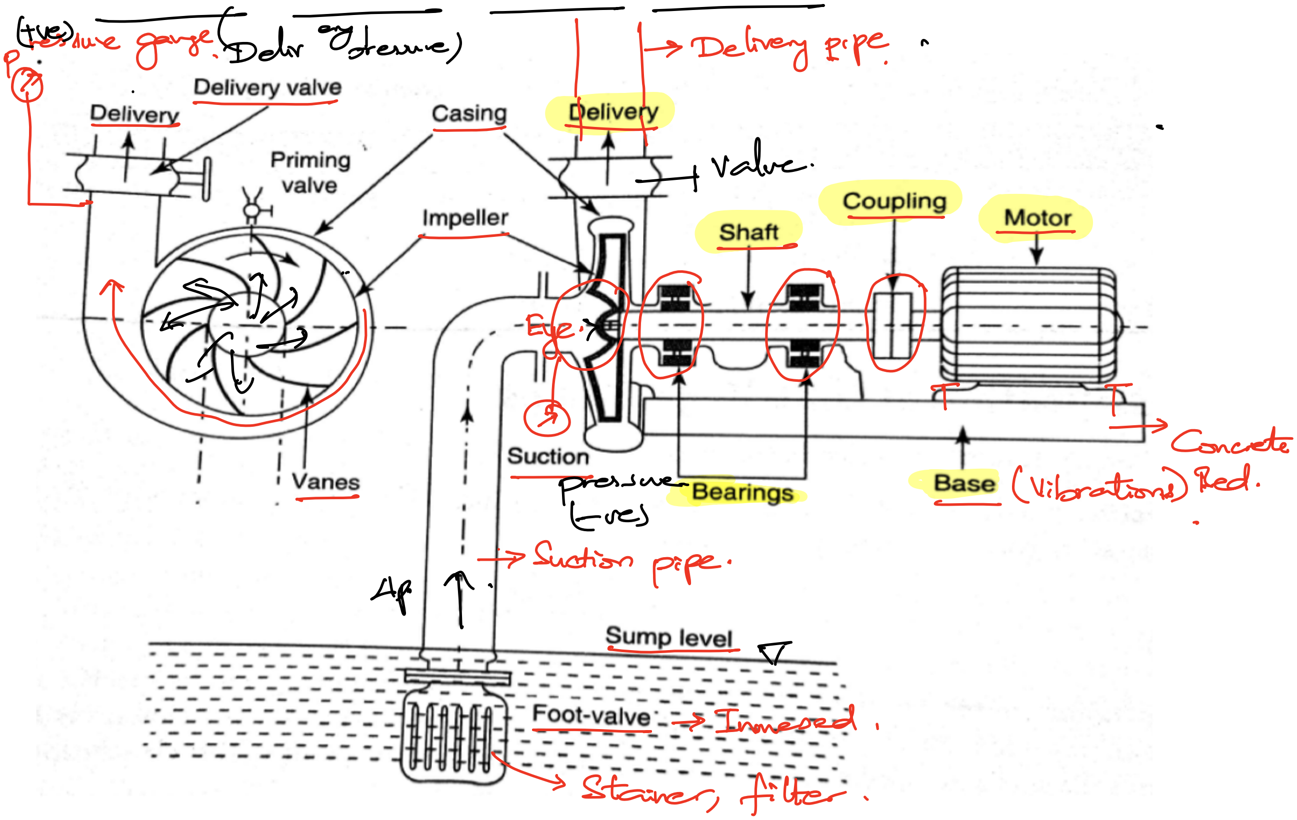
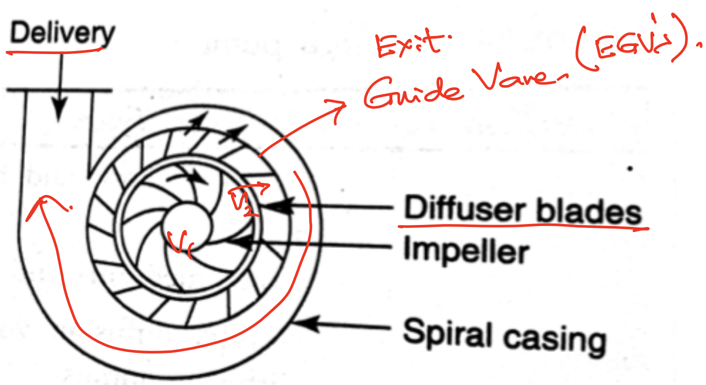
Centrifugal Pumps: Schematic diagram, suction head, delivery head, manometric head, pressure rise, manometric efficiency, hydraulic efficiency, volumetric efficiency, overall efficiency, minimum starting speed, slip, priming, cavitation, MSL, NPSH, multistage centrifugal pumps. **04 Hours**

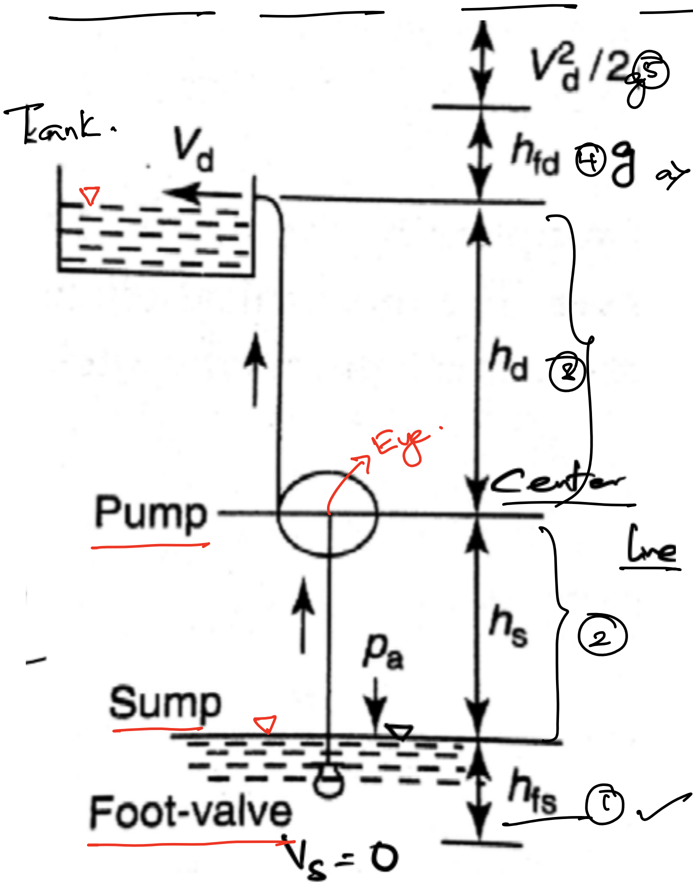
Construction of a Centrifugal pump.





High pressure liquid.

Pressure heads associated with Centrifugal pump.



1) Frictional head (h_f)

a) h_{fs} → Frictional loss on the suction side.

h_{fs} → Loss of head on the suction side of the pump.

b) h_{fd} → Frictional loss on the delivery side.

h_{fd} → Loss of head on the delivery side of the pump.

$h_f = h_{fs} + h_{fd}$.

2) Suction head (h_s): It is the head from the surface of the sump to the eye of the impeller (centre line of pump)

3) Delivery head (h_d): It is the head from the eye of the impeller (centre line of pump) to the water level in the delivery tank.

4) Kinetic head loss $\left(\frac{V_d^2}{2g}\right)$: Exit loss of the fluid.

5) As the fluid is stationary in the sump ($V_s = 0$)

therefore $\frac{V_s^2}{2g} \simeq 0$.

Euler head (H_E)

$$H_E = \frac{W}{g} = \frac{V_{w_2}u_2 - V_{w_1}u_1}{g} \rightarrow \text{Euler's equation.}$$

If $V_{w_1} = 0 \rightarrow$ No whirl at the inlet.

$$H_E = \frac{V_{w_2}u_2}{g} \rightarrow$$

Manometric Head (H_m)

Head under which the pump will work

$$H_m = \underline{h_{fs}} + \underline{h_s + h_d} + \underline{h_{fd}} + \underbrace{\frac{V_d^2}{2g}}_{\text{losses}} \qquad \left[\because \frac{V_s^2}{2g} = 0\right]$$

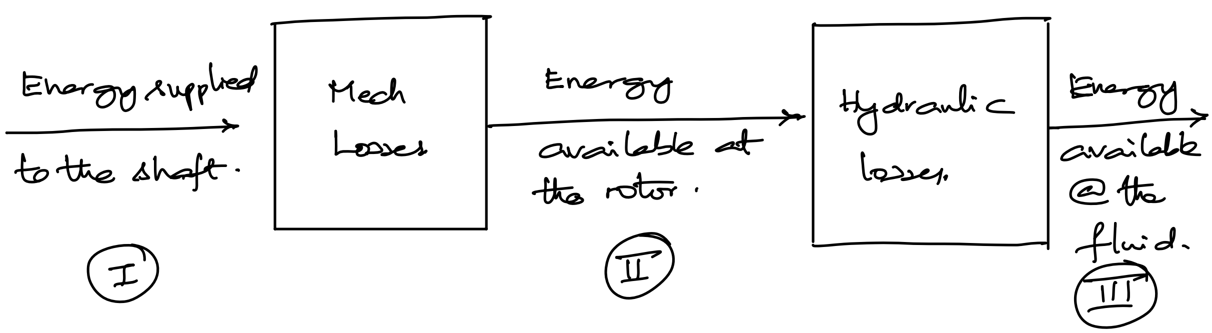
$h_f = h_{fs} + h_{fd}$ and $h = h_s + h_d$.

$$H_m = h_f + h + \frac{V_d^2}{2g} \rightarrow$$

Note: 1 → Suction side
2 → Delivery side.

$$H_m = \underbrace{h_{f_1} + h_{f_2}}_{h_f} + \underbrace{h_1 + h_2}_{h} + \frac{V_2^2}{2g}$$

Efficiencies associated with centrifugal pump.



Energy (OR) Power.

1) **Mechanical Efficiency (η_{mech}):**

$$\eta_{mech} = \frac{\text{Energy available @ the rotor [Impeller]}}{\text{Energy supplied to the shaft}}$$

$$\eta_{mech} = \frac{II}{I} < 1 \qquad 0 < \eta_{mech} < 1 \quad [0.85 \text{ to } 0.95]$$

2) **Hydraulic efficiency. (OR) Manometric Efficiency:**

$$\eta_{hyd} = \frac{\text{Energy (OR) Head available with the fluid}}{\text{Energy (OR) head available @ the rotor}}$$

$$\eta_{hyd} = \frac{III}{II} < 1 \qquad 0 < \eta_{hyd} < 1 \quad [0.85 \text{ to } 0.95]$$

$$\eta_{hyd} = \frac{\text{Manometric head} \rightarrow H_M}{\text{Euler head} \rightarrow H_E} = \frac{H_M}{H_E}$$

$$\eta_{hyd} = \frac{H_M}{\frac{V_{w_2}u_2 - V_{w_1}u_1}{g}} \rightarrow \frac{H_M}{\frac{V_{w_2}u_2}{g}} \quad \text{if } V_{w_1} = 0.$$

$$\eta_{hyd} = \frac{gH_M}{V_{w_2}u_2 - V_{w_1}u_1} \rightarrow \frac{gH_M}{V_{w_2}u_2}.$$

3) **Volumetric Efficiency (η_{vol}):**

$$\eta_{vol} = \frac{\text{Mass (OR) Volume of the fluid exiting the pump}}{\text{Mass (OR) Volume of the fluid entering the pump}}$$

$$\eta_{vol} = \frac{\dot{m}}{\dot{m} + \Delta\dot{m}} \quad \text{(OR)} \quad \frac{Q}{Q + \Delta Q}. \qquad 0 < \eta_{vol} < 1 \rightarrow [\eta_{vol} = 0.96 \text{ to } 0.98]$$

4) Overall efficiency ($\eta_{overall}$)

$$\eta_{overall} = \frac{\text{Energy (or) Head available with the fluid}}{\text{Energy supplied to the shaft.}}$$

$$= \frac{III}{I}$$

$$\eta_{overall} = \frac{\text{Manometric head}}{\text{Power input to the shaft.}} = \frac{H_m}{P}$$

$$\eta_{overall} = \eta_{mech} \times \eta_{vol} \times \eta_{hyd}.$$

Analysis of Centrifugal pump:

i) Effect of 'β_2' on 'H'

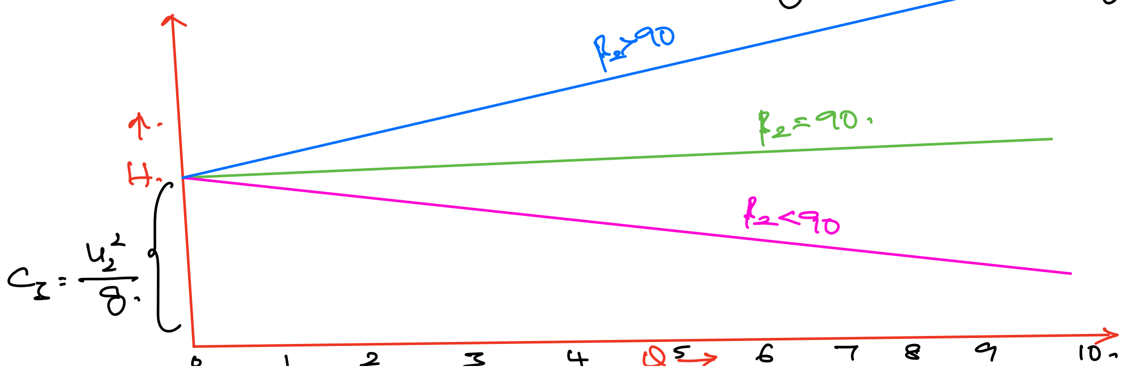
W V/s Q and H V/s Q. → for varying 'β_2' Exit blade angle.

$$W = u_2^2 - \frac{u_2 \cot\beta_2 Q}{A_2} = C_1 - C_2 Q. \qquad C_1 = u_2^2 \qquad C_2 = \frac{u_2 \cot\beta_2}{A_2}.$$

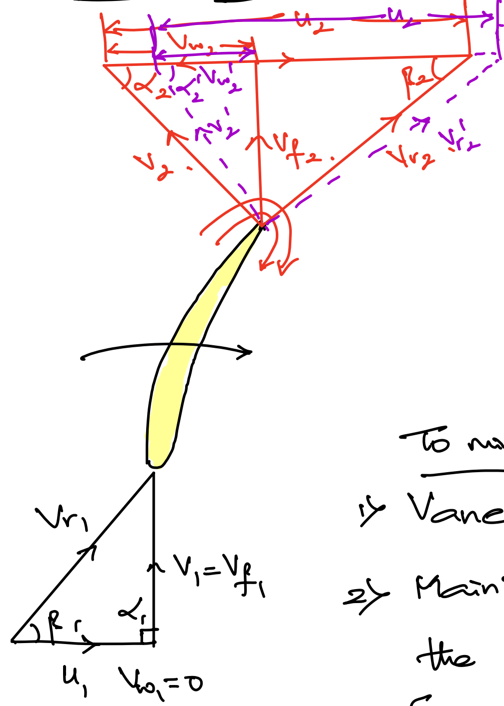
In terms of Pressure head. $H = \frac{W}{g}$.

$$H = \frac{u_2^2}{g} - \frac{u_2 \cot\beta_2 Q}{A_2 g}.$$

$$H = C_3 - C_4 Q. \qquad \because C_3 = \frac{u_2^2}{g} \quad C_4 = \frac{u_2 \cot\beta_2}{A_2 g}.$$



2> Slip and slip factor



$u_2 = \frac{\pi d_2 N}{60}$

Slip $= V_{w_2} - V_{w_2}' = \Delta V_{w_2}$

Slip factor 'σ' $= \frac{V_{w_2} - V_{w_2}'}{V_{w_2}}$

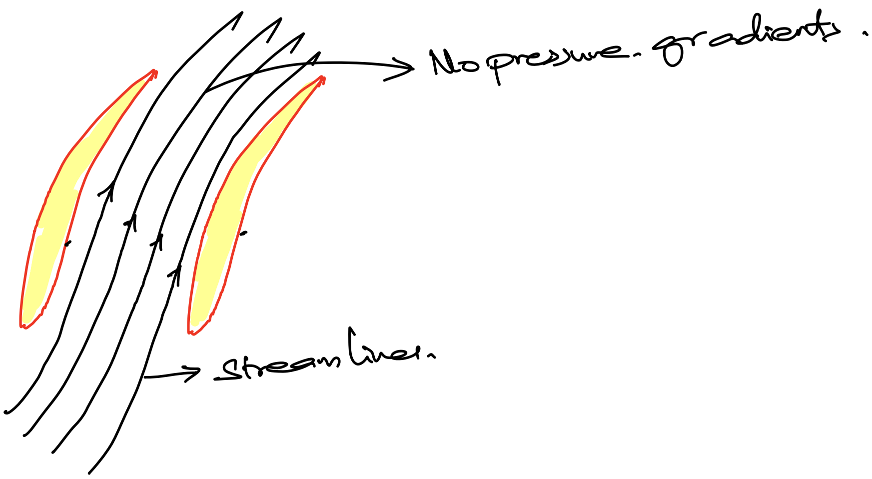
'σ' $= \frac{\Delta V_{w_2}}{V_{w_2}} < 1$

To minimize the slip:

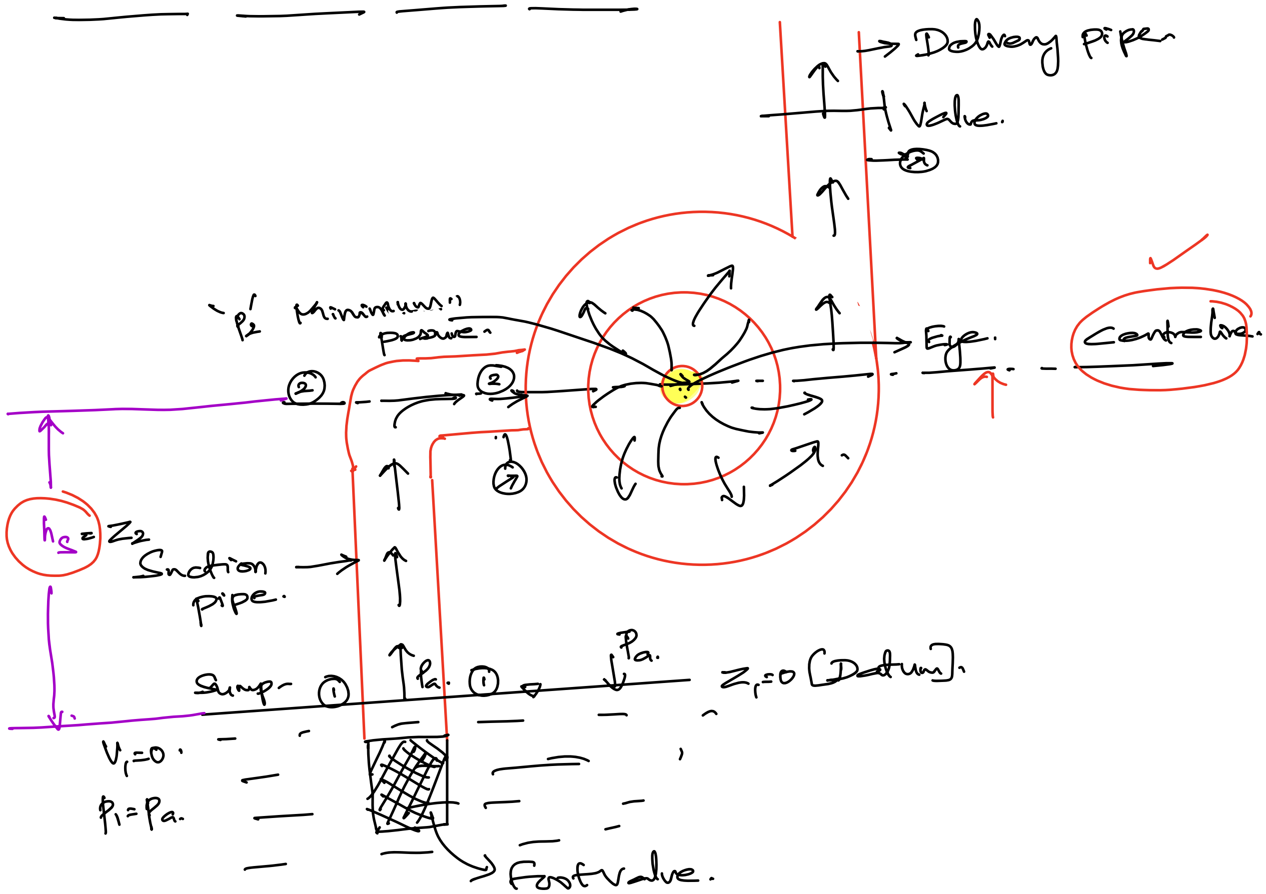
1> Vane surface has to be smooth.

2> Maintaing a streamlined flow b/n the blades (or) Vanes.

[Vane congruent flow]



Maximum Suction Lift (MSL):



$h_s \rightarrow$ Maximum height under which the pump is placed above the sump. ⓧ critical. [Maximum Suction Lift].

P_a = Atmospheric pressure.

$h_a = \frac{P_a}{\rho g} \rightarrow$ Atmospheric head.

ⓧ P_2 is the minimum pressure.

Ensure that $P_2 > P_{vapour}$ [Pressure @ which the fluid changes its phase].

Applying Bernoulli's equation ① – ②

$$\frac{p_1}{\rho g} + \cancel{\frac{V_1^2}{2g}}^{\,0} + \cancel{Z_1}^{\,0} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + \text{Losses}.(h_{fs})$$

$$\frac{p_a}{\rho g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_{fs}. \rightarrow ①.$$

$h_2 = \frac{p_2}{\rho g}$. $> h_v = \frac{p_v}{\rho g}$.

$h_a = \frac{p_a}{\rho g}$.

$$Z_2 = h_a - h_2 - \frac{V_2^2}{2g} - h_{fs}. \rightarrow ②.$$

$$MSL = h_a - h_v - \frac{V_2^2}{2g} - h_{fs} \rightarrow ③$$

$$MSL = h_a - \left(h_v + \frac{V_2^2}{2g} + h_{fs}\right) \rightarrow \text{minimum}$$

Factors affecting M.S.L

1) Altitude → Height at which centrifugal pump has to be installed. → $h_a \downarrow$.

2) Pipe parameters → h_{fs} → Friction, length, dia, fittings.

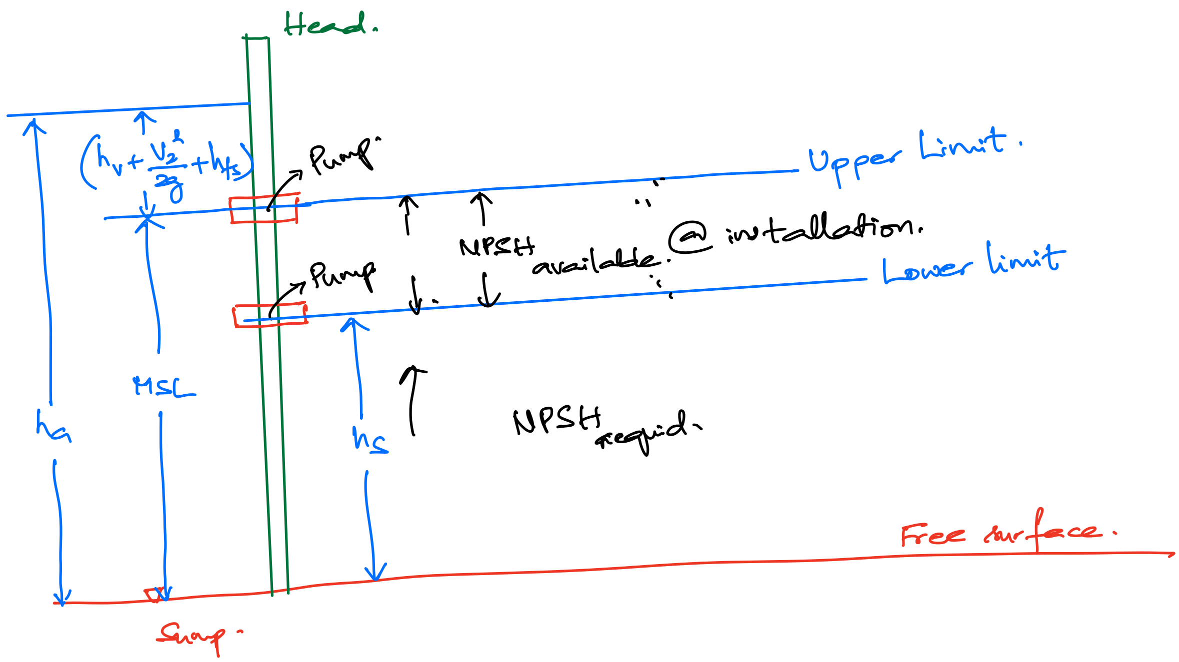
3) Fluid velocity at the eye of the impeller → $V_2 \downarrow$.

4) Ambient (OR) atmospheric temperature → h_v. → Temp↑ h_v↑

NET POSITIVE SUCTION HEAD (NPSH)

NPSH → manufacturer (Required)
NPSH → Available @ the installation point

$NPSH_{available} > NPSH_{required}$. ✗ critical.



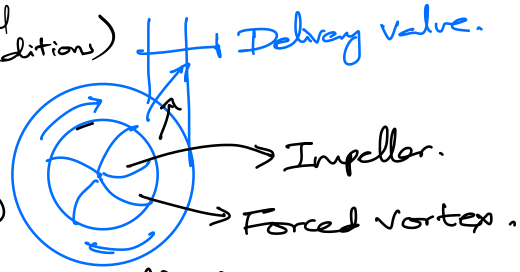
$$NPSH = MSL - h_s.$$

$$= h_a - h_v - \frac{V_2^2}{2g} - h_{fs} - h_s.$$

$$NPSH_{\text{available}} = h_a - \left(h_v + \frac{V_2^2}{2g} + h_{fs} + h_s\right) \checkmark$$

<u>Minimum</u> <u>starting</u> <u>speed</u> of a <u>centrifugal</u> <u>pump</u>. (N_{min}).

It is the minimum speed required for the pump to start delivering the fluid. (Ideal conditions)



Manometric head is given by.

$$\frac{W}{mg} = H_m = \cancel{\frac{V_2^2 - V_1^2}{2g}} + \frac{u_2^2 - u_1^2}{2g} + \cancel{\frac{V_{r_1}^2 - V_{r_2}^2}{2g}}. \rightarrow (1)$$

$H_p \geq H_m. \rightarrow$ Pump starts delivering fluid.

$$H_p \geq \frac{u_2^2 - u_1^2}{2g}. \geq H_m.$$

H_p = Head required by the pump to deliver fluid.

$$H_p \geq \frac{\pi^2 d_2^2 N^2 - \pi^2 d_1^2 N^2}{60^2 \times 2g}.$$

$$u_1 = \frac{\pi d_1 N}{60} \; ; \; u_2 = \frac{\pi d_2 N}{60}.$$

For $N = N_{min} \rightarrow$ Minimum speed required.

$$H_p \geq \frac{\pi^2 d_2^2 N_{min}^2 - \pi^2 d_1^2 N_{min}^2}{60^2 \times 2g} \geq H_m.$$

$$N_{min}^2 \geq \frac{60^2\, 2g\, H_m}{\pi^2 (d_2^2 - d_1^2)}$$

$$N_{min} \geq \sqrt{\frac{60^2 \times 2g\, H_m}{\pi^2 (d_2^2 - d_1^2)}} \rightarrow (2)$$

Home work

Find the N_{min} for

1) d_1 = const, d_2 = varying
2) d_2 = const, d_1 = varying.
3) d_1 & d_2 = const, H_E = varying.

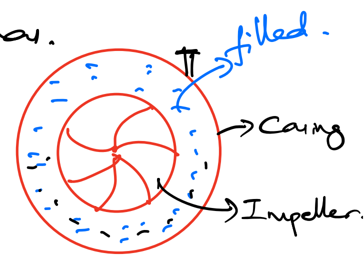
$$\eta_{mano} = \frac{H_m}{H_E} \rightarrow H_m = \eta_{mano} \times H_E$$

$$N_{min} \geq \sqrt{\frac{60^2 \times 2g\, \eta_{mano}\, H_E}{\pi^2 (d_2^2 - d_1^2)}} \rightarrow (3)$$

Priming:

i) Before starting the pump, the impeller has to be filled with the working fluid (H_2O)

2) The process of filling the fluid into the casing through the priming valve, to ensure the impeller is merged in the fluid (H_2O).



Cavitation: Eye of the impeller $\rightarrow$ Lowest pressure.

i) $Pr_{eye} < Pr_{vapour} \rightarrow$ i) Formation of bubbles (air)

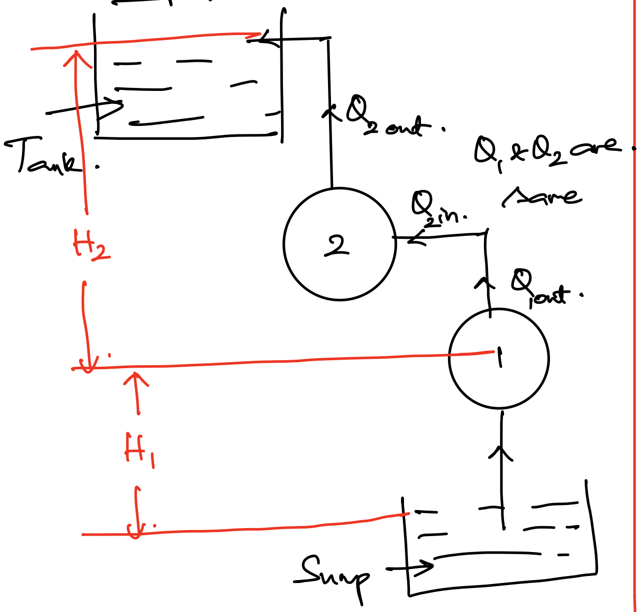
ii) Stick onto the surface of pipe/casing

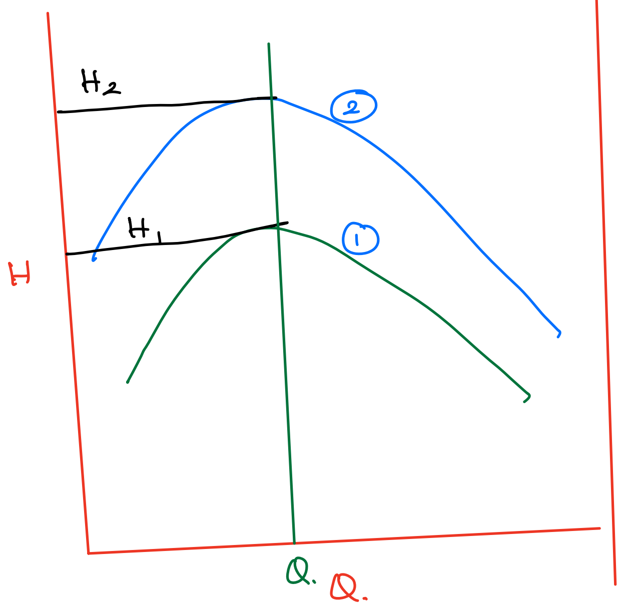
iii) Fluid flows over these bubbles they collapse.

iv) Collapsing of bubbles $\rightarrow$ continuously $\rightarrow$ material get eroded.

Multistaging of centrifugal pumps.

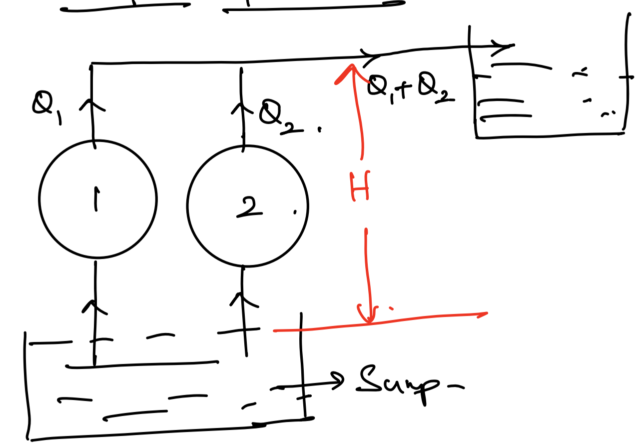
① Pumps in Series.

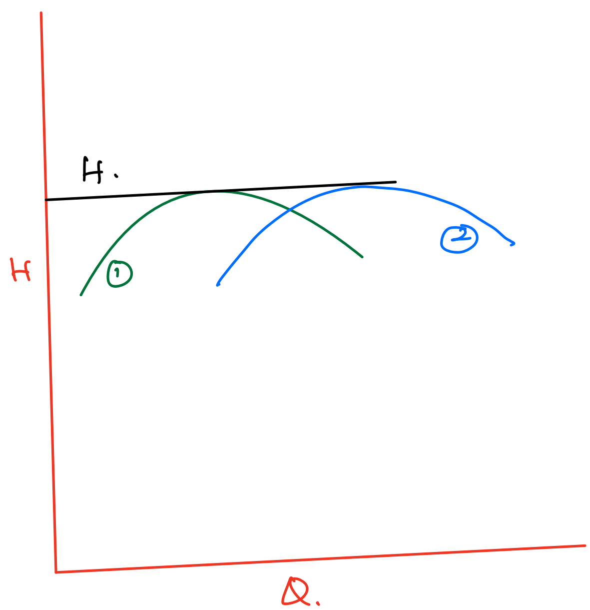




1) Increase the pressure head at the same discharge.

② Pumps in parallel





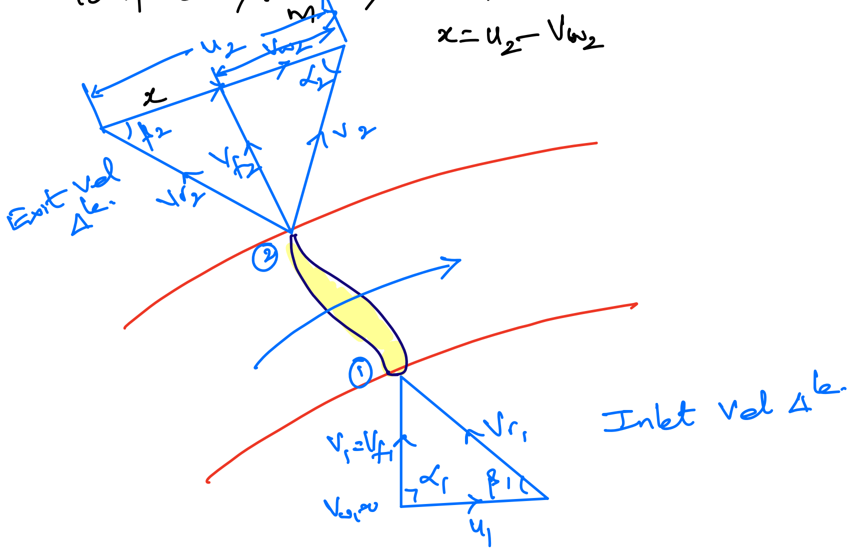
2) Increase the discharge at the same pressure head.

1. The blade angles at the inlet and outlet of the impeller of a centrifugal pump are 55^o and 75^o, and the corresponding diameters are 3 cm and 6 cm respectively. The blade width at the outlet is 0.75 cm. The speed is 1500 rpm. The entry of water is radial without any whirl component. The flow component of fluid velocity remains constant in the impeller. Draw the velocity triangles and calculate (a) the specific work, (b) flow rate (c) power of the machine and (d)the manometric head. The hydraulic efficiency may be taken as 0.85.

Sol: Given: $\beta_1 = 55^\circ$; $\beta_2 = 75^\circ$; $d_1 = 0.03$ m; $d_2 = 0.06$ m; $b_2 = 0.0075$ m; $N = 1500$ rpm

$\alpha_1 = 90^\circ$; $V_{w_1} = 0$; $V_{f_1} = V_{f_2} = V_f$; $\eta_{hyd} = 0.85$.

To find i) $\frac{W}{m}$ ii) Q iii) P iv) H_m.



Blade velocities

$$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.03 \times 1500}{60} = 2.35 \text{ m/s}$$

$$u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.06 \times 1500}{60} = 4.71 \text{ m/s}$$

Consider the inlet velocity triangle.

$$\tan\beta_1 = \frac{V_{f_1}}{u_1} \rightarrow \left\{ \tan\beta_1 = \frac{V_1}{u_1} \right\}$$

$$V_{f_1} = u_1 \tan\beta_1$$

$$= 2.35 \times \tan 55$$

$$V_{f_2} = V_1 = V_{f_1} = 3.35 \text{ m/s}.$$

Consider the exit velocity triangle.

$$\tan\beta_2 = \frac{V_{f2}}{u_2 - V_{w2}} \rightarrow u_2 - V_{w2} = \frac{V_{f2}}{\tan\beta_2}.$$

$$V_{w2} = u_2 - \frac{V_{f2}}{\tan\beta_2}$$

$$= 4.71 - \frac{3.35}{\tan 75}$$

$$V_{w2} = 3.81\ m/s$$

i) Specific work

$$\frac{W}{m} = V_{w2}u_2 - \cancel{V_{w1}u_1} = V_{w2}u_2.$$

$$\frac{W}{m} = 3.81 \times 4.71$$

$$\frac{W}{m} = 17.94\ J/kg.$$

ii) Discharge $Q = A_1 V_{f1} = A_2 V_{f2}$. $\qquad A_1 = \pi d_1 b_1;\ A_2 = \pi d_2 b_2$.

$$\therefore A_2 = \pi \times 0.06 \times 0.0075$$

$$A_2 = 0.00141\ m^2$$

$$\therefore Q = A_2 V_f = 0.00141 * 3.35$$

$$Q = 4.73 \times 10^{-3}\ m^3/s.$$

iii) Power $P = \dot{m}\frac{W}{m}$ $\qquad \dot{m} = \rho Q$ $\qquad \rho_{H_2O} = 1000\ kg/m^3$

$$= 1000 \times 4.73 \times 10^{-3}$$

$$= 4.73\ kg/s.$$

$$= 4.73 * 17.94$$

$$P = 84.85\ W.$$

iv) $\eta_{mano} = \frac{H_m}{H_E}$ $\qquad H_E = \frac{V_{w2}u_2}{g}.$

$$H_m = \eta_{mano} * \frac{V_{w2}u_2}{g}.$$

$$= 0.85 * \frac{3.81 * 4.71}{9.81}$$

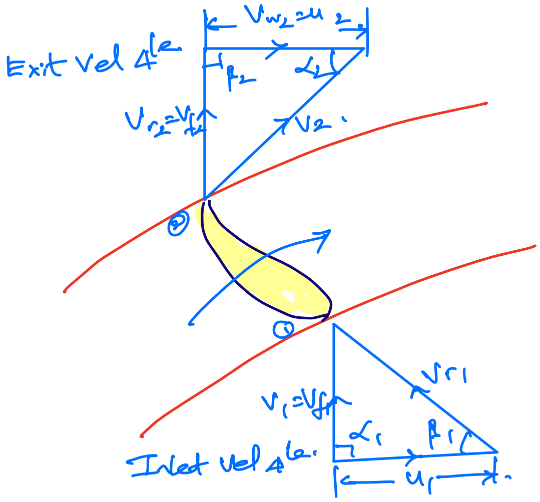
$$H_m = 1.55\ m.$$

2. A centrifugal pump is driven by an induction motor at 960 rpm. The flow rate of water is 50 lps against a head of 10 m. The flow velocity is constant at 6.5 m/s through the impeller. The blades are radial at the outlet and the losses are estimated as 15% of the output. Assume that water enters the rotor at $\alpha_1 = 90^o$. Calculate (a)tip diameter of the impeller, (b) width of the blades at the outlet, (c) fluid delivery angle (d) power and (e) specific speed.

Sol: Given: $N = 960$ rpm; $H_m = 10$ m; $Q = 50\times10^{-3}\ m^3/s$; $V_f = 6.5$ m/s; $\eta_{hyd} = (1-0.15) = 0.85$

$\alpha_1 = 90°$; $\beta_2 = 90°$; $V_{w_1} = 0$

To find: i) d_2 ii) b_2; iii) α_2 iv) P v) N_{sp}



Hydraulic/Manometric efficiency

$$\eta_{mano} = \frac{H_m}{H_E} = \frac{H_m}{\frac{V_{w_2}u_2}{g}}$$

$$\eta_{mano} = \frac{gH_m}{u_2^2} \quad \because V_{w_2} = u_2.$$

$$0.85 = \frac{9.81\times10}{u_2^2}$$

$$u_2 = \sqrt{\frac{9.81\times10}{0.85}}$$

$$u_2 = 10.74\ m/s = V_{w_2}.$$

i) Blade speed @ tip.

$$u_2 = \frac{\pi d_2 N}{60} \rightarrow d_2 = \frac{60\,u_2}{\pi N} = \frac{60\times10.74}{\pi\times960} \rightarrow d_2 = 0.21\ m.$$

ii) Discharge $Q = A_1V_{f_1} = A_2V_{f_2}$.

$A_1 = \pi d_1 b_1$ & $A_2 = \pi d_2 b_2$.

$$50\times10^{-3} = A_2 \times 6.5$$

$$A_2 = \frac{50\times10^{-3}}{6.5} = 7.69\times10^{-3}\ m^2$$

$$A_2 = \pi d_2 b_2$$

$$b_2 = \frac{A_2}{\pi d_2} = \frac{7.69\times10^{-3}}{\pi\times0.21}$$

$$b_2 = 0.0116\ m$$

iii) From the exit velocity triangle.

$$\tan\alpha_2 = \frac{V_{f_2}}{u_2}.$$

$$\alpha_2 = \tan^{-1}\left[\frac{V_{f_2}}{u_2}\right]$$

$$\alpha_2 = \tan^{-1}\left[\frac{6.5}{10.74}\right]$$

$$\alpha_2 = 31.18°$$

iv) $P = \frac{\rho g Q H_m}{1000} = \frac{1000 \times 9.81 \times 50 \times 10^{-3} \times 10}{1000} = 4.91$ kW. $\quad w = \rho g$

v) Specific speed $N_{sp} = \frac{N\sqrt{Q}}{H_m^{3/4}}$

$$= \frac{960\sqrt{50 \times 10^{-3}}}{10^{3/4}}$$

$$N_{sp} = 38.17 \text{ rpm.}$$

4. A centrifugal pump has its impeller with inlet and outlet diameters of 35 cm and 70 cm respectively. It is used in a setup with a manometric head of 28 m. Determine the minimum starting speed.

Sol: Given: $d_1 = 0.35$ m; $d_2 = 0.7$ m; $H_m = 28$ m. To find: N_{min}.

$$N_{min} \geq \sqrt{\frac{60^2 \times 2gH_m}{\pi^2(d_2^2 - d_1^2)}}$$

$$\geq \sqrt{\frac{60^2 \times 2 \times 9.81 \times 28}{\pi^2(0.7^2 - 0.35^2)}}$$

$$N_{min} \geq 738.42 \text{ rpm.}$$

Pump has to run @ atleast 738.42 rpm to deliver water

5. The basic design of a centrifugal pump has a dimensionless specific speed of 0.075 rev. 'the blades are forward facing on the impeller and the outlet angle is 120° to the tangent,with an impeller passage width at the outlet being equal to one-tenth of the diameter. The pump is to be used to raise water through a vertical distance of 35 m at a flow rate of $0.04m^3/s$. The suction and delivery pipes are each of 150 mm diameter and have a combined length of 40 m with a friction factor of 0.005. Other losses at the pipe entry, exit, bends,etc. are three times the velocity head in the pipes. If the blades occupy 6% of the circumferential area and the hydraulic efficiency (neglecting slip) is 76%, what will be the diameter of the pump impeller?

Sol: Given: $N_{sp} = 0.075$ rev; $\beta_2 = 120°$; $H = 35$ m; $Q = 0.04$ m³/s; $d_p = 0.15$ m; $L_p = 40$ m; $f = 0.005$; $h_f = 3.\frac{\vec{V}_p^2}{2g}$; $A_{b1} = 6\%$ of A_f; $\eta_{hyd} = 76\%$. To find: d_2

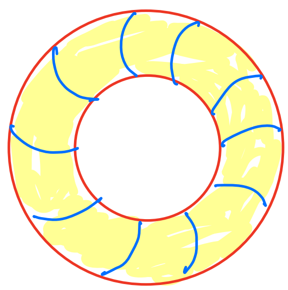
$b_2 = \frac{1}{10}$th $d_2 = 0.1 d_2$.

$\{H = H_s + H_d\}$

$Q = A_p \vec{V}_p \longrightarrow$ $A_p = \frac{\pi d_p^2}{4}$

$$= \frac{\pi \times 0.15^2}{4}$$

$$A_p = 0.0176 \text{ m}^2.$$



$$\vec{V}_p = \frac{Q}{A_p} = \frac{0.04}{0.0176}$$

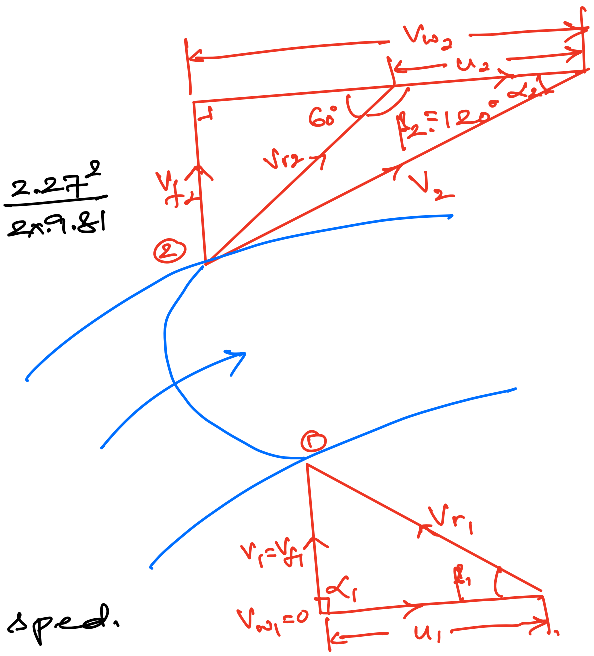
$$\vec{V}_p = 2.27 \text{ m/s.}$$

Head loss in the pipe.

$$h_l = \frac{4 f L_p V_p^2}{2 g d_p} + 3 \cdot \frac{\vec{V_p}^2}{2g}$$

$$= \frac{4 \times 0.005 \times 40 \times 2.27^2}{2 \times 9.81 \times 0.15} + 3 * \frac{2.27^2}{2 \times 9.81}$$

$$h_l = 2.18 \text{ m}.$$



Manometric head.

$$H_m = H + h_l$$
$$= 35 + 2.18.$$

$$H_m = 37.18 \text{ m}.$$

Non dimensional Specific sped.

$$N_{sp} = \frac{N \sqrt{Q}}{(g H_m)^{3/4}}.$$

$$0.075 = \frac{N \sqrt{0.04}}{(9.81 * 37.18)^{3/4}}.$$

$$N = 31.29 \text{ rps}.$$

$$N = 31.29 \times 60 = 1877.87 \text{ rpm}.$$

Flow area $A_{f_2} = \pi d_2 b_2 \rightarrow$ without blocking.

$$A_{f_2} = \pi d_2 b_2 - \frac{6}{100} \pi d_2 b_2$$

$A_{f_2} = 0.94 \pi d_2 b_2 \rightarrow$ With blocking.

$\because b_2 = 0.1 d_2.$

$$= 0.94 \pi d_2 \times 0.1 d_2$$

$$A_{f_2} = 0.295 d_2^2 \rightarrow (1).$$

Discharge. $Q = A_{f_2} V_{f_2} \rightarrow V_{f_2} = \frac{Q}{A_{f_2}}.$

$$V_{f_2} = \frac{0.04}{0.295 d_2^2} \rightarrow (2).$$

Blade velocity $u_2 = \frac{\pi d_2 N}{60}$

$$= \frac{\pi \times d_2 * 1877.87}{60}$$

$$u_2 = 98.32\, d_2 \rightarrow (3)$$

Hydraulic/Manometric efficiency.

$$\eta_{mano} = \frac{H_m}{H_E} = \frac{H_m}{\frac{V_{w_2} u_2}{g}} = \frac{g H_m}{V_{w_2} u_2}.$$

$$V_{w_2} = \frac{g H_m}{\eta_{mano} u_2} = \frac{9.81 * 37.18}{0.76 * 98.32 d_2}.$$

$$V_{w_2} = \frac{4.88}{d_2} \rightarrow (4)$$

From the exit velocity triangle.

$$\tan(180 - \beta_2) = \frac{V_{f2}}{V_{w_2} - u_2}. \qquad \because \beta_2 = 120^\circ$$

$$\tan 60 = \frac{\left[0.04/0.295 d_2^2\right]}{\left[\frac{4.88}{d_2} - 98.32 d_2\right]}.$$

By trial and Error method.

$$d_2 = 0.23 \text{ m}.$$

$V_{f2} = 2.56$ m/s.

$V_{w_2} = 21.2$ m/s.

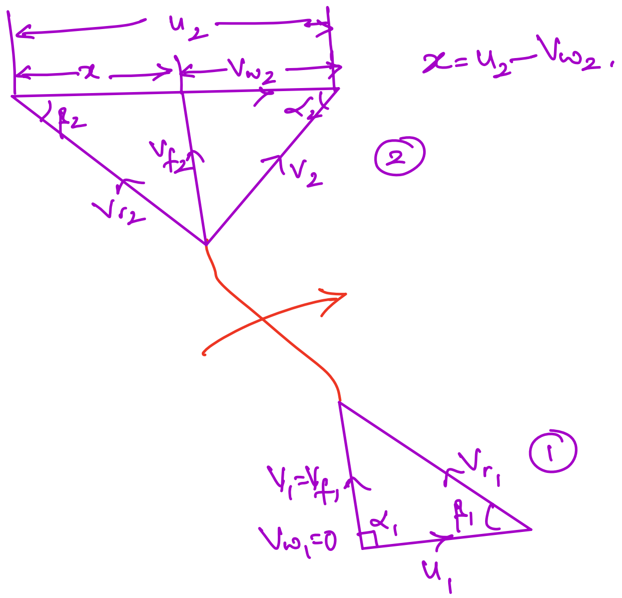
$u_2 = 22.6$ m/s.

20. A centrifugal pump having outer diameter equal to two times the inner diameter and running at 1200 rpm works against a total head of 32 m. The velocity of flow through the impeller is constant and equal to 3 m/s. The vanes are set back at an angle of 30° at the outlet. If the outer diameter of the impeller is 600 mm and width of outlet is 50 mm. Determine (i) Vane angle at inlet , (ii) Work done per second by impeller impeller and (iii) Manometric ~~Manometric~~ efficiency ~~efficiency.~~

Sol: Given: $N = 1200$ rpm; $H_m = 32$ m; $V_f = 3$ m/s; $\beta_2 = 30°$; $d_2 = 2d_1$

$d_2 = 0.6$ m; $\therefore d_1 = 0.3$ m; $b_2 = 0.05$ m;

To find: a) β_1 b) $\frac{W}{m}$ c) η_{hyd} (or) η_{mano}.



Tangential velocities of the impeller

$$u_1 = \frac{\pi d_1 N}{60} = \frac{\pi \times 0.3 \times 1200}{60} = 18.85 \text{ m/s}.$$

$$u_2 = 2u_1 = 37.7 \text{ m/s} \quad (\text{OR}) \quad u_2 = \frac{\pi d_2 N}{60} = \frac{\pi \times 0.6 \times 1200}{60} = 37.7 \text{ m/s}.$$

From the inlet velocity Δle.

$$\tan\beta_1 = \frac{V_f}{u_1} \rightarrow \beta_1 = \tan^{-1}\left[\frac{3}{18.85}\right]$$

$$\beta_1 = 9.04°.$$

From the exit velocity triangle.

$$\tan\beta_2 = \frac{V_f}{x} \rightarrow \tan\beta_2 = \frac{V_f}{u_2 - V_{w_2}}$$

$$\therefore V_{w_2} = u_2 - \frac{V_f}{\tan\beta_2}$$

$$= 37.7 - \frac{3}{\tan 30}$$

$$V_{w_2} = 32.5 \ m/s.$$

$$\frac{W}{m} = V_{w_2} u_2$$

$$= 32.5 \times 37.7$$

$$= 1213.94 \ J/kg.$$

$$\frac{W}{m} = 1.214 \ KJ/kg.$$

$$Q = \pi d_2 b_2 V_f = \pi \times 0.6 \times 0.05 \times 3 = 0.283 \ m^3/s.$$

$$\dot{m} = \rho Q = 1000 \times 0.283 = 283 \ kg/s.$$

$$\therefore \text{Power } P = \dot{m}\frac{W}{m} = 283 \times 1.214$$

$$P = 343.25 \ KW.$$

Euler head $H_E = \frac{V_{w_2} u_2}{g}$

$$= \frac{32.5 \times 37.7}{9.81}$$

$$H_E = 124.89 \ m.$$

c) Manometric Efficiency $\eta_{mano} = \frac{H_m}{H_E}$

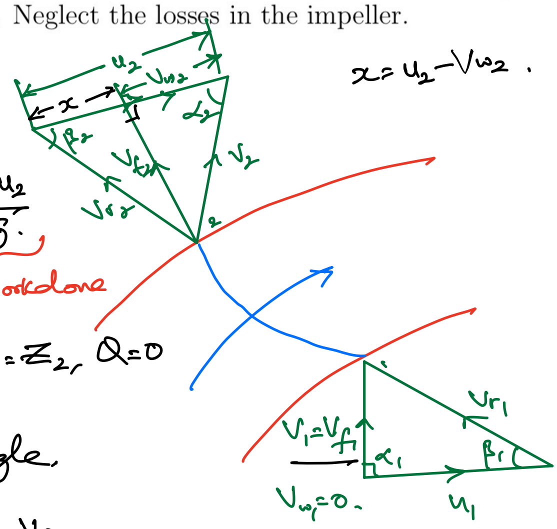
$$= \frac{32}{124.89}$$

$$\eta_{mano} = 25.62\%.$$

13. Show that the pressure rise in the impeller of a centrifugal pump with backward curved vane can be expressed as

$$\frac{p_2}{\rho g} - \frac{p_1}{\rho g} = \frac{1}{2g}[V_{f1}^2 + U_2^2 - V_{f2}^2 Cosec^2\beta_2]$$

where the subscripts 1 and 2 represents the inlet and outlet, β is the blade angle, U is the tangential velocity of the impeller and V_f is the flow velocity. Neglect the losses in the impeller.



$x = u_2 - V_{w2}$

Apply the energy equation ① & ②

$$\cancel{\dot{Q}}^{0} + \frac{P_1}{\rho g} + \frac{\vec{V}_1^{\,2}}{2g} + \cancel{Z_1} = \frac{P_2}{\rho g} + \frac{\vec{V}_2^{\,2}}{2g} + \cancel{Z_2} - \underbrace{\frac{V_{w_2}u_2}{g}}_{\text{Workdone}}$$

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{V_1^2 - V_2^2}{2g} + \frac{V_{w_2}u_2}{g} \rightarrow ① \quad Z_1 = Z_2, \ Q = 0$$

From the outlet velocity triangle,

$$\tan\beta_2 = \frac{V_{f_2}}{u_2 - V_{w_2}} \rightarrow \cot\beta_2 = \frac{u_2 - V_{w_2}}{V_{f_2}}$$

$$\frac{WD}{m} = \frac{V_{w_2}u_2 - \cancel{V_{w_1}u_1}}{g}$$

$$u_2 - V_{w_2} = V_{f_2}\cot\beta_2$$

$$V_{w_2} = u_2 - V_{f_2}\cot\beta_2 \longrightarrow ②$$

$V_2^2 = V_{w_2}^2 + V_{f_2}^2 \rightarrow$ Substitute for V_{w_2} from ②.

$$V_2^2 = (u_2 - V_{f_2}\cot\beta_2)^2 + V_{f_2}^2$$

$$V_2^2 = u_2^2 - 2u_2V_{f_2}\cot\beta_2 + V_{f_2}^2\cot\beta_2^2 + V_{f_2}^2$$

$\cosec^2\theta = 1 + \cot^2\theta$

$$V_2^2 = u_2^2 - 2u_2V_{f_2}\cot\beta_2 + V_{f_2}^2(1 + \cot\beta_2^2)$$

$$V_2^2 = u_2^2 - 2u_2V_{f_2}\cot\beta_2 + V_{f_2}^2\cosec^2\beta_2 \longrightarrow ③$$

Substitute for V_1, V_2 and V_{w_2} in equation ①

$$\frac{P_2}{\rho g} - \frac{P_1}{\rho g} = \frac{V_{f_1}^2}{2g} - \frac{\left(u_2^2 - 2u_2V_{f_2}\cot\beta_2 + V_{f_2}^2\cosec^2\beta_2\right)}{2g} + \frac{(u_2 - V_{f_2}\cot\beta_2)u_2}{g}$$

$$= \frac{1}{2g}\left[V_{f_1}^2 - u_2^2 + 2u_2V_{f_2}\cot\beta_2 - V_{f_2}^2\operatorname{cosec}^2\beta_2\right] + \frac{u_2^2 - u_2V_{f_2}\cot\beta_2}{g}.$$

$$= \frac{1}{2g}\left[V_{f_1}^2 - u_2^2 + \cancel{2u_2V_{f_2}\cot\beta_2} - V_{f_2}^2\operatorname{cosec}^2\beta_2 + 2u_2^2 - \cancel{2u_2V_{f_2}\cot\beta_2}\right]$$

$$= \frac{1}{2g}\left[V_{f_1}^2 - u_2^2 - V_{f_2}^2\operatorname{cosec}^2\beta_2 + 2u_2^2\right]$$

$$\left.\begin{aligned}\frac{p_2}{\rho g} - \frac{p_1}{\rho g} &= \frac{1}{2g}\left[V_{f_1}^2 + u_2^2 - V_{f_2}^2\operatorname{cosec}^2\beta_2\right]\\ \frac{p_2}{\rho g} - \frac{p_1}{\rho g} &= \frac{V_{f_1}^2}{2g} + \frac{u_2^2}{2g} - \frac{V_{f_2}^2\operatorname{cosec}^2\beta_2}{2g}.\end{aligned}\right\} \rightarrow (4)$$