UNIT – 4

Availability and Exergy: Available and unavailable energy, concept of availability, availability of heat source at constant and variable Temperatures, Dead state, Exergy balance equation and Exergy analysis for non-flow and steady flow systems, Helmholtz and Gibbs function, second law efficiency. **07 hours**

① Any real process → Irreversible

② Two approaches are used for analysis of such processes [qualitative].

ⓐ Concept of entropy → Entropy generation
→ Lost work [Cannot be recovered]

ⓑ Concept of availability (Exergy) and unavailability (Anergy) → Exergy destruction

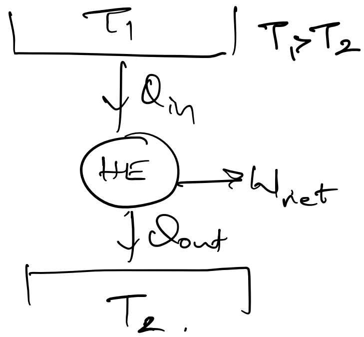
Energy (Work and Heat)

High grade energy | Low grade energy

High grade energy → Complete conversion from one form to another. eg: Work, Electricity, Water power, Tidal power,

Low grade energy → Partial conversion.
eg: Heat, Heat from combustion.

We have looked into efficiencies.



$\eta_{act} = \dfrac{W_{net}}{Q_{in}}$ [Actual efficiencies]. → ①

Carnot efficiencies for a reversible processes $\eta_{th_{rev}} = 1 - \dfrac{T_2}{T_1}$ → ②.

$\eta = 1 - \dfrac{Q_2}{Q_1}$

Efficiencies of many components such as turbines, compressor, condenser etc, they are called first law efficiencies.

Will look into more meaningful analysis is called as Second law efficiency [exergy].

Exergy [Availability].

Exergy is a property that enables us to determine the useful work potential of a given amount of energy at some state.

This state is referred as dead state @ $p_0 = 1.01325$ bar, $T_0 = 25°C$ $\underbrace{[P_0, T_0, h_0, u_0, S_0]}_{\text{Properties @ this dead state.}}$.

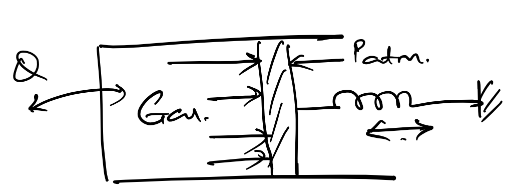
[Surroundings].

Exergy → European term.

Availability → MIT term.

$$W_{surr} = P_0(V_2 - V_1)$$

$$\left.\begin{array}{l} W_{useful} = \underset{\substack{\downarrow \\ \text{System.} \\ \text{[Reversible]}}}{\underline{W}} - \underset{\substack{\downarrow \\ \text{Surrounding.}}}{\underline{P_0(V_2 - V_1)}} \\ \text{[Availability].} \end{array}\right\}.$$

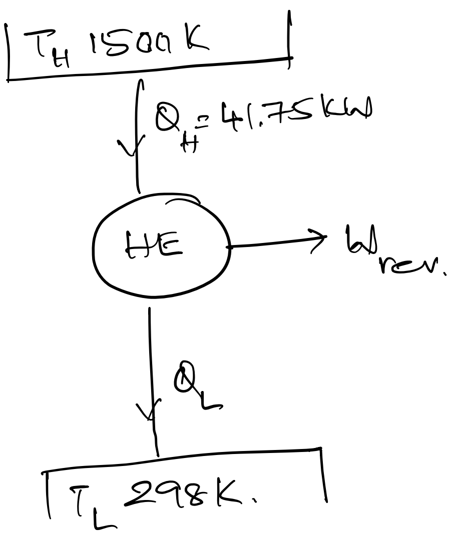


" Availability is the amount of useful work that could be get out of a system at a specified state by placing a reversible heat engine between given state point and the dead state."

It allows us to examine a process (or cycle and determine the condition for improvement.

$$\eta_{th\,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{Q_L}{Q_H} \Big\}\ \text{reversible.}$$

Example



$$\eta_{th\,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{298}{1500} = 0.8013.$$

$$W_{th\,rev} = \eta_{th\,rev} * Q_H \qquad \left[\eta_{th\,rev} = \frac{W_{th\,rev}}{Q_H}\right].$$

$$(W_{max}) = 0.8013 * 41.75 = \underset{\substack{\downarrow \\ \text{Max}}}{\underline{33.4\ \text{kW}}}$$

$$\underset{\substack{\uparrow \\ \text{Actual}}}{\underline{W_{useful}}} = \underline{W_{th\,rev}} - \underline{W_{surrounding}}.$$

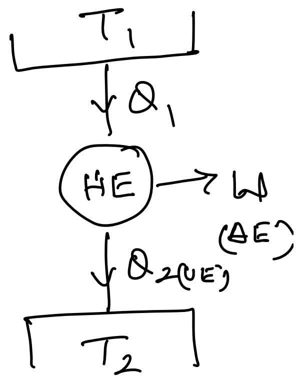
$$\eta_{th\,rev} = \frac{W_{th\,rev}}{Q_H}$$

Available energy referred to a cycle:

Heat supplied = Available Energy + Unavailable Energy

$Q_1 = W + Q_2.$

$$Q_1 = AE + UE$$



(or) $\underline{AE} = Q_1 - \underline{UE} \longrightarrow (1).$

W_{max} useful. $\qquad Q_2.$

For T_1 & T_2, $\eta_{rev} = 1 - \dfrac{T_2}{T_1} \longrightarrow (2)$

With the decrease in T_2 to T_0 [surroundings], → Dead state temp.

η_{rev} will increase.

$\therefore$ (2) becomes.

$$\eta_{rev} = 1 - \frac{T_0}{T_1} \quad \& \quad \eta_{rev} = \frac{W_{max}}{Q_1}$$

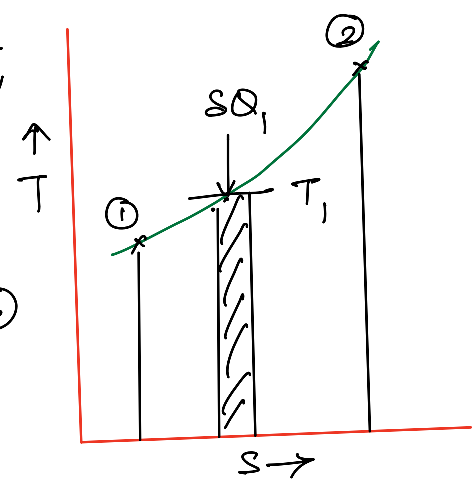
(or) $\quad W_{max} = \eta_{rev}\, Q_1$

$$W_{max} = \left(1 - \frac{T_0}{T_1}\right) Q_1 \longrightarrow (3).$$

If δQ_1 is the heat supplied @ T_1

then, $\delta W_{max} = \left(1 - \dfrac{T_0}{T_1}\right)\delta Q_1$

$$\delta W_{max} = \delta Q_1 - \frac{T_0}{T_1}\delta Q_1 \longrightarrow (4)$$



For the process from 1 to 2, integrating.

$$\int_1^2 \delta W_{max} = \int_1^2 \delta Q_1 - \int_1^2 \frac{T_0}{T_1}\delta Q_1 \qquad \frac{\delta Q}{T} = \Delta S.$$

$$\underbrace{W_{max}}_{AE} = \underbrace{Q_{1-2}}_{Q_1} - \underbrace{T_0(S_2 - S_1)}_{UE} \longrightarrow (5)$$

But $UE = Q_{1-2} - W_{max}$.

$$= Q_{1-2} - Q_{1-2} + T_0(S_2 - S_1)$$

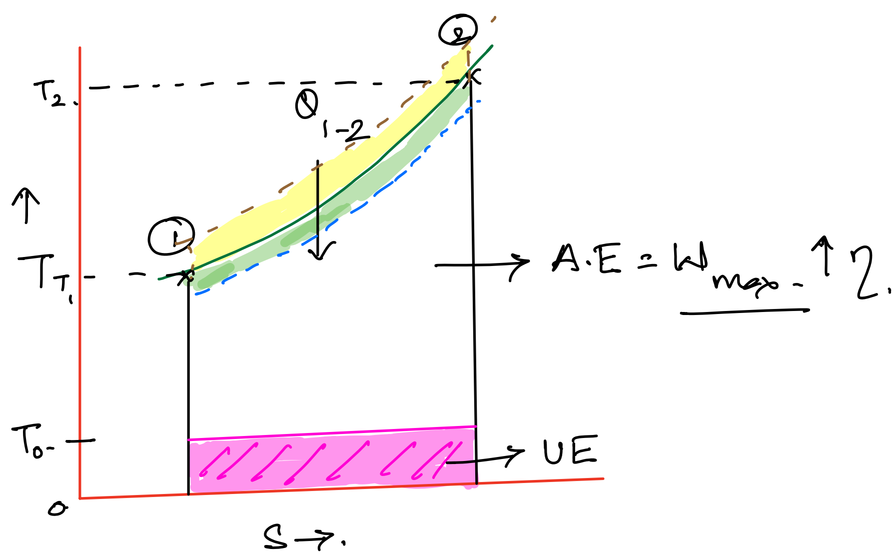
$$\left.\begin{aligned} UE &= T_0(S_2 - S_1) \\ UE &= T_0(S_f - S_i) \end{aligned}\right\} \longrightarrow (6)$$

S_f = Final Entropy point

S_i = Initial — " —

T_0 ↑ Lowest temp (surrounding)

So the unavailable energy is at the lowest possible temperature with change in entropy.

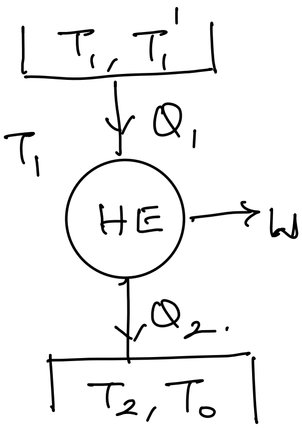


Available Energy due to finite temp. difference

$Q_1 = T_1 \Delta S = T_1' \Delta S' \longrightarrow (1)$ $\quad \because \frac{Q}{T} = \Delta S$

$Q_2 = T_2 \Delta S = T_0 \Delta S \longrightarrow (2)$.

$T_1' < T_1$

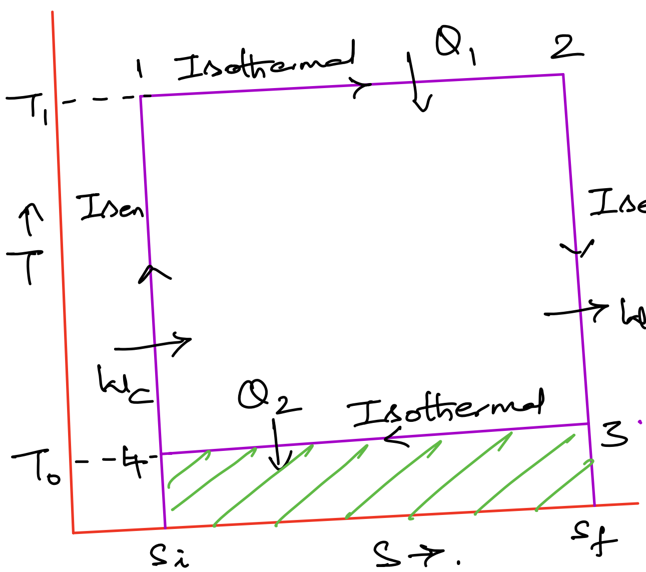


For the same $Q_1 \rightarrow \Delta S' > \Delta S$.

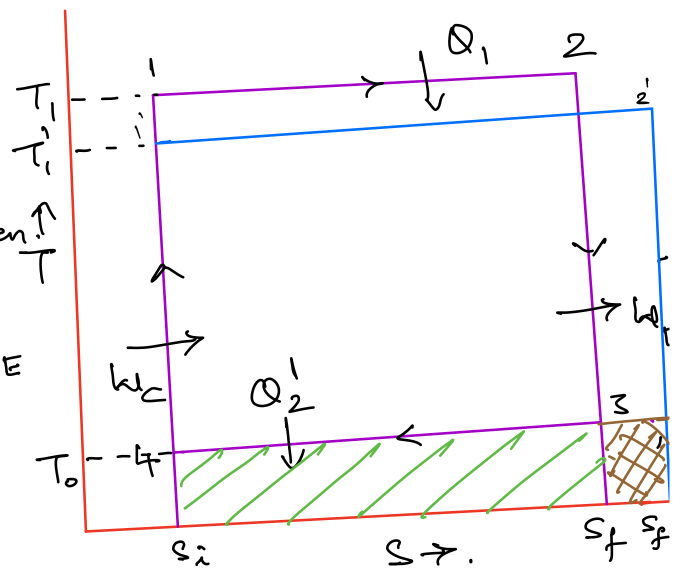
Compare (1) & (2), we conclude,

for $Q_1 > Q_2 \longrightarrow T_1' \Delta S' > T_0 \Delta S$

Carnot Cycle



Actual cycle



$W = Q_1 - Q_2 = T_1 \Delta S - T_0 \Delta S \rightarrow (3)$

$W' = Q_1 - Q_2' = T_1' \Delta S' - T_0 \Delta S' \longrightarrow (4)$.

$\Delta S = S_f - S_i$

$\Delta S' = S_f' - S_i$

As $W' < W$, because $Q_2' > Q_2$.

So loss of available energy due to irreversible heat transfer through finite temperature difference between the source and the working fluid during the heat addition process is given by;

$$W - W' = (Q_1 - Q_2) - (Q_1 - Q_2')$$

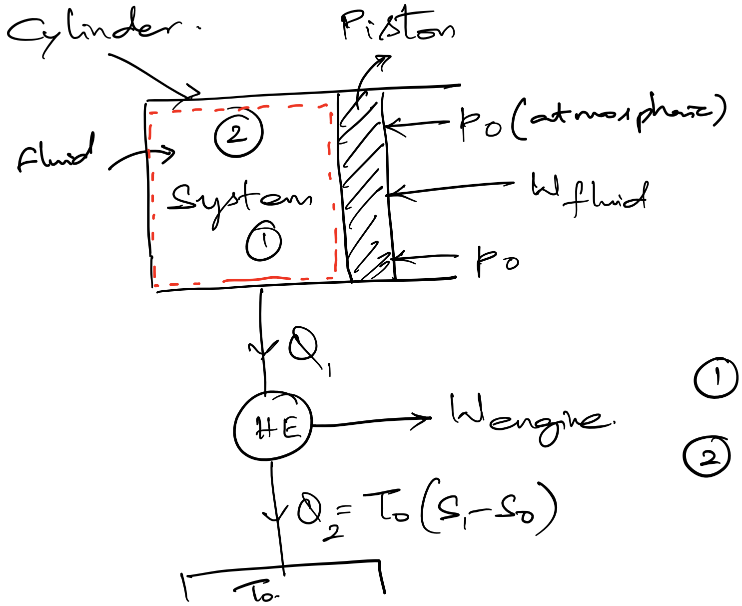
$$= \cancel{Q_1} - Q_2 - \cancel{Q_1} + Q_2'$$

$$= Q_2' - Q_2 \qquad \text{(Increase in UE)}$$

$$W - W' = T_0(\Delta s' - \Delta s) \rightarrow \text{Decrease in A.E}$$

Thus the decrease in A.E is the product of the lowest feasible temperature of heat rejection and the entropy change in the system. So greater the temperature difference $(T_1 - T_1')$, more the heat rejection Q_2' and an increase in unavailable part of the energy.

Availability in a non-flow systems [Exergy].



mass will remain same.

$W_{net} = \pm W_{fluid} \pm W_{engine}$.

$W_{atm} = p_0(V_1 - V_0) \rightarrow$ Constant pressure.

$1 \rightarrow p_1, V_1, T_1, u_1$

$2 \rightarrow p_2, V_2, T_2, u_2$.

For the engine.

$$W_{Eng} = Q_1 - Q_2$$

$$= Q_1 - T_0(S_1 - S_0) \longrightarrow 1$$

Piston cylinder arrangement [Heat balance]

Heat supplied to the engine = Heat rejected by the fluid

So. $-Q_1 = (U_0 - U_1) + W_{fluid}$. [I law thermody] [$-$ve, heat is rejected by the piston-cylinder arrangement].

$$W_{fluid} = (U_1 - U_0) - Q_1 \longrightarrow 2$$

Adding 1 & 2, we get .

$$\underbrace{W_{fluid} + W_{engine}}_{W_{net}} = (U_1 - U_0) - \cancel{Q_1} + \cancel{Q_1} - T_0(S_1 - S_0)$$

$$= (U_1 - U_0) - T_0(S_1 - S_0) \longrightarrow 3$$

The net workdone by the fluid on the piston is less than the total workdone by the fluid.

Work done on atmosphere $= p_0(V_0 - V_1) \rightarrow$ ④

Hence, maximum work available is [useful].

W_{max} [useful]. $= \underbrace{(U_1 - U_0) - T_0(S_1 - S_0)}_{W_{net}} - \underbrace{p_0(V_0 - V_1)}_{W_{atm}}$

$p_0, T_0 \rightarrow$ Dead state pr & temp.

$= U_1 - U_0 - T_0S_1 + T_0S_0 - p_0V_0 + p_0V_1$

$= \underbrace{(U_1 + p_0V_1 - T_0S_1)} - \underbrace{(U_0 + p_0V_0 - T_0S_0)} \rightarrow$ ⑤

W_{max} (useful) $= \alpha_1 - \alpha_0$. [OR] $\phi_1 - \phi_0$

∴ $\alpha = U + pV - Ts$ is called non-flow availability function. [ϕ].

If there exist @ state point ② with surroundings. then

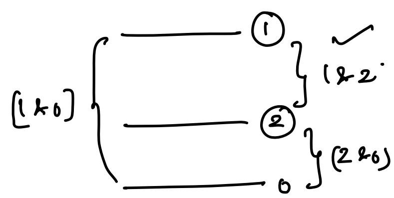
$\phi_2 - \phi_0 = \underbrace{(U_2 + p_0V_2 - T_0S_2)}_{\phi_2} - \underbrace{(U_0 + p_0V_0 - T_0S_0)}_{\phi_0} \rightarrow$ ⑥

Between system 1 & System 2. [⑥ − ⑤].

$\phi_2 - \phi_1 = (U_2 + p_0V_2 - T_0S_2) - (U_1 + p_0V_1 - T_0S_1)$

[If system 2 is @ higher energy level.]

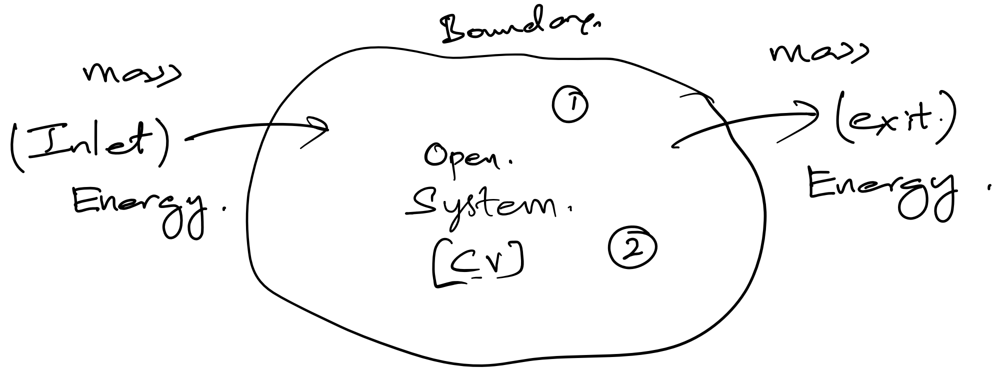
$\phi_2 - \phi_1 = (u_2 + p_0V_2 - T_0S_2) - \cancel{(u_0 + p_0V_0 - T_0S_0)} - (u_1 + p_0V_1 - T_0S_1)$
$+ \cancel{(u_0 + p_0V_0 - T_0S_0)}$



$= \underbrace{(u_2 + p_0V_2 - T_0S_2)} - \underbrace{(u_1 + p_0V_1 - T_0S_1)}$

$= (\alpha_2 - \alpha_1)$ (or) $(\phi_2 - \phi_1)$

Availability in a Steady flow system [Control Volume].



$$W_{useful} = \underbrace{(E_1 - E_0)}_{\text{Max Energy}} - \underbrace{T_0(s_1 - s_0)}_{UE} \qquad h = u + pv.$$

$\Delta KE = 0$

$\Delta PE = 0$.

$$E_1 = \underbrace{u_1 + p_1 v_1}_{} + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1} = h_1 + \cancel{\frac{V_1^2}{2}} \cancel{+ gz_1}$$

$$E_0 = \underbrace{u_0 + p_0 v_0}_{} + \cancel{\frac{V_0^2}{2}} + \cancel{gz_0} = h_0. \ [\text{@ dead state PE, KE} = 0$$

$$W_{useful} = \left(h_1 + \cancel{\frac{V_1^2}{2}} + \cancel{gz_1}\right) - h_0 - T_0(s_1 - s_0) \longrightarrow 6$$

If KE & PE are neglected depending upon the type of control volume (system)

$$AE = W_{useful} = \underbrace{(h_1 - T_0 s_1)}_{\varphi_1} - \underbrace{(h_0 - T_0 s_0)}_{\varphi_0} \longrightarrow 7$$

$\longrightarrow 1 \to 0$: $\varphi_1 - \varphi_0 = (h_1 - T_0 s_1) - (h_0 - T_0 s_0) \longrightarrow 8$

$2 \to 0$.

$$\varphi_2 - \varphi_0 = (h_2 - T_0 s_2) - (h_0 - T_0 s_0). \longrightarrow 9$$

For the process $1 \to 2$ $\qquad \varphi = h - Ts$.

$$\varphi_2 - \varphi_1 = \underbrace{(h_2 - T_0 s_2)}_{\varphi_2} - \underbrace{(h_1 - T_0 s_1)}_{\varphi_1} \longrightarrow 10.$$

φ = Availability function for flow system.

Helmholtz and Gibbs functions

Workdone in a non-flow reversible system (per unit mass) is given by

$$W_{1-0} = Q_{1-0} - (u_0 - u_1) \qquad [\text{I law } Q = W + \Delta u]$$

$$= T ds - (u_0 - u_1) \qquad \left[\text{II law } ds = \int_1^0 \frac{\delta Q}{T}\Big|_{rev.}\right]$$

$$= T(s_0 - s_1) - (u_0 - u_1)$$

ds = change in Entropy.

$$W_{1-0} = (u_1 - Ts_1) - (u_0 - Ts_0)$$

The term $(u - Ts)$ is called as Helmholtz function,

This gives the maximum possible output when the heat 'Q' is transferred at constant temperature and is the case with large heat source.

For work against atmosphere $\underbrace{p_0(v_0 - v_1)}$ → Constant pressure.

then the maximum work available;

$$\underbrace{W_{max}}_{AE} = W - p_0(v_0 - v_1) \rightarrow W_{atm.}$$

$$= (u_1 - Ts_1) - (u_0 - Ts_0) - p_0(v_0 - v_1) \qquad h = u + pv.$$

$$= (u_1 + p_0 v_1 - Ts_1) - (u_0 + p_0 v_0 - Ts_0)$$

$$= (h_1 - Ts_1) - (h_0 - Ts_0)$$

$$= g_1 - g_0$$

∴ $g = h - Ts$ is known as Gibb's function (or)

Free energy function

∴ The maximum possible available work when system changes from 1 to 2 is given by,

$$W_{max} = (g_1 - g_0) - (g_2 - g_0) = g_1 - g_2$$

For a steady flow system,

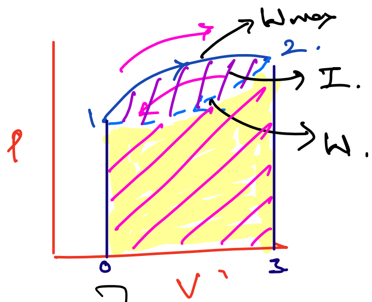
$$W_{max} = (g_1 - g_2) + \overbrace{(KE_1 - KE_2)}^{\Delta KE} + \overbrace{(PE_1 - PE_2)}^{\Delta PE}$$

Note: When state 1 proceeds to dead state (zero) then $\phi = \varphi = g$.

Irreversibility:

The actual work which a system does is always less than the idealized reversible work and the difference between the two is called the irreversibility of the process.

$I = W_{max} - W$. [Degradation (OR) dissipation]



For a non-flow process:-

(Intensive) $i = \frac{I}{m}$ → Extensive [per unit mass].

$$i = \underbrace{[(U_1 - U_2) - T_0(S_1 - S_2)]}_{W_{max}} - \underbrace{[(U_1 - U_2) + Q_1]}_{W}.$$

PV ≃ insignificant [Flow work]

$$i = T_0(s_2 - s_1) - Q$$

$$i = T_0 \Delta s_{sys} + T_0 \Delta s_{surr}$$

$$i = T_0 \left(\underbrace{\Delta s_{sys}}_{\text{System}} + \underbrace{\Delta s_{surr}}_{\text{Surrounding}}\right]$$

$$i \geq 0$$

$$\left[-Q = T_0 \Delta s_{surr} \right]$$

$$\Delta s_{surr.} = \frac{-Q}{T} \rightarrow \text{Interaction.}$$

Univ = Sys + Surroundings.

For steady flow-process.

$$\varphi = h - Ts.$$

$$\varphi_1 = h_1 - T_0 s_1$$

$$\varphi_2 = h_2 - T_0 s_2.$$

$$i = W_{max} - W$$

$$i = \Big[\underbrace{\left(\varphi_1 + \frac{\vec{V}_1^2}{2} + gz_1\right) - \left(\varphi_2 + \frac{\vec{V}_2^2}{2} + gz_2\right)}_{W_{max}}\Big]$$

$$- \Big[\underbrace{\left(h_1 + \frac{\vec{V}_1^2}{2} + gz_1\right) - \left(h_2 + \frac{\vec{V}_2^2}{2} + gz_2\right) + Q}_{W.}\Big]$$

$$i = T_0(s_2 - s_1) - Q$$

$$i = T_0 \Delta s_{sys} + T_0 \Delta s_{surr}$$

$$i = T_0 \left[\underbrace{\Delta s_{sys}}_{sys} + \underbrace{\Delta s_{surr}}_{surr.}\right], \quad i \geq 0.$$

Expression is same for both
- Non-flow
- Steady flow. (Control Volume)

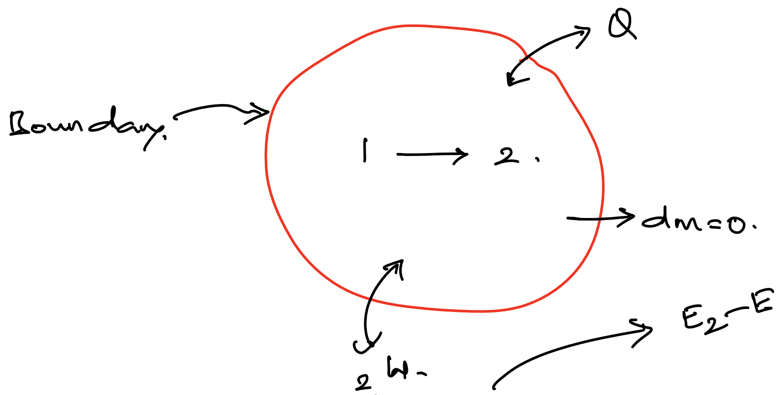
Exergy balance:

1) Exergy in − Exergy out = 0.

2) Exergy in − Exergy out = Exergy destroyed.

(a) Closed system (b) Control volume (or open system.

Exergy balance for a closed system



$E_2 - E_1 = Q_{1-2} - W_{1-2}$.

I law $\longrightarrow$ $E_2 - E_1 = \int_1^2 \delta Q - W_{1-2} \longrightarrow$ (1). $\quad \left[\because W_{1-2} = \int_1^2 \delta W\right]$.

II law $\longrightarrow$ $S_2 - S_1 - \int_1^2 \frac{\delta Q}{T} = S_{gen}$.

Multiply by T_0 on both sides.

$$T_0(S_2 - S_1) - T_0 \int_1^2 \frac{\delta Q}{T} = T_0 S_{gen} \longrightarrow (2').$$

(1) − (2'). [Substract (2) from (1)].

$$E_2 - E_1 - T_0(S_2 - S_1) + T_0 \int_1^2 \frac{\delta Q}{T} = \int_1^2 \delta Q - W_{1-2} - T_0 S_{gen}.$$

$$E_2 - E_1 - T_0(S_2 - S_1) = \int_1^2 \delta Q - T_0 \int_1^2 \frac{\delta Q}{T} - W_{1-2} - T_0 S_{gen}.$$

$$E_2 - E_1 - T_0(S_2 - S_1) = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q - W_{1-2} - T_0 S_{gen}. \rightarrow (3)$$

If the Available energy $1 \rightarrow 2$.

$$A_2 - A_1 = E_2 - E_1 + P_0(V_2 - V_1) - T_0(S_2 - S_1). \quad \text{from (3).}$$

$$A_2 - A_1 = \int_1^2 \left(1 - \frac{T_0}{T}\right) \delta Q - W_{1-2} + P_0(V_2 - V_1) - T_0 S_{gen}.$$

$$\underbrace{A_2 - A_1}_{\text{Change in Availability (or) Exergy}} = \underbrace{\int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q}_{\text{Exergy transfer with heat}} - \underbrace{\left[W_{1-2} - P_0(V_2 - V_1)\right]}_{\text{Exergy transfer with work}} - \underbrace{T_0 S_{gen}}_{\text{Exergy lost [destroyed]}} \rightarrow (4)$$

From equation (4) we can write a generalize equation,

$$\underbrace{\frac{dA}{dt}}_{\text{Rate of change of Exergy}} = \underbrace{\sum_j \left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j}_{\text{Rate of Exergy with heat transfer}} - \underbrace{\left[\dot{W} - P_0 \frac{dV}{dt}\right]}_{\text{Rate of Exergy with work transfer}} - \underbrace{\dot{I}}_{\text{Rate of exergy destroyed [Irreversibility]}} \rightarrow (5)$$

(X)

Special case:

An Isolated system. $dA = -I$ (or) $\frac{dA}{dt} = -\dot{I}$.

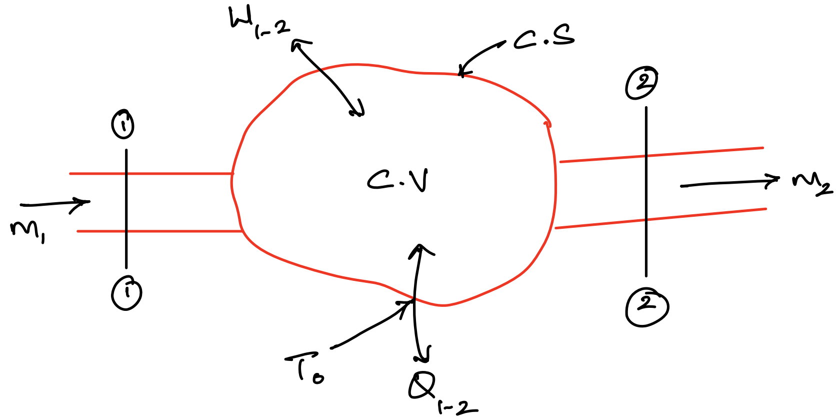
Comment:

1) $I > 0 \longrightarrow$ obeys II law.

2) $I = 0 \longrightarrow$ ———— " ————

3) $I < 0 \longrightarrow$ It violate II law & Impossible.

Exergy balance for a steady flow system [C.V].



(SFEE)

I law $\rightarrow$ $$Q_{1-2} + H_1 + \frac{m\vec{V}_1^2}{2} + mgZ_1 = H_2 + m\frac{\vec{V}_2^2}{2} + mgZ_2 + \dot{W}_{1-2} \rightarrow ①$$

II law $\rightarrow$ $$(S_2 - S_1) - \int_1^2 \frac{\delta Q}{T} = S_{gen}.$$

T_0 = Dead state absolute Temperature.

Multiply by T_0 on both sides.

$$T_0(S_2 - S_1) - T_0\int_1^2 \frac{\delta Q}{T} = T_0 S_{gen} \rightarrow ②$$

Add ① & ② for further simplification

$$H_1 + m\frac{V_1^2}{2} + mgZ_1 + Q_{1-2} + T_0(S_2 - S_1) - T_0\int_1^2 \frac{\delta Q}{T} = H_2 + m\frac{V_2^2}{2} + mgZ_2 + W_{1-2} + T_0 S_{gen}.$$

$$T_0(S_2 - S_1) - T_0\int_1^2 \frac{\delta Q}{T} + \int_1^2 \delta Q = H_2 - H_1 + m\left(\frac{V_2^2 - V_1^2}{2}\right) + mg(Z_2 - Z_1) + T_0 S_{gen} + W_{1-2}$$

$$T_0(S_2 - S_1) + \int_1^2 \left(1 - \frac{T_0}{T}\right)\delta Q = H_2 - H_1 + m\left(\frac{V_2^2 - V_1^2}{2}\right) + mg(Z_2 - Z_1) + W_{1-2} + T_0 S_{gen}$$

Rearranging the terms.

$$\underline{H_2 - H_1 - T_0(S_2 - S_1) + m\left(\frac{V_2^2 - V_1^2}{2}\right) + mg(Z_2 - Z_1)} = \int_1^2 \left(1 - \frac{T_0}{T_1}\right)\delta Q - W_{1-2} - T_0 S_{gen}.$$

$$\underline{A_2 - A_1} = \int_1^2 \underline{\left(1 - \frac{T_0}{T}\right)\delta Q} - \underline{W_{1-2}} - \underline{T_0 S_{gen}} \rightarrow ③$$

where $A_2 - A_1 = (H_2 - H_1) - T_0(S_2 - S_1) + m\left(\frac{V_2^2 - V_1^2}{2}\right) + mg(Z_2 - Z_1)$

In the form of rate equation at steady state & flow.

$$\sum_j \left[1 - \frac{T_0}{T_j}\right]\dot{Q}_j - \dot{W}_{cv} + \dot{m}(a_{f_1} - a_{f_2}) - T_0 S_{gen} \rightarrow (4)$$

where, $a_{f_1} - a_{f_2} = (h_1 - h_2) - T_0(s_1 - s_2) + \frac{V_1^2 - V_2^2}{2} + g(Z_1 - Z_2)$

In terms of intensive properties

Efficiency: $\eta = \frac{\text{Workdone} \rightarrow W_{max}}{\text{Heat supplied} \rightarrow Q}$ First law efficiency. Energy

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

From the absolute temp scale.

$$\eta_{max} = \eta_{rev} = 1 - \frac{T_2}{T_1} \quad [\text{Carnot efficiency}]$$

Exergy balance is carried out to increase the work O/p. of any device.

Second law efficiency: (Exergetic)

$$\eta_{II} = \frac{\eta_{th}}{\eta_{th\,rev.}} \quad \text{[Relative one]} \qquad \eta_{th} = \frac{W}{Q.} \qquad \eta_{th\,rev} = 1 - \frac{T_2}{T_1}$$

rev. → Carnot engine.

A ratio of the thermal efficiency of an actual heat engine to that of a reversible heat engine.

$$\eta_{II} = \frac{\text{Exergy recovered}}{\text{Exergy supplied.}} = \frac{\text{Exergy converted}}{\text{Exergy input.}}$$

Example: Heat engine:

$W_{max} = W_{rev}$ & W_{useful} = Available energy.

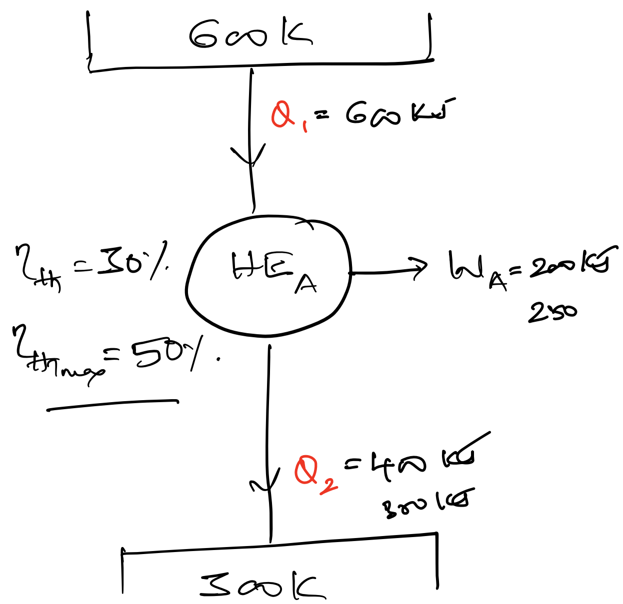
$$\eta_{II} = \frac{\left.\dfrac{W_{useful}}{W_{max}\ (\text{OR})\ W_{rev.}}\right\}\text{Actual.}}{\left.\dfrac{W_{max}\ (\text{OR})\ W_{rev.}}{Q_{supplied}}\right\}\text{Reversible. (ideal)}}$$

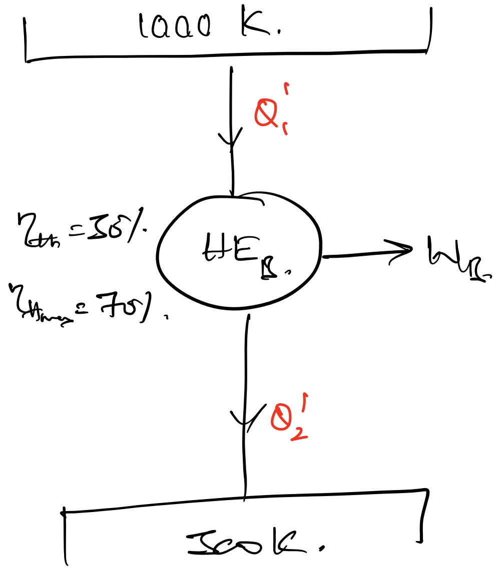
Ⓧ Thermodynamic cycles.

$$\eta_{relative} = \frac{\eta_{cycle.}}{\eta_{Carnot.}}$$

↑ Second law η.

{ 1. Otto cycle
Diesel cycle
Dual combustion cycle.
Brayton cycle }





(I) $\eta_{II} = \frac{0.30}{0.50} = 0.6$. ✓

(II) $\eta_{II} = \frac{0.30}{0.70} = 0.43$. ✓

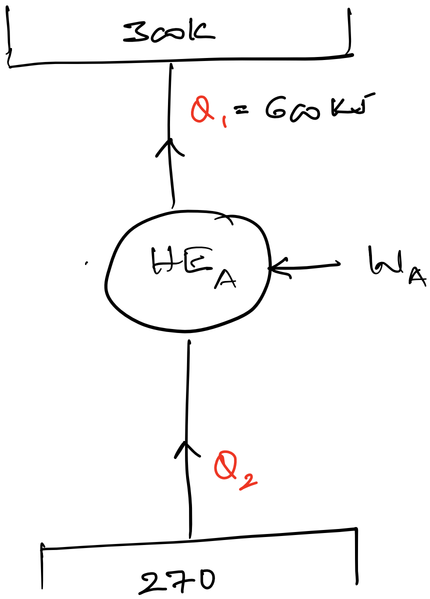
(*) Engine 'A' is converting 60% of the available work potential to useful work.

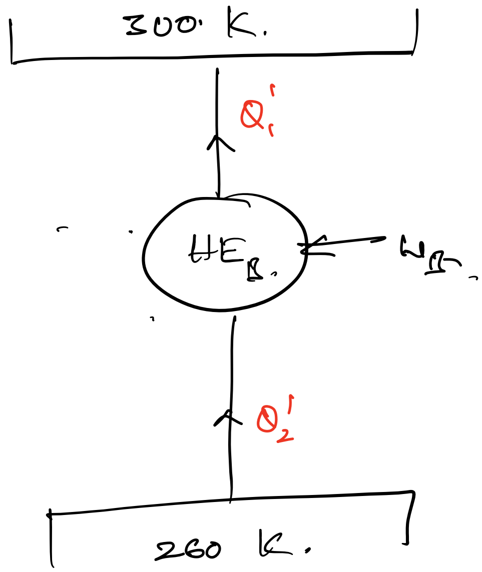
Engine 'B' is converting only 43% of the available work potential to useful work

Refrigerators and Heat pumps!

$$\eta_{II} = \frac{C.OP_{act}}{C.OP_{rev.}} < 1. \ [100 \text{ percent}].$$

$$\eta_{II} = \frac{1.5}{4.} < 1.$$





I $\quad \eta_{II} = \frac{5.0}{9.0} =$

$$c.op_{rev} = \frac{T_2}{T_1 - T_2}$$

$$= \frac{270}{(300-270)}$$

$$= 9$$

$$c.op_{act} = \frac{Q_2}{W_A}$$

$$9 > 5 = \frac{2.5}{0.5}$$

$$9 > 5 = \frac{2.0}{0.4}$$

4

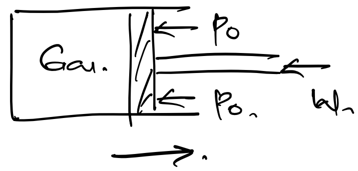
> 6.

II $\quad \eta_{II} = \frac{}{6.5} =$

$$c.op_{rev} = \frac{260}{300-260}$$

$$= 6.5$$

Numericals on Exergy analysis.



① $W_{surr} = p_0(V_1 - V_0)$

② $W_{max} = Q - \underbrace{T_0(S_1 - S_0)}_{U.E}$

③ Availability in non-flow system (ϕ)

AE $\begin{cases} W_{useful_{1-0}} = (U_1 + p_0V_1 - T_0S_1) - (U_0 + p_0V_0 - T_0S_0) \\ W_{useful_{1-2}} = (U_1 + p_0V_1 - T_0S_1) - (U_2 + p_0V_2 - T_0S_2). \end{cases}$ X $[2-1]$

④ Availability in steady-flow system (ψ)

AE $\begin{cases} W_{useful_{1-0}} = (h_1 - T_0S_1) - (h_0\ T_0S_0) \\ W_{useful_{1-2}} = (h_1 - T_0S_1) - (h_2 - T_0S_2) \end{cases}$ X $[2-1]$

⑤ Irreversibility $\Rightarrow i = \underbrace{[(U_1 - U_2) - T_0(S_1 - S_2)]}_{W_{max}} - \underbrace{[(U_1 - U_2) + Q_1]}_{W_{act}}$.

$$W_{max} = (U_1 - U_2) - T_0(S_1 - S_2)$$

(or) $\quad$ " $= T_0(S_2 - S_1) - (U_2 - U_1)$ $\Big\}$ Both are same.

⑥ $i = T_0[\Delta S_{sys} + \Delta S_{surr}]$.

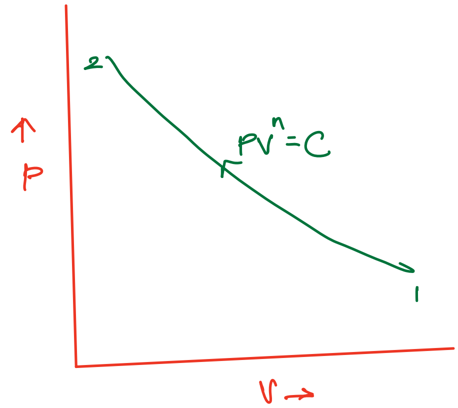
Numericals on Exergy analysis.

1. One kg of air is compressed polytropically from 1 bar pressure and temperature of 300 K to a pressure of 6.8 bar and temperature of 370 K. Determine the irreversibility if the sink temperature is 293 K. Assume R = 0.287 kJ/kg K, C_p= 1.005 kJ/kg K and C_v= = 0.716 kJ/kg K.

Sol: Given: $P_1 = 1\,bar$; $T_1 = 300K$; $P_2 = 6.8\,bar$; $T_2 = 370K$; $T_0 = 293K$;

$R = 0.287\ kJ/kg\text{-}K$; $C_p = 1.005\ kJ/kg\text{-}K$; $C_v = 0.716\ kJ/kg\text{-}K$. $m = 1kg$.

To find: i) I. ii) W_{max} iii) W_{actual}.



For a closed system:

$$W_{max} = (u_2 - u_1) - T_0(s_2 - s_1) \rightarrow Q.$$

$u = f(T)$

$$u_2 - u_1 = C_v(T_2 - T_1)$$
$$= 0.716(370 - 300)$$
$$u_2 - u_1 = 50.12\ kJ/kg \uparrow.$$

$$s_2 - s_1 = C_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{P_2}{P_1}\right)$$
$$= 1.005 \times \ln\left(\frac{370}{300}\right) - 0.287 \times \ln\left(\frac{6.8}{1}\right)$$
$$= -0.339\ kJ/kg\text{-}K.$$

I law $\rightarrow W_{1-2} = Q_{1-2} - \Delta u.$

$W_{1-2} = (u_2 - u_1) - Q_{1-2}$

$$\therefore W_{max} = 50.12 \times 10^3 - 293 \times -0.339 \times 10^3.$$

$W_{max} = 149.45\ kJ/kg.$ [Compressive work]

(X) As the index of compression is 'n'.

$P_1V_1^\gamma = P_2V_2^\gamma = C$; $P_1V_1^n = P_2V_2^n = C$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \qquad \left\{ \because \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \right\}$$

$$\frac{n-1}{n} = \frac{\ln\left(\frac{T_2}{T_1}\right)}{\ln\left(\frac{P_2}{P_1}\right)} = \frac{\ln\left(\frac{370}{300}\right)}{\ln\left(\frac{6.8}{1}\right)} \Rightarrow$$

$n = 1.123$

Actual work in a polytropic process.

$$W_{act} = \frac{P_2V_2 - P_1V_1}{n-1} = \frac{mR(T_2-T_1)}{n-1} = \frac{1 \times 0.287(370-300)}{1.123-1}$$

$$W_{act} = 163.33 \text{ KJ/kg.}$$

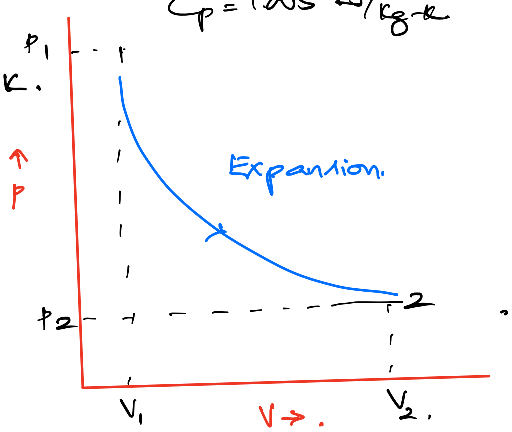
$$\therefore I_{rrev} = W_{max} - W_{act} = 149.45 - 163.33$$

$$= -13.8 \text{ KJ/kg. (Compressive).}$$

2. Calculate the maximum work developed when air expands in a piston-cylinder arrangement from 600 kpa and 150^oC to 150 kpa and 50^oC. Take $T_o = 300\ K$ and $p_o = 100\ kpa$. Also find the availablity at initial and final states. $m = 1\text{kg}$.

Sol: $P_1 = 600\text{kpa}$; $T_1 = 150+273 = 423K$; $P_2 = 150\text{kpa}$; $T_2 = 50+273 = 323K$.

$T_o = 300K$; $P_o = 100\text{kpa}$. $C_v = 0.72$ KJ/kgK. $C_p = 1.005$ KJ/kgK. $C_p = 1.005$ KJ/kg-K



$$W_{max} = \underline{(u_1-u_2)} - \underline{T_0(s_1-s_2)}$$

$C_v = f(T)$. ↑ P

$$u_1 - u_2 = C_v(T_1 - T_2)$$

$$= 0.72(423-323)$$

$$u_1 - u_2 = 72 \text{ KJ/kg}$$

$$s_1 - s_2 = C_p \ln\left(\frac{T_1}{T_2}\right) - R\ln\left(\frac{P_1}{P_2}\right)$$

$$= 1.005 \ln\left(\frac{423}{323}\right) - 0.287 \ln\left(\frac{600}{150}\right)$$

$$s_1 - s_2 = -0.126 \text{ KJ/kg-K.}$$

$$W_{max} = 72 - 300 \times -0.126$$

$$= 109.8 \text{ KJ/kg. (Expansion).}$$

$$P_1V_1 = mRT_1 \rightarrow V_1 = \frac{mRT_1}{P_1} = \frac{1 \times 0.287 \times 10^3 \times 423}{600 \times 10^3} = 0.2 \text{ m}^3/\text{kg.}$$

$$P_2V_2 = mRT_2 \rightarrow V_2 = \frac{mRT_2}{P_2} = \frac{1 \times 0.287 \times 323}{150} = 0.618 \text{ m}^3/\text{kg.}$$

$$P_oV_o = mRT_o \rightarrow V_o = \frac{mRT_o}{P_o} = \frac{1 \times 0.287 \times 300}{100} = 0.861 \text{ m}^3/\text{kg.}$$

Availability @ 1 [Initial state].

$$(\phi_1-\phi_0)=(u_1+p_0V_1-T_0s_1)-(u_0+p_0V_0-T_0s_0)$$

$$=(u_1-u_0)+p_0(V_1-V_0)-T_0(s_1-s_0)$$

$$u_1-u_0=C_v(T_1-T_0)=0.72(423-300)=88.56\ \text{kJ/kg}.$$

$$s_1-s_0=C_p\ln\left(\frac{T_1}{T_0}\right)-R\ln\left(\frac{p_1}{p_0}\right)=1.005\ln\left(\frac{423}{300}\right)-0.287\ln\left(\frac{600}{100}\right)=-0.169\ \text{kJ/kg}$$

$$\phi_1-\phi_0=88.56+100(0.202-0.861)-300\times-0.169.$$

$$\phi_1-\phi_0=73.36\ \text{kJ/kg}.$$

Availability @ 2. [Final state]

$$\phi_2-\phi_0=(u_2+p_0V_2-T_0s_2)-(u_0+p_0V_0-T_0s_0)$$

$$=(u_2-u_0)+p_0(V_2-V_0)-T_0(s_2-s_0)$$

$$=C_v(T_2-T_0)+p_0(V_2-V_0)-T_0\left[C_p\ln\left(\frac{T_2}{T_0}\right)-R\ln\left(\frac{p_2}{p_0}\right)\right]$$

$$=0.72(323-300)+100(0.618-0.861)-300\left[1.005\ln\left(\frac{323}{300}\right)-0.287\ln\left(\frac{150}{100}\right)\right]$$

$$=4.86\ \text{kJ/kg}.$$

(X) Availability between 1 & 2.

$$\therefore\ \phi_1-\phi_2=(\phi_1-\phi_0)-(\phi_2-\phi_0)$$

$$=73.76-4.86$$

$$\phi_1-\phi_2=68.5\ \text{kJ/kg}.$$

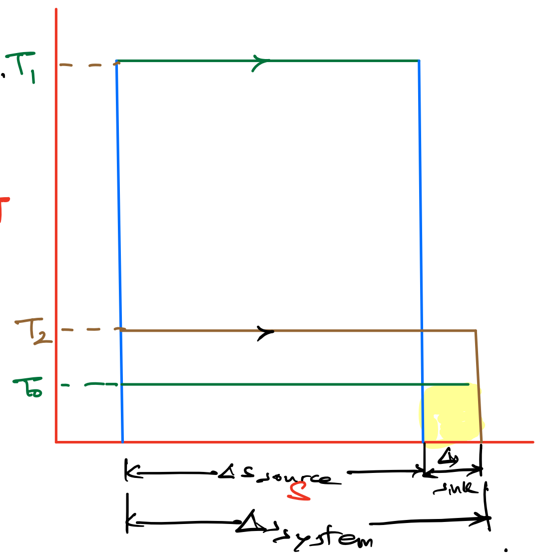
3. A system at 500 K receives 7200 kJ/min from a source at 1000 K. The temperature of atmosphere is 300 K. Assuming that the temperatures of system and source remain constant during heat transfer find out:

   (a) The entropy produced during heat transfer

   (b) The decrease in available energy after heat transfer

Sol: $T_2 = 500\,K$; $T_1 = 1000\,K$; $Q = 7200\ kJ/min$; $T_0 = 300\,K$.

To find i) Δs ii) Decrease in AE



a) Entropy @ source due to heat transfer

$$= -\frac{Q}{T_1} = -\frac{7200}{1000} = -7.2\ kJ/min\text{-}K.$$

Entropy @ sink due to heat transfer

$$= \frac{Q}{T_2} = \frac{7200}{500} = 14.4\ kJ/min\text{-}K.$$

$$\Delta S_{net} = -7.2 + 14.4 = +7.2\ kJ/min\text{-}K.$$

b) Available Energy @ source

$$= -\frac{Q}{T_1}(T_1 - T_0) = -\frac{7200}{1000}(1000 - 300)$$

$$= -5040\ kJ/min.$$

Available Energy @ sink

$$= \frac{Q}{T_2}(T_2 - T_0) = \frac{7200}{500}(500 - 300)$$

$$= 2880\ kJ/min.$$

Change in available energy $= -5040 + 2880 = -2160\ kJ/min.$

Comment: There is a decrease in available energy of 2160 kJ/min due to heat transfer.

4. A centrifugal compressor handles 25 kg/min of air. Air enters the compressor at 1 bar and 15^oC and leaves the cmpressor at 2 bar and 94^oC. The environment temperature is 21^oC. What is the actual and minimum power required to drive the compressor? Neglect heat interaction and changes in kinetic and potential energies between inlet and exit of compressor.

Sol: Given: $\dot{m} = 25$ kg/min $= \frac{25}{60} = 0.42$ kg/s ; $P_1 = 1$ bar; $T_1 = 15 + 273 = 288$ K.

$P_2 = 2$ bar; $T_2 = 94 + 273 = 367$ K; $T_0 = 21 + 273 = 294$ K; $\dot{Q} = 0$; $\Delta KE = \Delta PE = 0$

To find ① W_{act} ② W_{min} [Work absorbing m/c]

From the SFEE from the I law; ∵ Neglecting ΔKE & ΔPE

$$\cancel{\dot{Q}} + \dot{m}\left[h_1 + \cancel{\frac{\vec{V}_1^2}{2}} + \cancel{gZ_1}\right] = \dot{m}\left[h_2 + \cancel{\frac{\vec{V}_2^2}{2}} + \cancel{gZ_2}\right] + W_{act}$$

$h = C_p T$

$h = f(T)$

$$W_{act} = \dot{m}\left[h_1 - h_2\right]$$
$$= \dot{m} C_p (T_1 - T_2)$$
$$= 0.42 \times 1.005 (288 - 367)$$
$$W_{act} = -33.34 \text{ kW} \quad [\text{Compression}]$$

Exergy analysis. ∵ Neglecting ΔKE & ΔPE

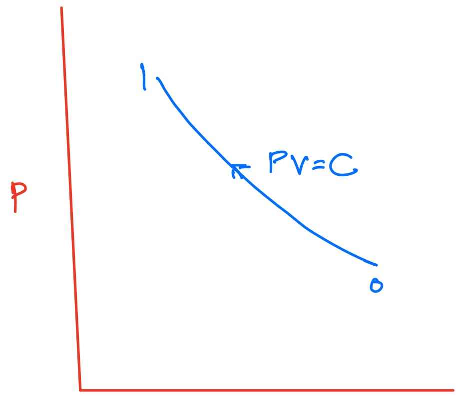
$$W_{min} = \dot{m}\left[(h_1 - h_2) - T_0(s_1 - s_2)\right]$$
$$= \dot{m}\left[C_p(T_1 - T_2) - T_0\left[C_p \ln\left(\frac{T_1}{T_2}\right) - R\ln\left(\frac{P_1}{P_2}\right)\right]\right]$$
$$= 0.42\left[1.005(288 - 367) - 294\left[1.005\ln\left(\frac{288}{367}\right) - 0.287\ln\left(\frac{1}{2}\right)\right]\right]$$
$$W_{min} = -27.82 \text{ kW} \quad [\text{Compression}]$$

5. A 200 m^3 rigid tank contains compressed air at 1 Mpa and 300 K. Determine how muuch work can be obtained from this air if the environment conditions are 100 kpa and 300 K.

Sol: Given: $V_1 = 200\,m^3$; $P_1 = 1\,Mpa$; $T_1 = 300K$.

$P_0 = 100\,kpa$; $T_0 = 300K$

To find W_{min} [Availability]



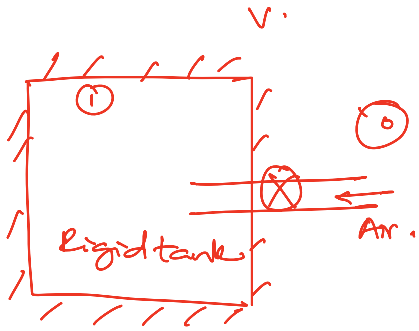
The temperature $T_0 = T_1$, so it is a constant temperature process, the change in internal energy is zero.

$P_0V_0 = P_1V_1$

$P_1V_1 = mRT_1$

$$m = \frac{P_1V_1}{RT_1} = \frac{1\times10^6\times200}{0.287\times10^3\times300}.$$

$m = 2323.\ kg.$



$$V_0 = \frac{P_1V_1}{P_0} = \frac{1\times10^6\times200}{100\times10^3}.$$

$V_0 = 2000\,m^3.$

$[\phi_1-\phi_0]$

$$W_{min} = m\left[\cancel{(u_1-u_0)} + P_0(V_1-V_0) - T_0(s_1-s_0)\right]$$

$$= m\left[P_0(V_1-V_0) - T_0\left[\cancel{C_p\ln\left(\frac{T_1}{T_0}\right)} - R\ln\left(\frac{P_1}{P_0}\right)\right]\right]$$

$$= m\left[P_0(V_1-V_0) + T_0R\ln\left(\frac{P_1}{P_0}\right)\right]$$

$$= 2323\left[100(200-2000) + 300\times0.287\ln\left(\frac{1\times10^6}{100\times10^3}\right)\right]$$

$W_{min} = -4.18\times10^8\ KJ$ [Compression].

9. 15 kg of water is heated in an insulated tank by a churning process from 300 K to 340 K. If the surrounding temperature is 300 K, find the loss in availability for the process

Sol: Given: $m = 15$ kg; $T_1 = 300K$; $T_2 = 340K$; $T_0 = 300K$

To find i) Unavailable energy $C_v = 4.187$ KJ/Kg-K.

Available energy [2-0]

$$AE = m\left[(u_2 - u_0) - T_0(s_2 - s_0)\right]$$

$$= m\left[C_v(T_2 - T_0) - T_0\left[C_v \ln\left(\frac{T_2}{T_0}\right) + \cancel{R\ln\left(\frac{V_2}{V_0}\right)}\right]\right]$$

∵ Constant Volume process.

$$= m\left[C_v(T_2 - T_0) - T_0 C_v \ln\left(\frac{T_2}{T_0}\right)\right]$$

$$= 15\left[4.187(340 - 300) - 300 \times 4.187 \ln\left(\frac{340}{300}\right)\right]$$

$$AE = 154.2 \text{ KJ}$$

Heat added due to churning process.

$$= mC_v(T_2 - T_1)$$

$$= 15 \times 4.187(340 - 300)$$

$$= 2512.2 \text{ KJ}$$

Loss in availability (Unavailable Energy) = Work/Heat added due to churning $- (\phi_2 - \phi_0)$

$$= 2512.2 - 154.2.$$

$$UE = 2358 \text{ KJ} \quad [\text{Unavailability}].$$

10. A closed system contains 2 kg of air during an adiabatic expansion process there occurs a change in its pressure from 500kPa to 100 kPa and in its temperature from 350 K to 320 K. if the volume doubles during the process make calculations for maximum work, the change in availability and irreversibility. Take for air Cv = 0.718 kJ/kg K and R = 0.287 kJ/kgK. The surrounding conditions may be assumed to be 100 kPa and 300 K.

Sol: $m = 2kg$; $PV^{\gamma} = C$; $p_1 = 500kpa$; $p_2 = 100kpa$; $T_1 = 350k$; $T_2 = 320k$.
$V_2 = 2V_1$; $C_v = 0.718\ KJ/kg\text{-}k$; $R = 0.287\ KJ/kg\text{-}k$: $P_0 = 100kpa$; $T_0 = 100kpa$.

To find: i) W_{max} ii) $\phi_1 - \phi_2$ iii) I.

i) Maximum work is given by;

$$W_{max} = m\left[(u_1-u_2) - T_0(s_1-s_2)\right].$$
$$= m\left[C_v(T_1-T_2) - T_0\left[C_v\ln\left(\frac{T_1}{T_2}\right) + R\ln\left(\frac{V_1}{V_2}\right)\right]\right]$$
$$= 2\left[0.718(350-300) - 300\left[0.718\ln\left(\frac{350}{300}\right) + 0.287\ln\left(\frac{V_1}{2V_1}\right)\right.\right.$$
$$W_{max} = 123.78\ KJ$$

ii) Change in availability.

$$\phi_1 - \phi_2 = m\left[(u_1-u_2) - T_0(s_1-s_2)\right] + P_0(V_1-V_2)$$
$$= m\left[C_v(T_1-T_2) - T_0\left[C_v\ln\left(\frac{T_1}{T_2}\right) + R\ln\left(\frac{V_1}{V_2}\right)\right]\right] + P_0(V_1-V_2)$$
$$= 123.78 + 100(0.402 - 2\times0.402)$$
$$\phi_1 - \phi_2 = 83.67\ KJ.$$

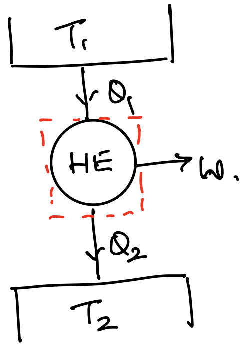
$$P_1V_1 = mRT_1$$
$$V_1 = \frac{mRT_1}{P_1} = \frac{2\times0.287\times350}{500}$$
$$V_1 = 0.402m^3.$$

iii) Irreversibility = $W_{max} - W_{actual}$.

$$W_{act} = \frac{P_1V_1 - P_2V_2}{\gamma-1} = \frac{500\times10^3\times0.402 - 100\times10^3\times0.804}{1.4-1}$$
$$W_{act} = 301.5\ KJ.$$

$$\therefore\ I = 123.78 - 301.5$$
$$I = -177.72\ KJ$$

11. A heat engine receives heat from a source at 1500 K at a rate of 600 kJ/s and rejects the waste heat to a sink at 300 K. If the power output of the engine is 400 kW. Determine the second-law efficiency of this heat engine.



Sol: $T_1 = 1500K$; $Q_1 = 600\ kJ/s$; $T_2 = 300K$; $W_o = 400\ kW$.

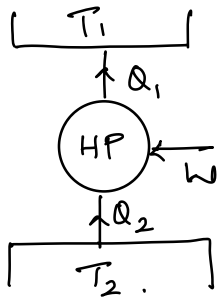
To find η_{II}

$$\eta_{rev} = \eta_{carnot} = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1500} = 0.8.$$

$$\eta_{act} = \frac{W}{Q_1} = \frac{400}{600} = 0.667$$

$$\eta_{rel} \Rightarrow \eta_{II} = \frac{\eta_{act}}{\eta_{carnot}} = \frac{0.667}{0.8} = 0.83.$$

12. Electric resistance heaters for residential buildings that have a COP of 1.0. Assuming an indoor temperature of 21°C and outdoor temperature of 10°C, determine the second-law efficiency of these heaters.



Sol: $C.OP_{HP} = 1.0$; $T_1 = 21°C = 294K$; $T_2 = 10°C = 283K$

$$C.OP_{rev.HP} = \frac{T_1}{T_1 - T_2} = \frac{294}{294-283} = 26.72.$$

$$\eta_{II} = \frac{C.OP_{HP}}{C.OP_{rev\ HP}} = \frac{1.0}{26.72} = 0.037 = 3.7\%.$$

Comment: As the efficiency of the heater is very less, it is not preferred.