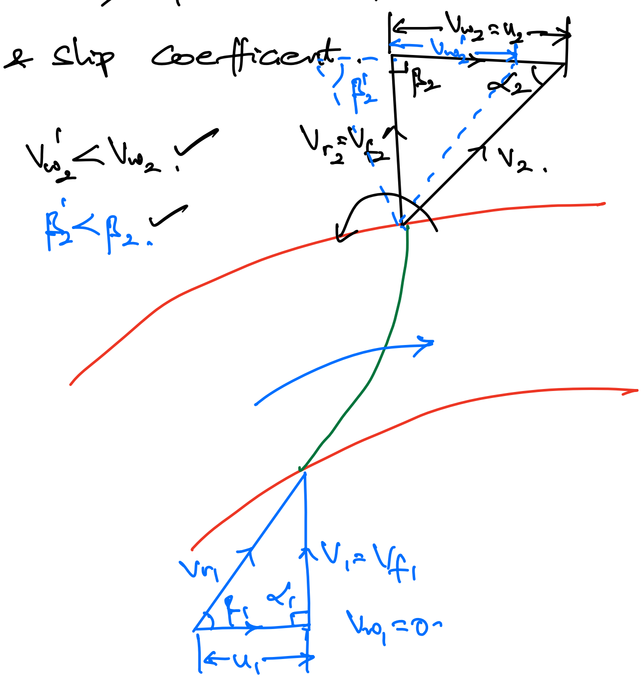
3. A centrifugal compressor runs at 15000 rpm and produces a stagnation pressure ratio of 4 between the impeller inlet and outlet. Stagnation conditions of air at the intake are 100 kPa and 300 K. The absolute velocity at the compressor intake is without any whirl component. At the exit of the impeller, the flow component of the velocity is 135 m/s and the blades are radial. The total-to-total efficiency of the compressor is 0.78. Draw the velocity triangles and find the blade angle at the inlet. Also compute the slip and slip coefficient. Take exit rotor diameter as 0.58 m and inlet rotor diameter as 0.25m.

Sol: Data: $N = 15000$ rpm; $\frac{P_{02}}{P_{01}} = 4$; $P_{01} = 100$ kPa; $T_{01} = 300$ K; $V_{w_1} = 0$

$V_{f_2} = 135$ m/s; $\eta_c = 0.78$, $d_2 = 0.58$ m; $d_1 = 0.25$ m; $\beta_2 = 90°$

To find i) β_1 ii) Slip & slip coefficient





From the h-s diagram:

$$\frac{T_{02}'}{T_{01}} = \left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_{02}' = T_{01}\left(\frac{P_{02}}{P_{01}}\right)^{\frac{\gamma-1}{\gamma}}.$$

$$T_{02}' = 300\,(4)^{\frac{1.4-1}{1.4}}$$

$$T_{02}' = 445.8\text{ K}.$$

$$\eta_c = \eta_{t\text{-}t} = \frac{C_p(T_{02}' - T_{01})}{C_p(T_{02} - T_{01})} \rightarrow 0.78 = \frac{445.8 - 300}{T_{02} - 300}.$$

$$T_{02} = 486.92\text{ K}.$$

$$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.25 \times 15000}{60} = 196.34\text{ m/s}.$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.58 \times 15000}{60} = 455.5\text{ m/s}.$$

$V_{f_1} = V_{f_2} = 135$ m/s.

From the inlet velocity triangle

$$\tan\beta_1 = \frac{V_{f_1}}{u_1} \rightarrow \beta_1 = \tan^{-1}\left[\frac{135}{196.34}\right] = 34.5^\circ$$

$W_{act} = C_p(T_{02} - T_{01}) \longrightarrow$ Actual work done.

$W_{ideal} = C_p(T_{02}' - T_{01}) \longrightarrow$ Ideal work done.

$W_{act} = 1.005(486.92 - 300)$

$W_{act} = 187.83$ kJ/kg $= V_{w_2}' u_2$. ($\because$ Due to slip)

$$V_{w_2}' = \frac{W_{act}}{u_2} = \frac{187.83 \times 10^3}{455.5} = 412.4 \text{ m/s}$$

$W_{ideal} = V_{w_2} u_2 = V_{w_2}^2$ (OR) $u_2^2 = 198.72$ kJ/kg

$V_{w_2} = 445.5$ m/s

i) Slip $\sigma = V_{w_2} - V_{w_2}' = 445.5 - 412.4 = 33.1$.

ii) Slip coefficient $\phi = \dfrac{V_{w_2} - V_{w_2}'}{V_{w_2}} = \dfrac{445.5 - 412.4}{445.4}$

$\phi = 0.074$