Gas Power cycles: Air standard cycles: Otto cycle, Diesel cycle and Dual cycle, Brayton cycle.

06 hours

Laws of Thermodynamics.

1) Zeroth law → Thermal equilibrium.

A ⟷ B ⟷ C } Applications

Reference system   Thermometer   Body.

2) First law: →

Thermodynamic process → $Q = W + \Delta E$; $\delta Q = \delta W + dE$

Thermodynamic cycle → $\oint Q = \oint W$.

3) Second law: →

Thermodynamic process → $Q = W + \Delta E \,|$ realistic.

Thermodynamic cycle → $\oint Q = \oint W \,|$ realistic

Q → Q supplied, Q rejected.

[Q rejection] ↓ not recovered.

Expressions for Work transfer [PdV – Non flow work].

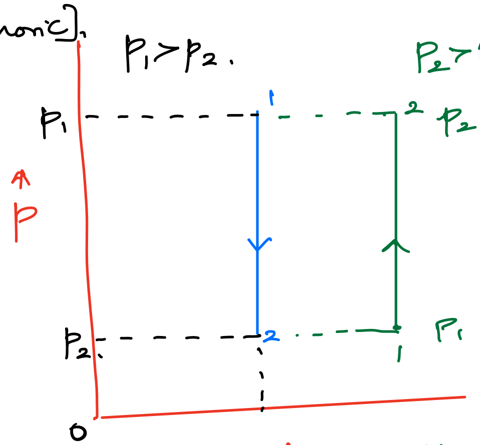
1) Constant Volume process: [Isochoric].

$$\int_1^2 \delta W = \int_1^2 p \cancel{dV}^{\,0}$$

[∵ $dV = V_1 - V_2 = 0$]   ∵ $V_1 = V_2$

$$W_{1-2} = 0$$

$P_1 > P_2$. $P_2 > P_1$



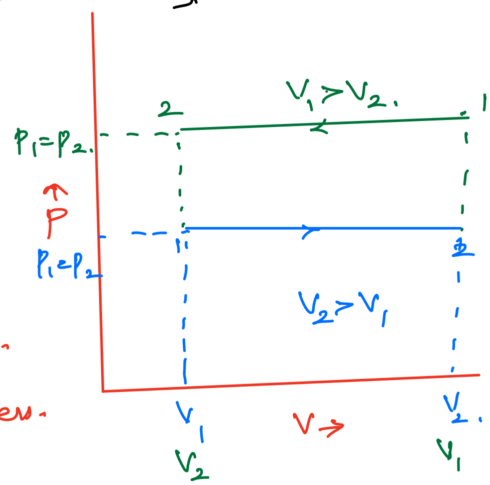
$V_1 = V_2$. $V_1 = V_2$.

2) Constant Pressure process: [Isobaric]

$$\int_1^2 \delta W = \int_1^2 p\,dV.$$

$$W_{1-2} = p\int_1^2 dV.$$

$$W_{1-2} = p\left[V_2 - V_1\right].$$



positive → Expansion process.

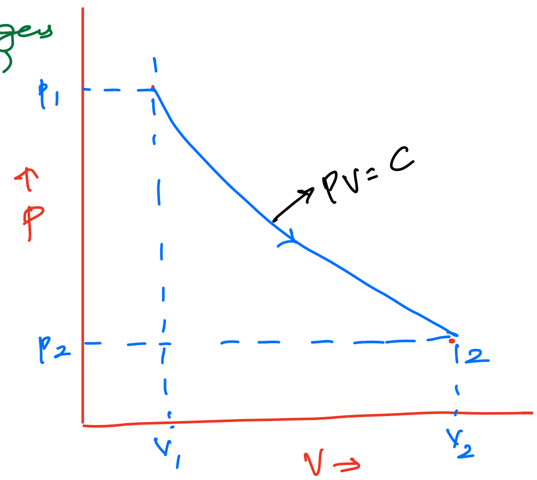
Negative → Compression process.

3) Constant temperature / Isothermal process.

$PV = mRT = C$ (Characteristic gas equation)

$P_1V_1 = P_2V_2 = PV = C \rightarrow (1)$

$$P = \frac{P_1V_1}{V} = \frac{P_2V_2}{V} \rightarrow (2)$$



$$\int_1^2 \delta W = \int_1^2 p\,dV.$$

$$W_{1-2} = \int_1^2 \frac{P_1V_1}{V}\,dV.$$

$$= P_1V_1 \int_1^2 \frac{1}{V}\,dV.$$

$$= P_1V_1 \ln\left[V\right]_{V_1}^{V_2}.$$

$$= P_1V_1 \ln\left[V_2 - V_1\right]$$

from (1) $P_1V_1 = P_2V_2$.

$$\frac{P_1}{P_2} = \frac{V_2}{V_1}$$

$$\left.\begin{aligned} W_{1-2} &= P_1V_1 \ln\frac{V_2}{V_1} = P_2V_2 \ln\frac{V_2}{V_1} \\ W_{1-2} &= P_1V_1 \ln\frac{P_1}{P_2} = P_2V_2 \ln\frac{P_1}{P_2} \end{aligned}\right\} (3)$$

④ Adiabatic process. [$Q=0$]

[If the process is reversible & adiabatic → Isentropic]

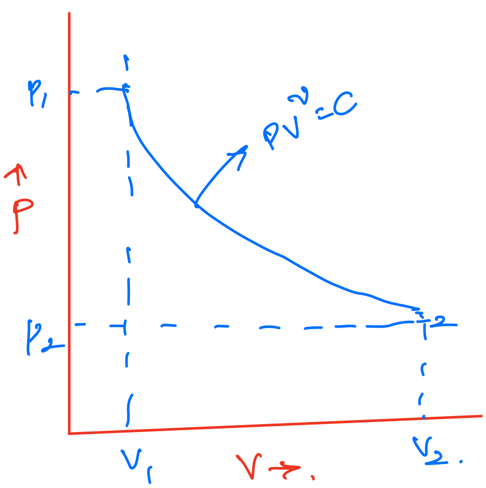
$\gamma = \frac{C_p}{C_v}$ for any fluid @ NTP.

Air → $C_p = 1.005$ kJ/kgK. $C_v = 0.718$ kJ/kgK.

$\gamma = 1.4$

$pV^{\gamma} = p_1 V_1^{\gamma} = p_2 V_2^{\gamma} = C \rightarrow ①$

$p = \frac{p_1 V_1^{\gamma}}{V^{\gamma}} \rightarrow ②$.



$$\int_1^2 \delta W = \int_1^2 p\,dV$$

$$W_{1-2} = \int_1^2 \frac{p_1 V_1^{\gamma}}{V^{\gamma}}\,dV.$$

$$= p_1 V_1^{\gamma} \int^2 V^{-\gamma}\,dV.$$

$$W_{1-2} = p_1 V_1^{\gamma} \left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_1}^{V_2}$$

$$= \frac{p_1 V_1^{\gamma}}{1-\gamma}\left[V_2^{(1-\gamma)} - V_1^{(1-\gamma)}\right].$$

$$W_{1-2} = \frac{p_1 V_1^{\gamma} V_2^{1-\gamma} - p_1 V_1^{\gamma} V_1^{1-\gamma}}{1-\gamma}.$$

$p_1 V_1^{\gamma} = p_2 V_2^{\gamma}$.

$$W_{1-2} = \frac{p_2 V_2^{\gamma} V_2^{1-\gamma} - p_1 V_1^{\gamma} V_1^{1-\gamma}}{1-\gamma}.$$

$$W_{1-2} = \frac{p_2 V_2 - p_1 V_1}{1-\gamma}. \rightarrow ③ \because \gamma > 1$$

$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} \rightarrow (4)$$

5) Polytropic process. [n] $PV^n = C \rightarrow$ Actual process.

X. $1 < n < \gamma$. $\rightarrow PV^n = P_1V_1^n = P_2V_2^n = C \rightarrow (1)$

$$P = \frac{P_1V_1^n}{V^n} \rightarrow (2)$$

$$\int_1^2 \delta W = \int_1^2 p\,dV.$$

$$W_{1-2} = \int_1^2 \frac{P_1V_1^n}{V^n}\,dV$$

$$= P_1V_1^n \int_1^2 V^{-n}\,dV.$$

$$= P_1V_1^n \left[\frac{V^{-n+1}}{-n+1}\right]_{V_1}^{V_2}$$

$$= \frac{P_1V_1^n}{1-n}\left[V_2^{1-n} - V_1^{1-n}\right]$$

$$= \frac{P_1V_1^n V_2^{1-n} - P_1V_1^n V_1^{1-n}}{1-n}. \qquad P_1V_1^n = P_2V_2^n.$$

$$W_{1-2} = \frac{P_2V_2 - P_1V_1}{1-n} \rightarrow (3) \quad n > 1$$

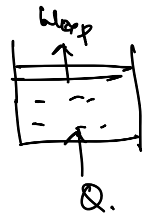
$$W_{1-2} = \frac{P_1V_1 - P_2V_2}{n-1} \rightarrow (4).$$

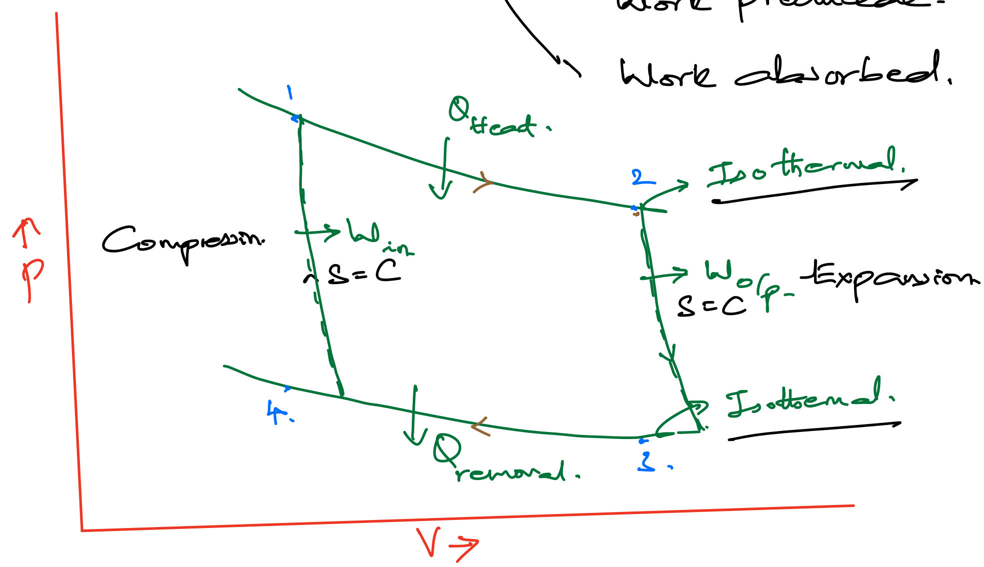
X. Home work $\rightarrow$ Draw the P-V diagram with all the thermodynamic processes super imposed on a single P-V diagram.

Thermodynamic Cycle.

Interactions → Heat → Heat addition.
Heat removal.

Interactions → Work. → Work produced.
Work absorbed.





Otto cycle: Q_H → Const volume process.
Q_L → Const Volume process. } Mixture = Air + fuel

Diesel Cycle: Q_H → Constant pressure
Q_L → Constant volume. } Intake → Air. dispersion. → Fuel

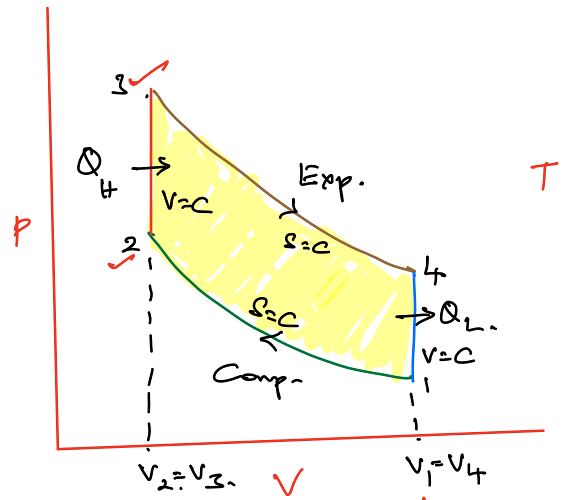
Dual: Q_H → Constant pressure & Const Volume
Q_L → Constant Volume

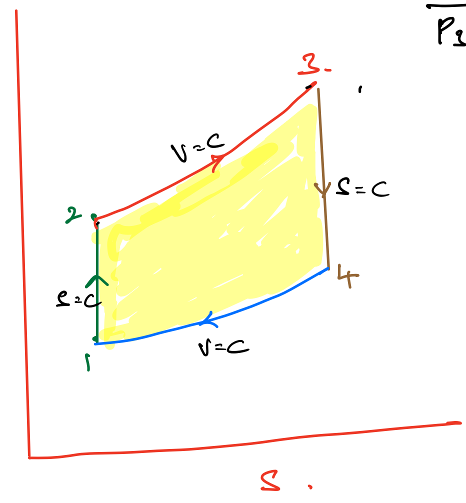
Brayton → Q_H → Constant pressure.
Q_L → Constant pressure

W_{Exp} & W_{comp} → Isentropic processes, it is common for all cycles.

Otto cycle:

$P_2 v_2 = mRT_2$ $\quad$ $P_3 v_3 = mRT_3$, $\rightarrow \dfrac{P_2 v_2 = mRT_2}{P_3 v_2 = mRT_3}$.



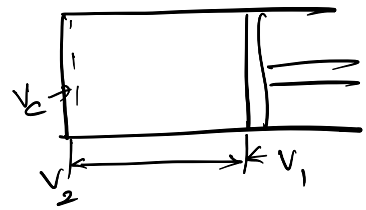


$|\leftarrow V_c \rightarrow|\leftarrow V_2 - V_1 \rightarrow|$

Compression ratio $\rightarrow r_c = \dfrac{V_2}{V_1} = \dfrac{V_3}{V_4} = r$

[Expansion ratio]

Expression for the workdone.



$$Q_H = m.C_v(T_3 - T_2) \rightarrow ①$$

$$Q_L = m.C_v(T_4 - T_1) \rightarrow ②$$

Workdone $= Q_H - Q_L$. $\rightarrow$ [Conversion Q to W]

$$W. = m.C_v(T_3 - T_2) - m.C_v(T_4 - T_1) \rightarrow ③$$

Thermal efficiency is given by.

$$\eta_{th} = \frac{\text{Workdone.}}{\text{Heat supplied.}}$$

$$\eta_{th} = \frac{m.C_v(T_3 - T_2) - m.C_v(T_4 - T_1)}{m.C_v(T_3 - T_2)}$$

$$\eta_{th} = \frac{(T_3 - T_2) - (T_4 - T_1)}{(T_3 - T_2)}$$

$$\eta_{th} = 1 - \frac{(T_4 - T_1) \rightarrow \Delta T_{rejection}}{(T_3 - T_2) \rightarrow \Delta T_{addition}} \longrightarrow (4)$$

Thermal efficiency in terms of compression/Expansion ratio.

$r_c = r_e = r \rightarrow$ Compression ratio.

Consider the isentropic compression & expansion process 1-2 & 3-4 respectively.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \rightarrow \frac{T_2}{T_1} = r^{\gamma-1} \rightarrow T_2 = T_1 r^{\gamma-1} \longrightarrow (5)$$

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} \rightarrow \frac{T_3}{T_4} = r^{\gamma-1} \rightarrow T_3 = T_4 r^{\gamma-1} \longrightarrow (6)$$

Substitute for T_2 & T_3 in (4)

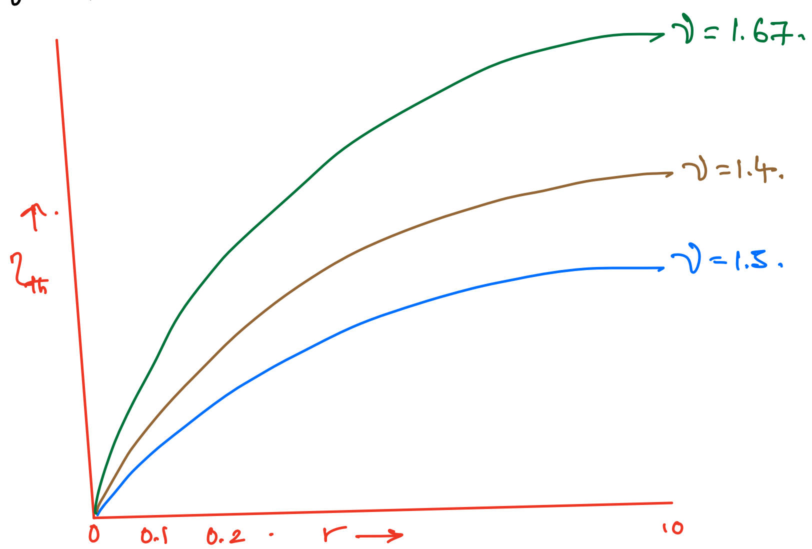
$$\therefore \eta_{th} = 1 - \frac{(T_4 - T_1)}{(T_4 r^{\gamma-1} - T_1 r^{\gamma-1})} = 1 - \frac{(T_4 - T_1)}{r^{\gamma-1}(T_4 - T_1)}$$

$$\eta_{th} = 1 - \frac{1}{r^{\gamma-1}} \longrightarrow (7)$$

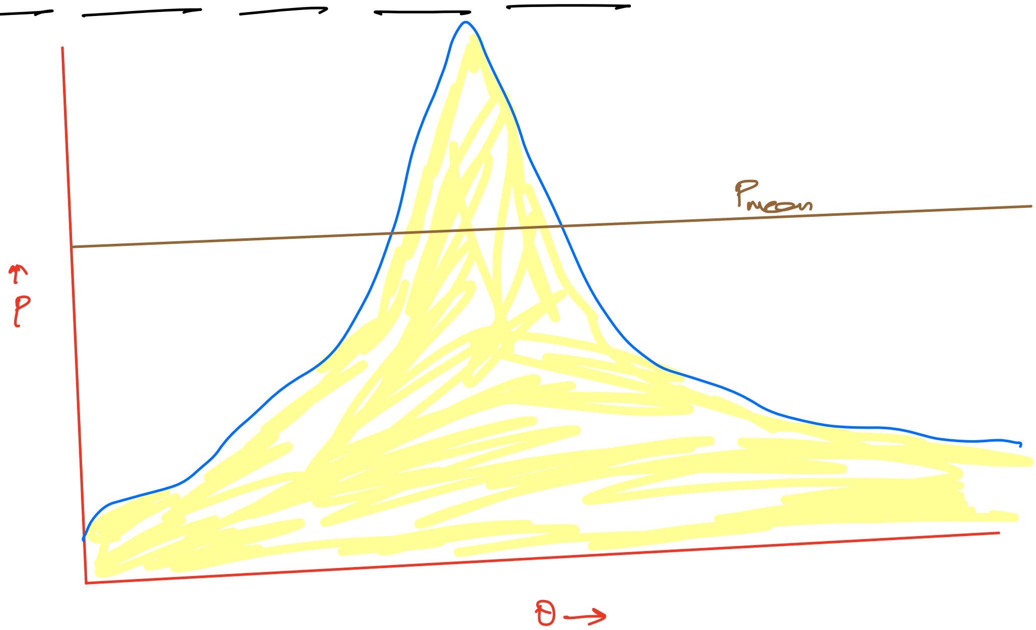
<u>Comment</u>: η_{th} depends on r & γ

Air $\rightarrow \gamma = 1.4$; Exhaust gas $\rightarrow \gamma = 1.3$; Monoatomic gas $\gamma = 1.67$

Home work.



Expression for mean effective pressure:



$$P_m = \frac{\text{Workdone}}{\text{Stroke volume}} = \frac{Q_H - Q_L}{V_1 - V_2} \rightarrow (1)$$

$$W = Q_H - Q_L = mC_v(T_3 - T_2) - mC_v(T_4 - T_1) \rightarrow (2)$$

$$V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right) = V_1\left(1 - \frac{1}{\left(\frac{V_1}{V_2}\right)}\right) = V_1\left(1 - \frac{1}{r}\right) \rightarrow (3)$$

$$r = \frac{V_1}{V_2}$$

From the ideal gas equation:

$$pV = mRT \rightarrow pv = RT \qquad \because v = \frac{V}{m}$$

$$v = \frac{RT}{p}$$

$$@1 \rightarrow v_1 = \frac{RT_1}{p_1} \rightarrow (4)$$

Substituting (4) in (3) we get,

$$V_1 - V_2 = \frac{RT_1}{p_1}\left(1 - \frac{1}{r}\right) \rightarrow (5)$$

$\therefore\ C_v = \dfrac{R}{\gamma-1} \rightarrow R = C_v(\gamma-1)$

$\therefore$ (5) becomes $V_1 - V_2 = \dfrac{C_v(\gamma-1)T_1}{p_1}\left(\dfrac{r-1}{r}\right) \rightarrow$ (6)

Substitute (6) in (1) to get. $m = 1\,kg$.

$$p_m = \frac{C_v(T_3-T_2) - C_v(T_4-T_1)}{\dfrac{C_v(\gamma-1)T_1}{p_1}\left(\dfrac{r-1}{r}\right)}$$

$$p_m = \frac{1}{(\gamma-1)} \times \frac{p_1}{T_1} \times \frac{[(T_3-T_2)-(T_4-T_1)]}{\left(\dfrac{r-1}{r}\right)} \rightarrow (7)$$

For any reversible adiabatic/isentropic process.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \rightarrow \frac{T_2}{T_1} = r^{\gamma-1} \longrightarrow T_2 = T_1 r^{\gamma-1} \rightarrow (8)$$

Let us define the pressure ratio $r_p = \dfrac{p_3}{p_2}$. $\quad r = \dfrac{V_1}{V_2}$.

For a constant volume process 2-3;

$$\frac{p_3}{p_2} = \frac{T_3}{T_2} = r_p. \longrightarrow T_3 = r_p T_2 = r_p T_1 r^{\gamma-1} \rightarrow (9)$$

(X) $p_2V_2 = RT_2$. $\quad p_3V_3 = RT_3$. $\rightarrow V_2 = V_3$, then. $\left.\begin{array}{l} p_2 = RT_2 \\ p_3 = RT_3. \end{array}\right]$

$$\text{Illy}\quad \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \longrightarrow \frac{T_4}{T_3} = \left[\frac{1}{\left(\frac{V_4}{V_3}\right)}\right]^{\gamma-1} \rightarrow \frac{T_4}{T_3} = \frac{1}{r^{\gamma-1}}$$

$r = \dfrac{V_1}{V_2} = \dfrac{V_4}{V_3}$.

$$\therefore\ T_4 = T_3 \cdot \frac{1}{r^{\gamma-1}} = \frac{r_p \cdot T_1 r^{\gamma-1}}{r^{\gamma-1}}$$

$$T_4 = r_p \cdot T_1 \rightarrow (10)$$

Substitute for T_2, T_3 & T_4 in (7) to get.

$$p_m = \frac{1}{\gamma-1} \times \frac{p_1}{T_1} \frac{\left[\left(r_p T_1 r^{\gamma-1} - T_1 r^{\gamma-1}\right) - \left(r_p T_1 - T_1\right)\right]}{\left(\frac{r-1}{r}\right)}$$

$$p_m = \frac{1}{\gamma-1} \times \frac{p_1 T_1}{T_1} \frac{\left[r^{\gamma-1}(r_p-1) - (r_p-1)\right]}{\left(\frac{r-1}{r}\right)}$$

$$p_m = \frac{p_1 r\left[(r_p-1)(r^{\gamma-1}-1)\right]}{(\gamma-1)(r-1)} \rightarrow (11)$$

Homework:

(1) Calculate p_m.

$r \rightarrow$ Varying

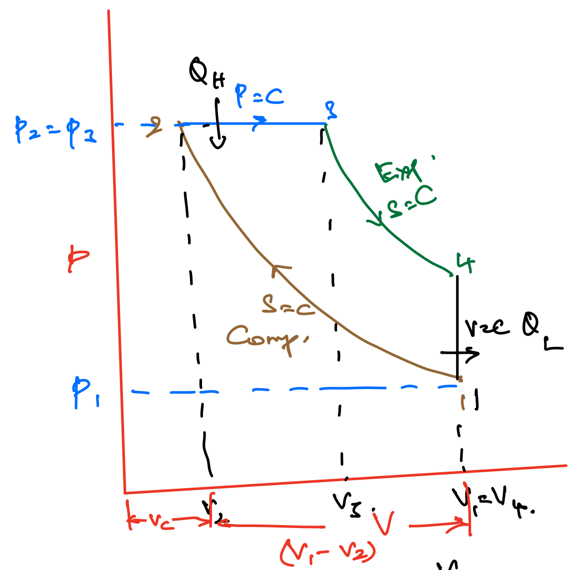
$r_p \rightarrow$ constant.

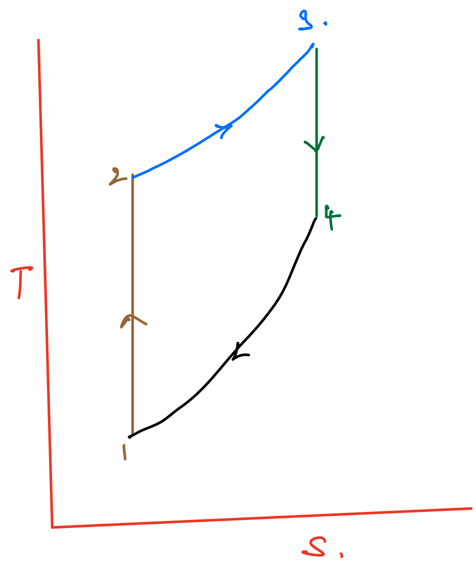
(2) Calculate p_m.

$r_p \rightarrow$ Varying.

$r \rightarrow$ Constant.

Diesel cycle:





Compression ratio $r_c = \frac{V_1}{V_2}$.

Cutoff ratio $\rho = \frac{V_3}{V_2} = \frac{T_3}{T_2}$ $\left[\because p=C \text{ @ } 2\text{-}3\right]$

$v_3 = \rho v_2$

$T_3 = \rho T_2$

$P_2 V_2 = mRT_2$

$P_3 V_3 = mRT_3$.

(X) $P_2 = \frac{mRT_2}{V_2}$

$P_3 = \frac{mRT_3}{V_3}$.

$$\frac{mRT_2}{V_2} = \frac{mRT_3}{V_3} \rightarrow \frac{V_3}{V_2} = \frac{T_3}{T_2}.$$

Heat supplied $Q_H = mC_p(T_3 - T_2) \rightarrow ①$

Heat rejected $Q_L = mC_v(T_4 - T_1) \rightarrow ②$

Workdone $= Q_H - Q_L$

$= mC_p(T_3 - T_2) - mC_v(T_4 - T_1) \rightarrow ③$

$\therefore$ Thermal efficiency is given by: $m = 1\,kg$.

$$\eta_{th} = \frac{\text{Workdone}}{\text{Heat supplied}}.$$

$$\eta_{th} = \frac{Q_H - Q_L}{Q_H}$$

$$= \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{C_p(T_3 - T_2)}$$

$$= 1 - \frac{1}{\left(\frac{C_p}{C_v}\right)} \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Ratio of Specific heats.

$$\gamma = \frac{C_p}{C_v}.$$

$$\therefore \eta_{th} = 1 - \frac{1}{\gamma} \cdot \frac{(T_4 - T_1)}{(T_3 - T_2)} \rightarrow ④$$

$\rightarrow \Delta T_{rejection}$

$\rightarrow \Delta T_{addition}$.

Consider the process 1-2; (Compression)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \rightarrow \frac{T_2}{T_1} = r_c^{\gamma-1} \rightarrow T_2 = T_1 r_c^{\gamma-1} \rightarrow ⑤$$

$$T_3 = \rho T_2 \rightarrow T_3 = \rho T_1 r_c^{\gamma-1} \rightarrow ⑥$$

Consider the process 3-4: (Expansion)

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \rightarrow \frac{T_4}{T_3} = \left(\frac{\rho}{r}\right)^{\gamma-1} \rightarrow T_4 = T_3\left(\frac{\rho}{r}\right)^{\gamma-1} = \frac{\rho \cdot T_1 r_c^{\gamma-1} \cdot \rho^{\gamma-1}}{r_c^{\gamma-1}}$$

$$\rightarrow T_4 = \rho^{\gamma} T_1 \rightarrow ⑦$$

X

$$\frac{V_3}{V_4} = \frac{V_3}{V_2} \cdot \frac{V_2}{V_4} \rightarrow \frac{V_3}{V_4} = \frac{V_3}{V_2} \cdot \frac{V_2}{V_1} \rightarrow \frac{V_3}{V_4} = \rho \cdot \frac{1}{r_c} = \frac{\rho}{r_c}.$$

$V_4 = V_1$

Substitute for T_2, T_3 and T_4 in ④ to get

$$\eta_{th} = 1 - \frac{1}{\gamma}\left[\frac{(\rho^{\gamma} T_1 - T_1)}{(\rho T_1 r_c^{\gamma-1} - T_1 r_c^{\gamma-1})}\right]$$

$$\eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{(\rho^{\gamma}-1)}{\gamma(\rho-1)}\right] \longrightarrow ⑧$$

Homework

η_{th}
① $r_c \rightarrow$ constant, $\rho \rightarrow$ Vary
② $\rho \rightarrow$ constant, $r_c \rightarrow$ Vary.

r_c (const)

Expression for mean effective pressure:

$$P_m = \frac{\text{Workdone}}{\text{Stroke Volume.}} = \frac{W}{V_1 - V_2}$$

$$W = Q_H - Q_L = C_p(T_3 - T_2) - C_v(T_4 - T_1) \longrightarrow ①$$

$$V_1 - V_2 = V_1\left(1 - \frac{V_2}{V_1}\right) = V_1\left(1 - \frac{1}{\frac{V_1}{V_2}}\right) = V_1\left(1 - \frac{1}{r_c}\right) \longrightarrow ②$$

$$Pv = RT \quad @ \ 1 \rightarrow P_1 v_1 = RT_1 \longrightarrow v_1 = \frac{RT_1}{P_1}$$

$\therefore$ ② becomes.

$$V_{stroke} = V_1 - V_2 = \frac{RT_1}{P_1}\left(1 - \frac{1}{r}\right) \longrightarrow ③$$

$$C_v = \frac{R}{\gamma - 1} \longrightarrow R = C_v(\gamma - 1)$$

$$\left\{ C_p = \frac{\gamma}{\gamma-1}R. \quad C_v = \frac{R}{\gamma-1} \right\}$$

$$\therefore V_{stroke} = V_1 - V_2 = \frac{C_v(\gamma-1)T_1}{P_1}\left(1 - \frac{1}{r_c}\right) \longrightarrow ④$$

$$\therefore P_m = \frac{C_p(T_3 - T_2) - C_v(T_4 - T_1)}{\frac{C_v(\gamma-1)T_1}{P_1}\left(1 - \frac{1}{r_c}\right)}$$

$\div C_v.$

$$P_m = \frac{\frac{C_p}{C_v}(T_3 - T_2) - (T_4 - T_1)}{\frac{(\gamma-1)T_1}{P_1}\left(1 - \frac{1}{r_c}\right)} \qquad \left[\because \gamma = \frac{C_p}{C_v}.\right]$$

$$\therefore \; P_m = \frac{\gamma(T_3 - T_2) - (T_4 - T_1)}{\frac{(\gamma-1)T_1}{P_1}\left(1 - \frac{1}{r_c}\right)}$$

$$P_m = \frac{1}{\gamma-1} \cdot \frac{P_1}{T_1}\left[\frac{\gamma(T_3 - T_2) - (T_4 - T_1)}{\left(\frac{r_c - 1}{r_c}\right)}\right] \rightarrow (5)$$

Substitute for T_2, T_3 & T_4 in (5)

$$\left[T_2 = T_1 r_c^{\gamma-1}; \quad T_3 = \rho T_1 r_c^{\gamma-1}; \quad T_4 = \rho^{\gamma} T_1\right]$$

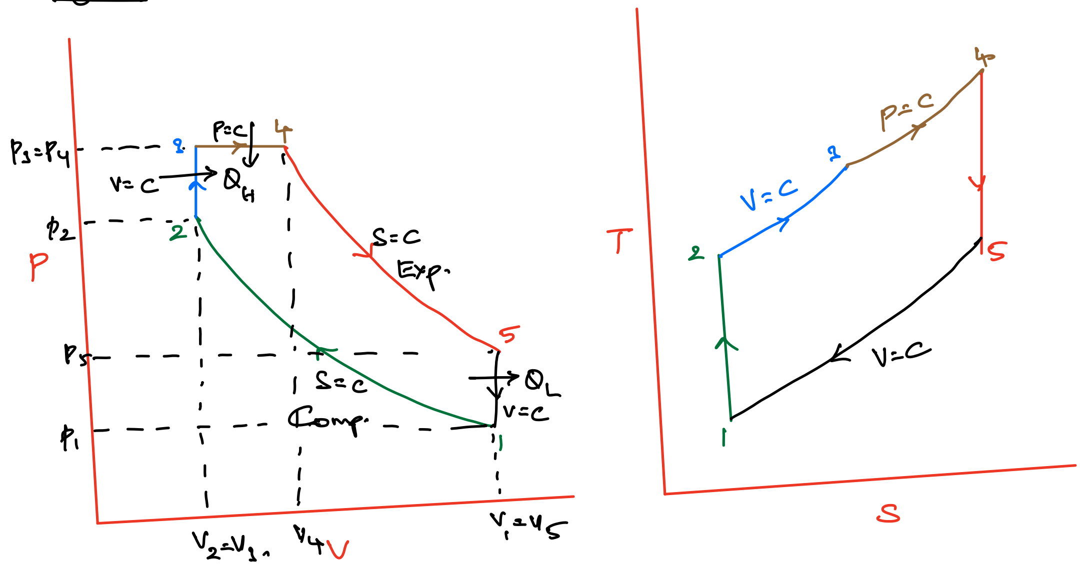
$$\therefore \; P_m = \frac{1}{\gamma-1} \cdot \frac{P_1}{T_1}\left[\frac{\gamma\left(\rho T_1 r_c^{\gamma-1} - T_1 r_c^{\gamma-1}\right) - \left(\rho^{\gamma} T_1 - T_1\right)}{\left(\frac{r_c - 1}{r_c}\right)}\right]$$

$$P_m = \frac{1}{(\gamma-1)} \cdot \frac{P_1}{\cancel{T_1}} \cdot \frac{r_c}{(r_c - 1)} \cdot \cancel{T_1}\left[\gamma r_c^{\gamma-1}(\rho - 1) - (\rho^{\gamma} - 1)\right]$$

$$P_m = \frac{P_1 \cdot r_c\left[\gamma r_c^{\gamma-1}(\rho - 1) - (\rho^{\gamma} - 1)\right]}{(\gamma-1)(r_c - 1)} \rightarrow (6)$$

(X) Homework, obtain a plot of η_{th} V/s r_c for different gases. Super impose the plot with the plot obtained for otto cycle and comment on it.

Dual Cycle:



Compression ratio $\rightarrow r_c = \dfrac{V_1}{V_2}$

Expansion ratio $\rightarrow r_e = \dfrac{V_5}{V_4}$.

Cut off ratio $\rightarrow \rho = \dfrac{V_4}{V_3}$.

Pressure ratio $\rightarrow r_p = \dfrac{P_3}{P_2}$

Workdone = $Q_H - Q_L$.

$$W. = C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1) \rightarrow (1)$$

Thermal efficiency $\eta_{th} = \dfrac{\text{Workdone}}{\text{Heat supplied}}$. $\gamma = \dfrac{C_p}{C_v}$.

$$\eta_{th} = \frac{C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$$\eta_{th} = 1 - \frac{C_v(T_5 - T_1)}{C_v(T_3 - T_2) + C_p(T_4 - T_3)}$$

$\div C_v$

$$\eta_{th} = 1 - \frac{\overset{\to \Delta T_{rejection}}{(T_5 - T_1)}}{\underbrace{(T_3 - T_2)}_{\Delta T_{addition ①}} + \gamma \underbrace{(T_4 - T_3)}_{\Delta T_{addition ②}}} \rightarrow ②$$

Consider the process 1–2.

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} \rightarrow \frac{T_2}{T_1} = r_c^{\gamma-1} \rightarrow T_2 = T_1 r_c^{\gamma-1} \rightarrow ③$$

$$\rho = \frac{V_4}{V_3} = \frac{T_4}{T_3} \rightarrow T_4 = \rho T_3 . \rightarrow ④$$

$$T_4 = \rho r_p T_1 r_c^{\gamma-1}$$

$$P_3V_3 = RT_3.$$
$$P_4V_4 = RT_4.$$
$$\frac{V_3}{V_4} = \frac{T_3}{T_4}$$
$p=C$

$$r_p = \frac{P_3}{P_2} = \frac{T_3}{T_2} \rightarrow T_3 = r_p . T_2 .$$

$$T_3 = r_p . T_1 r_c^{\gamma-1} \rightarrow ⑤$$

$$P_2V_2 = RT_2$$
$$P_3V_3 = RT_3$$
$$\frac{P_2}{P_3} = \frac{T_2}{T_3}$$
$V=C$

Consider the process 4–5

$$\frac{T_5}{T_4} = \left(\frac{V_4}{V_5}\right)^{\gamma-1} \rightarrow ⑥$$

$$\frac{V_4}{V_5} = \frac{V_4}{V_3} \cdot \frac{V_3}{V_5} \qquad \because V_3 = V_2 . \ \& \ V_5 = V_1 \ ⊗$$

$$\frac{V_4}{V_5} = \frac{V_4}{V_3} \cdot \frac{V_2}{V_1} \rightarrow \frac{V_4}{V_5} = \frac{V_4}{V_3} \cdot \frac{1}{\left(\frac{V_1}{V_2}\right)} = \frac{\rho}{r_c} \rightarrow ⑦$$

$$\therefore \frac{T_5}{T_4} = \left(\frac{\rho}{r_c}\right)^{\gamma-1}$$

$$T_5 = \frac{T_4 \rho^{\gamma-1}}{r_c^{\gamma-1}} = \frac{\rho r_p T_1 \cancel{r_c^{\gamma-1}} \cdot \rho^{\gamma-1}}{\cancel{r_c^{\gamma-1}}}$$

$$T_5 = \rho^{\gamma} . r_p . T_1 \rightarrow ⑧$$

Substitute for T_2, T_3, T_4 & T_5 in (2) simplify.

$$\eta_{th} = 1 - \frac{\left(\rho^{\gamma} r_p \cancel{T_1} - \cancel{T_1}\right)}{\left(r_p r_c^{\gamma-1} \cancel{T_1} - r_c^{\gamma-1} \cancel{T_1}\right) + \gamma\left(\rho r_p r_c^{\gamma-1} \cancel{T_1} - r_p r_c^{\gamma-1} \cancel{T_1}\right)}$$

$$\eta_{th} = 1 - \frac{\left(\rho^{\gamma} r_p - 1\right)}{r_c^{\gamma-1}\left[\left(r_p - 1\right) + \gamma r_p\left(\rho - 1\right)\right]} \rightarrow (9)$$

Expression for mean effective pressure.

$$p_m = \frac{\text{Workdone.}}{\text{Stroke Volume.}} = \frac{W}{V_1 - V_2} \rightarrow (1)$$

$$pv = RT \rightarrow @\,1 \quad p_1 v_1 = RT_1 \rightarrow v_1 = \frac{RT_1}{p_1} \rightarrow (2)$$

$$v_1 - v_2 = v_1\left(1 - \frac{v_2}{v_1}\right) = v_1\left(1 - \frac{1}{\frac{v_1}{v_2}}\right) = v_1\left(1 - \frac{1}{r_c}\right) \qquad \because r_c = \frac{v_1}{v_2}.$$

$$\therefore v_1 - v_2 = \frac{RT_1}{p_1}\left(1 - \frac{1}{r_c}\right) \rightarrow (3)$$

$$C_v = \frac{R}{\gamma - 1} \rightarrow R = C_v(\gamma - 1)$$

$$\therefore v_1 - v_2 = \frac{C_v(\gamma-1) T_1}{p_1}\left(1 - \frac{1}{r_c}\right) \rightarrow (4)$$

$$\therefore W = \underbrace{c_v(T_3 - T_2) + C_p(T_4 - T_3)}_{Q_H} - \underbrace{C_v(T_5 - T_1)}_{Q_L.} \rightarrow (5)$$

$\therefore$ (1) becomes.

$\div C_v$.

$$p_m = \frac{C_v(T_3 - T_2) + C_p(T_4 - T_3) - C_v(T_5 - T_1)}{\frac{C_v(\gamma - 1)T_1}{p_1}\left(1 - \frac{1}{r_c}\right)}$$

$$\left[\because \gamma = \frac{C_p}{C_v}\right]$$

$$p_m = \frac{(T_3 - T_2) + \gamma(T_4 - T_3) - (T_5 - T_1)}{\frac{(\gamma - 1)T_1}{p_1}\left(1 - \frac{1}{r_c}\right)} \rightarrow (6)$$

We have obtained temperatures T_2, T_3, T_4, T_5.

$T_2 = T_1 r_c^{\gamma-1}$; $T_3 = r_p T_1 r_c^{\gamma-1}$; $T_4 = \rho r_p T_1 r_c^{\gamma-1}$; $T_5 = \rho^{\gamma} r_p T_1$

Substitute for T_2, T_3, T_4 & T_5 in (6)

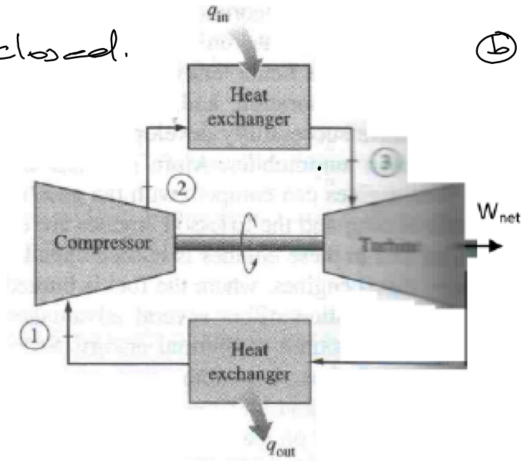
$$p_m = \frac{(r_p T_1 r_c^{\gamma-1} - T_1 r_c^{\gamma-1}) + \gamma(\rho r_p T_1 r_c^{\gamma-1} - r_p T_1 r_c^{\gamma-1}) - (\rho^{\gamma} r_p T_1 - T_1)}{\frac{(\gamma-1)T_1}{p_1}\left(\frac{r_c - 1}{r_c}\right)}$$

$$p_m = \frac{r_c^{\gamma-1}(r_p - 1) + \gamma r_p r_c^{\gamma-1}(\rho - 1) - (\rho^{\gamma} r_p - 1)}{\frac{(\gamma-1)}{p_1}\left(\frac{r_c-1}{r_c}\right)}$$

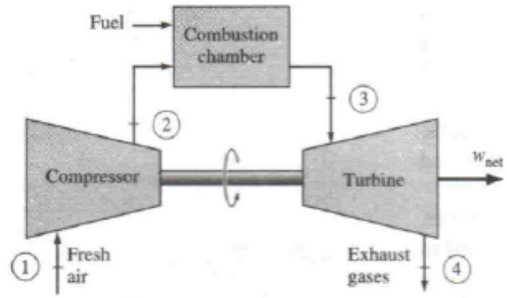
$$p_m = \frac{r_c}{r_c - 1} \cdot \frac{p_1}{\gamma - 1}\left[r_c^{\gamma-1}(r_p - 1) + \gamma r_p r_c^{\gamma-1}(\rho - 1) - (\rho^{\gamma} r_p - 1)\right] \rightarrow (7)$$

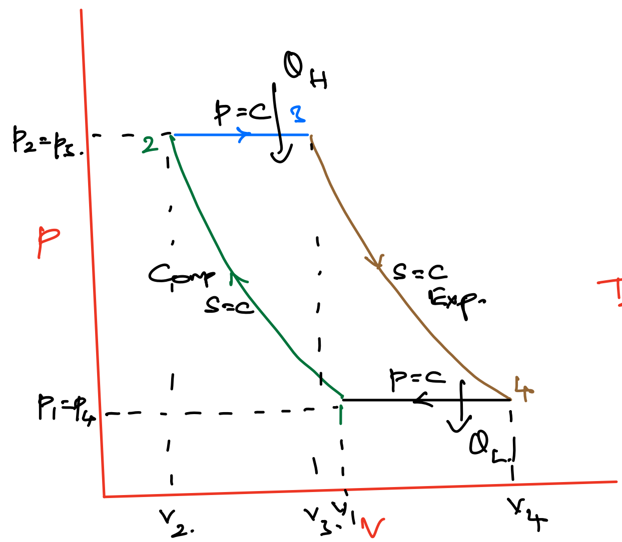
Brayton cycle (Joule / Gas turbine)

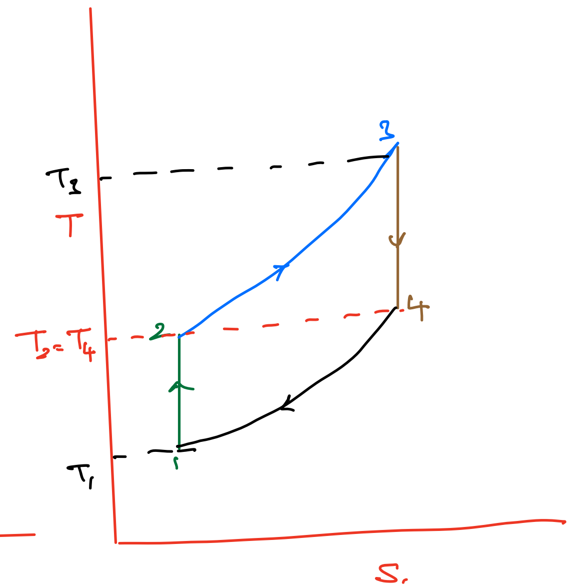
ⓐ closed.



ⓑ Open.







Expression for thermal efficiency of a Brayton cycle.

$\therefore$ Workdone $= Q_H - Q_L$.

$$W = C_p(T_3-T_2) - C_p(T_4-T_1) \rightarrow (1).$$

Thermal efficiency $\rightarrow \eta_{th} = \dfrac{W}{Q_H}$

$$= \frac{C_p(T_3-T_2) - C_p(T_4-T_1)}{C_p(T_3-T_2)}$$

$$\eta_{th} = 1 - \frac{\overbrace{(T_4 - T_1)}^{\Delta T \text{ rejection}}}{\underbrace{(T_3 - T_2)}_{\Delta T \text{ addition.}}} \longrightarrow (2)$$

Let r_p = pressure ratio $\rightarrow$ $r_p = \frac{p_2}{p_1} = \frac{p_3}{p_4} \longrightarrow (3)$

Consider the process 1-2: (Compression)

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \longrightarrow \text{Thermodynamics.}$$

$$\therefore \frac{T_2}{T_1} = r_p^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1 r_p^{\frac{\gamma-1}{\gamma}} \longrightarrow (4)$$

Consider the process 3-4 (Expansion),

$$\text{lly} \quad \frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}.$$

$$\therefore \frac{T_3}{T_4} = r_p^{\frac{\gamma-1}{\gamma}}.$$

$$T_3 = T_4 r_p^{\frac{\gamma-1}{\gamma}} \longrightarrow (5)$$

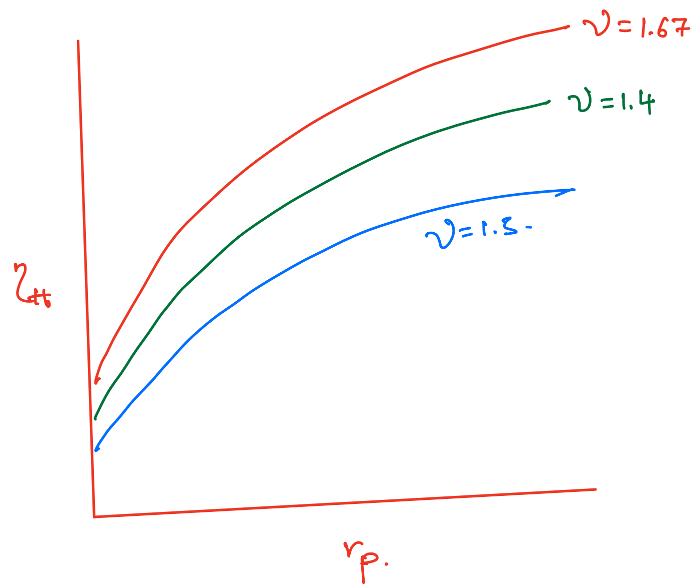
Substitute for T_2 & T_3 in (2)

$$\therefore \eta_{th} = 1 - \frac{(T_4 - T_1)}{\left(T_4 r_p^{\frac{\gamma-1}{\gamma}} - T_1 r_p^{\frac{\gamma-1}{\gamma}}\right)}$$

$$\eta_{th} = 1 - \frac{(T_4 - T_1)}{r_p^{\frac{\gamma-1}{\gamma}}(T_4 - T_1)}$$

$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} \longrightarrow (6)$$

⊗ Homework.



Expression for W_{max} for a given pressure ratio

The workdone is given by.

$$W = C_p(T_3 - T_2) - C_p(T_4 - T_1)$$

$$W = C_p T_3\left(1 - \frac{T_2}{T_3}\right) - C_p T_1\left(\frac{T_4}{T_1} - T_1\right) \longrightarrow (1)$$

For an ideal case with equal compression & expansion. ⊗

$$T_2 = T_4 \longrightarrow (2)$$

$$\therefore W = C_p T_3\left(1 - \frac{T_4}{T_3}\right) - C_p T_1\left(\frac{T_2}{T_1} - 1\right) \longrightarrow (3)$$

$$W = C_p T_3\left(1 - \frac{1}{\frac{T_3}{T_4}}\right) - C_p T_1\left(\frac{T_2}{T_1} - 1\right)$$

$$W = C_p T_3\left(1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}}\right) - C_p T_1\left(r_p^{\frac{\gamma-1}{\gamma}} - 1\right) \longrightarrow (4)$$

To maximize the workdone, (4) has to differentiated w.r.to r_p and equate it to zero.

$$\frac{dW}{dr_p} = 0 \; : \qquad \text{Let } z = \frac{\gamma - 1}{\gamma}.$$

$\therefore$ (4) can be written as

$$W = C_p T_3 \left(1 - \frac{1}{r_p^z}\right) - C_p T_1 \left(r_p^z - 1\right)$$

$$\frac{dW}{dr_p} \rightarrow \frac{d}{dr_p}\left[T_3\left(1 - \frac{1}{r_p^z}\right) - T_1\left(r_p^z - 1\right)\right] = 0.$$

$$T_3\left(0 - \frac{-z}{r_p^{z+1}}\right) - T_1\left(z \cdot r_p^{z-1} - 0\right) = 0$$

$$T_3 \cdot \frac{z}{r_p^{z+1}} - T_1 z r_p^{z-1} = 0$$

$$T_3 \cdot \frac{z}{r_p^{z+1}} = T_1 z r_p^{z-1}$$

$$\frac{T_3}{T_1} = r_p^{2z}$$

$$r_p = \left(\frac{T_3}{T_1}\right)^{1/2z}$$

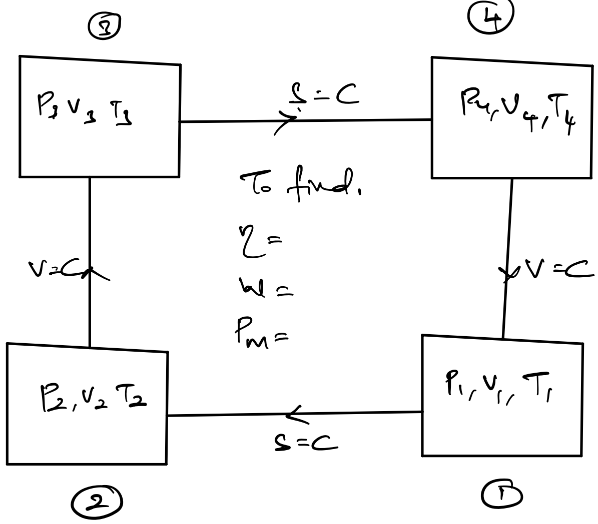
$$r_p = \left(\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)} \rightarrow (5) \quad \left[\because z = \frac{\gamma-1}{\gamma}\right]$$

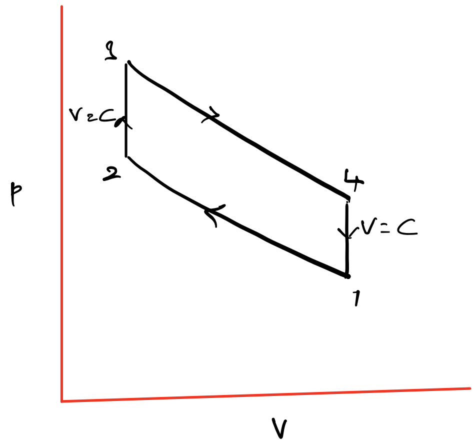
Substitute for r_p in equation (4) to get.

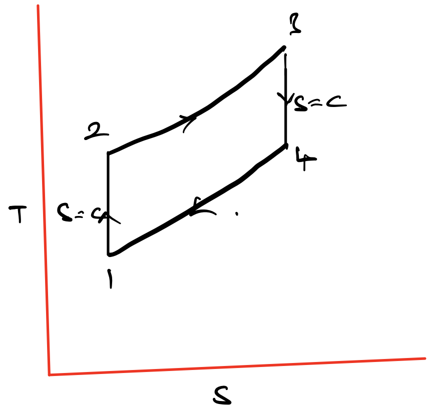
$$W_{max} = C_p \cdot T_3\left(1 - \frac{1}{\left[\left(\frac{T_3}{T_1}\right)^{\gamma/2(\gamma-1)}\right]^{\frac{\gamma-1}{\gamma}}}\right) - C_p T_1 \left(\left(\frac{T_3}{T_1}\right)^{\frac{\gamma}{2(\gamma-1)} \cdot \frac{\gamma-1}{\gamma}} - 1\right)$$

$$W_{max} = C_p T_3 \left(1 - \frac{1}{\left(\frac{T_3}{T_1}\right)^{1/2}}\right) - C_p T_1 \left(\left(\frac{T_3}{T_1}\right)^{1/2} - 1\right) \longrightarrow (6)$$

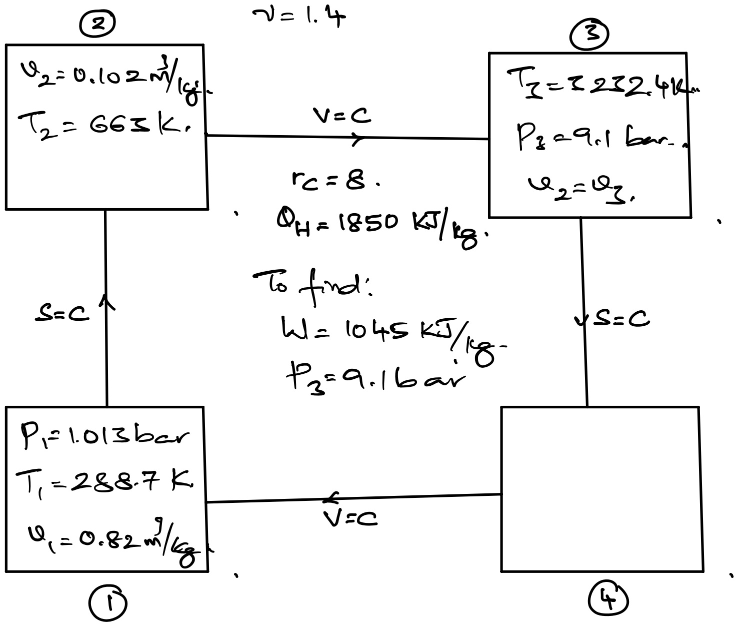
Numericals on Gas Power Cycles

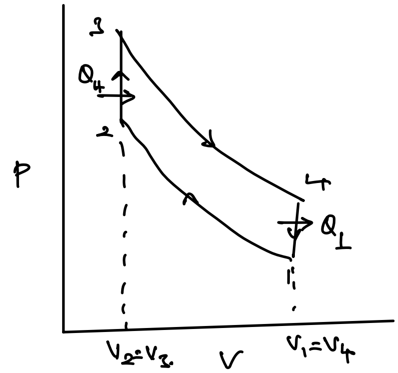


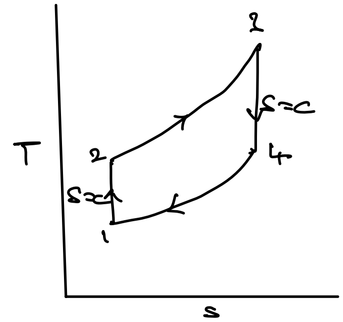




1. A spark-ignition engine with a compression ratio of 8 operates on an Otto cycle using air with a low temperature of 288.7 K and a low pressure of 1.013 bar. If the energy addition during combustion is 1850 kJ/kg. Determine the work output and the maximum pressure.







For an otto cycle, thermal efficiency

$$\eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} = 1 - \frac{1}{8^{1.4-1}} = 56.47\%$$

Also $\eta_{th} = \frac{W}{Q_H} \rightarrow 0.5647 = \frac{W}{1850} \rightarrow W = 1045 \text{ kJ/kg}$

From the characteristic gas equation.

$$P_1 v_1 = RT_1 \rightarrow v_1 = \frac{RT_1}{P_1}$$

For air: $R = 287$ J/kg-K.

$$= \frac{287 \times 288.7}{1.013 \times 10^5}$$

$$v_1 = 0.82 \text{ m}^3/\text{kg}$$

Compression ratio

$$r_c = \frac{v_1}{v_2} \rightarrow v_2 = \frac{v_1}{r_c} = \frac{0.82}{8} = 0.102 \text{ m}^3/\text{kg}$$

For the process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = r_c^{\gamma-1} \rightarrow T_2 = T_1 r_c^{\gamma-1}$$

$$= 288.7 \times 8^{1.4-1}$$

$$T_2 = 663 \text{ K}$$

Heat supplied is given by

$C_v = 0.72\ kJ/kg\text{-}K$ (Air)

$$Q_H = C_v(T_3 - T_2)$$

$$1850 = 0.72(T_3 - 663)$$

$$T_3 = 3232.4K.$$

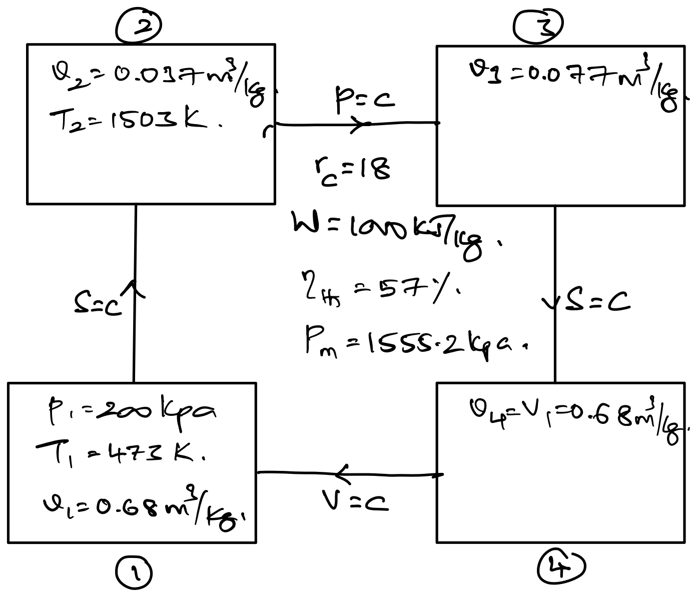
For the process 2-3, $\quad P_3 = \frac{RT_3}{v_3} = \frac{RT_3}{v_2}$

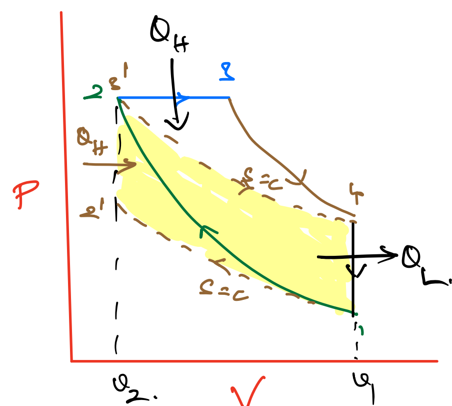
$v_2 = v_3$

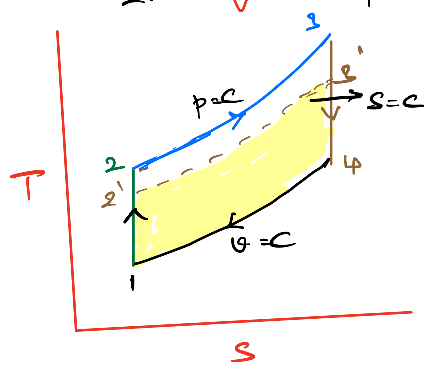
$$P_3 = \frac{287 \times 3232.4}{0.102}$$

$$P_3 = 9.1\ bar.$$

4. A diesel cycle, with a compression ratio of 18 operates on air with a low pressure of 200 kPa and a low temperature 200^0C. If the work output is 1000 kJ/ kg, determine the thermal efficiency and the MEP. Also compare with the efficiency of an Otto cycle operating with the same maximum pressure.







From the characteristic gas equation

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 473}{200 \times 10^3} = 0.68\ m^3/kg.$$

$$r_c = \frac{v_1}{v_2} \rightarrow v_2 = \frac{v_1}{r_c} = \frac{0.68}{18} = 0.037\ m^3/kg.$$

For the process 1-2;

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \rightarrow T_2 = T_1 r_c^{\gamma-1} = 473 \times 18^{1.4-1} = 1503K.$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \rightarrow P_2 = P_1 (r_c)^{\gamma} = 200 \times 18^{1.4} = 11.44 \text{ Mpa.}$$

For the process 3-4;

$$\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{V_3}{V_4}\right)^{\gamma-1}$$

$$T_4 = 40621.6\, v_3 \left(\frac{v_3}{0.68}\right)^{1.4-1}$$

$$T_4 = 47397.3\, v_3^{1.4}$$

$$W = C_p (T_3 - T_2) - C_v (T_4 - T_1)$$

$$1000 = 1.05 (40621.6\, v_3 - 1503) - 0.72 (47397.3\, v_3^{1.4} - 473)$$

$$1000 = 42658.68\, v_3 - 1578.15 - 34126.056\, v_3^{1.4} + 340.56$$

$$2237.6 = 42658.68\, v_3 - 34126.056\, v_3^{1.4}.$$

For the process 2-3.

$$\frac{T_3}{v_3} = \frac{T_2}{v_2} = \frac{1503}{0.027} = 40621.6.$$

$$T_3 = 40621.6\, v_3.$$

By trial and error, $v_3 = 0.077\ m^3/kg$.

$\therefore T_3 = 3127.8\,K$; $T_4 = 1308.7\,K$.

$$\therefore \rho = \frac{v_3}{v_2} = \frac{0.077}{0.027} = 2.08.$$

$$\therefore \eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right] = 1 - \frac{1}{18^{1.4-1}} \left[\frac{2.08^{1.4} - 1}{1.4(2.08-1)}\right] = 62.78\%.$$

$$P_M = \frac{W}{v_1 - v_2} = \frac{1000}{0.68 - 0.027} = 1555.2 \text{ Kpa.}$$

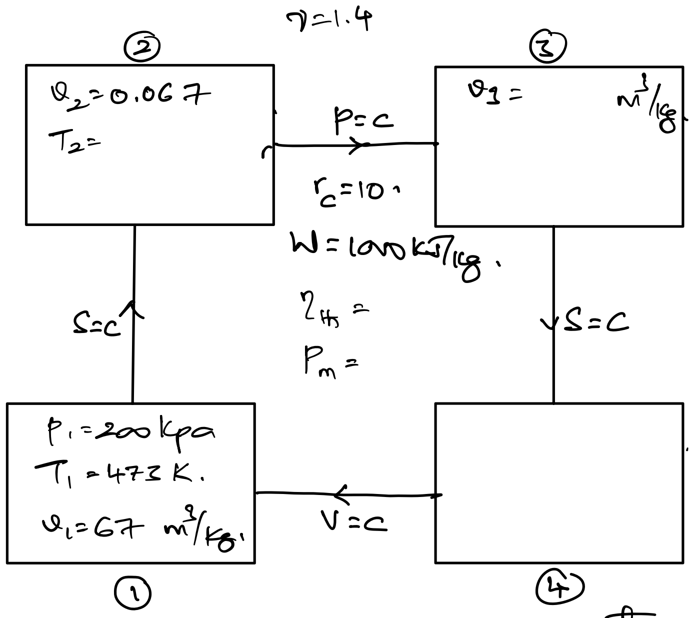
For the comparison otto cycle.

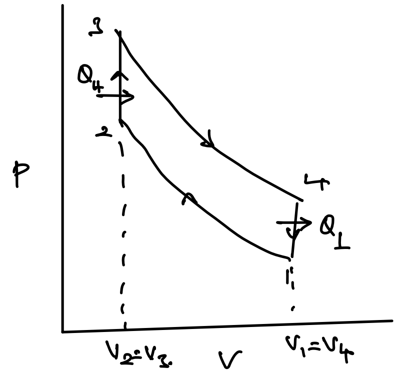
$$r_c = \frac{v_1}{v_3} = \frac{0.68}{0.077} = 8.83.$$

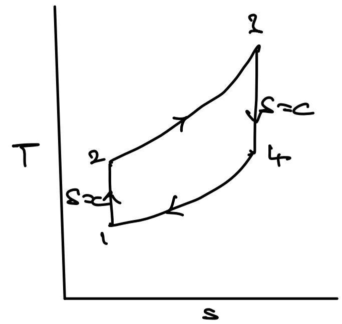
$$\eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} = 1 - \frac{1}{8.83^{1.4-1}}$$

$$\eta_{th} = 58.15\%.$$

3. A spark-ignition engine is proposed to have a compression ratio of 10 while operating with a low temperature of 200^0C and a low pressure of 200 kPa. If the work output is to be 1000 kJ/kg, calculate the maximum possible thermal efficiency and compare with that of a Carnot cycle. Also calculate the MEP.







From the characteristic gas equation

$$P_1 v_1 = RT_1 \rightarrow v_1 = \frac{RT_1}{P_1} = \frac{288 \times 473}{200 \times 10^3}$$

$$v_1 = 0.67 \ m^3/kg$$

$$r_c = \frac{v_1}{v_2} \rightarrow v_2 = \frac{v_1}{r_c} = \frac{0.67}{10} = 0.067 \ m^3/kg.$$

Thermal efficiency $\eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} = 1 - \frac{1}{10^{1.4-1}} = 60.1\%.$

$$\eta_{th} = \frac{W}{Q} \rightarrow Q = \frac{W}{\eta_{th}} = \frac{1000}{0.601} = 1663.8 \ kJ/kg.$$

For the process 1-2

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \rightarrow T_2 = T_1 r_c^{\gamma-1} = 473 \times 10^{1.4-1} = 1188.12K.$$

$$Q = c_v(T_3 - T_2)$$

$$1663.8 = 0.72(T_3 - 1188.12)$$

$$T_3 = 3498.91K.$$

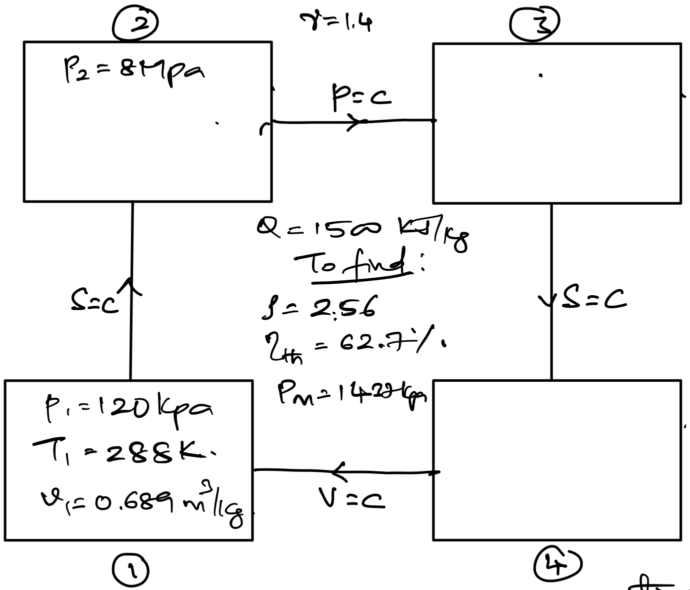
Carnot efficiency.

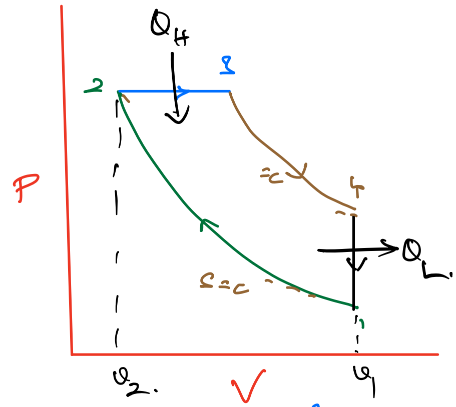
$$\eta_{carnot} = \frac{(T_3 - T_1)}{T_3}$$

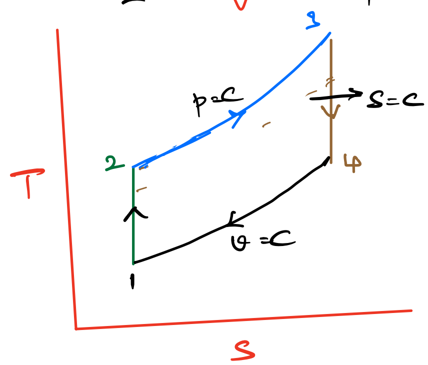
$$= \frac{3498.91 - 473}{3498.91}$$

$$\eta_{carnot} = 86.48\%.$$

8. Air enters the compression process of a diesel cycle at 120 kPa and 15^0C. The pressure after compression is 8 MPa and 1500 kJ/kg is added during combustion. What are (a) the cutoff ratio, (b)the thermal efficiency, and (c)the MEP







From the characteristic gas equation

$$P_1 v_1 = RT_1 \longrightarrow v_1 = \frac{RT_1}{P_1} = \frac{287 \times 288}{120 \times 10^3}.$$

$$v_1 = 0.689\ m^3/kg$$

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}. \longrightarrow T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}.$$

$$= 288 \times \left(\frac{8 \times 10^6}{120 \times 10^3}\right)^{\frac{1.4-1}{1.4}}$$

$$T_2 = 956.11\ K.$$

$$P_2 v_2 = RT_2 \longrightarrow v_2 = \frac{RT_2}{P_2.} = \frac{287 \times 956.11}{8 \times 10^6}$$

$$v_2 = 0.034\ m^3/kg.$$

$\therefore$ Compression ratio $r_c = \frac{v_1}{v_2} = \frac{0.689}{0.034}$

$$r_c = 20.6.$$

Heat addition:

$$Q = C_p(T_3 - T_2)$$

$$1500 = 1.005\,(T_3 - 956.11)$$

$$T_3 = 2448.64\ K.$$

For the process 2-3; $P_2 = P_3$.

$$\left.\begin{aligned} P_2 v_2 &= RT_2 \\ P_3 v_3 &= RT_3 \end{aligned}\right\} \quad \frac{v_2}{v_3} = \frac{T_2}{T_3}.$$

$$v_3 = \frac{v_2}{T_2} * T_3.$$

$$= \frac{0.034}{956.11} * 2448.64$$

$$v_3 = 0.087 \ m^3/kg.$$

a) Cut off ratio $\rho = \frac{v_3}{v_2} = \frac{0.087}{0.034} = 2.56.$

Thermal efficiency.

b) $$\therefore \eta_{th} = 1 - \frac{1}{r_c^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\gamma(\rho - 1)}\right]$$

$$= 1 - \frac{1}{20.6^{1.4-1}} \left[\frac{2.56^{1.4} - 1}{1.4(2.56-1)}\right]$$

$$\eta_{th} = 62.7\%.$$

Workdone $W = \eta_{th} * Q.$

$$= 0.627 * 1500.$$

$$W = 941.$$

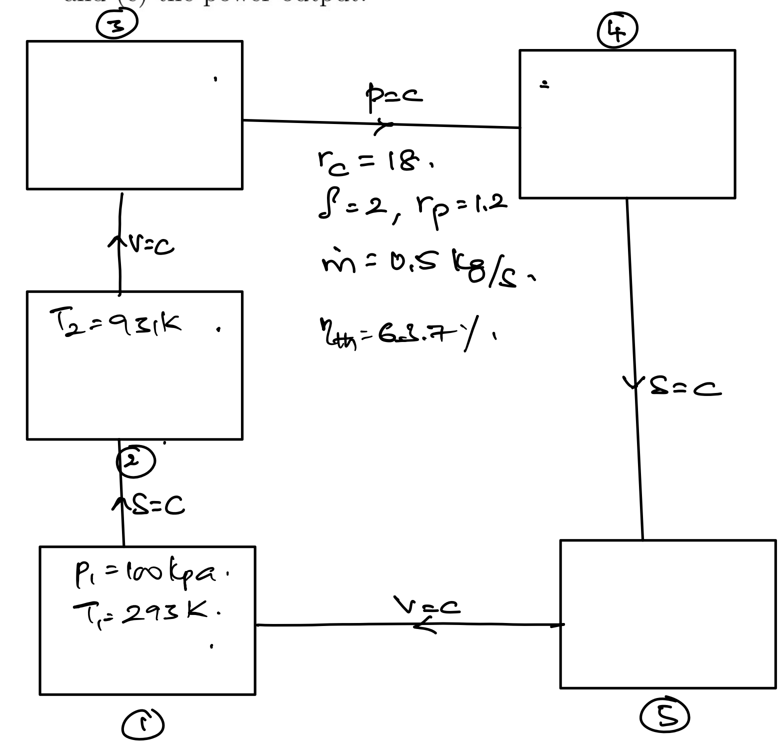
c) Mean effective pressure.

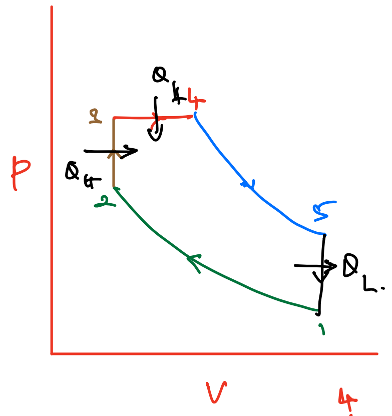
$$P_m = \frac{W}{v_1 - v_2}.$$

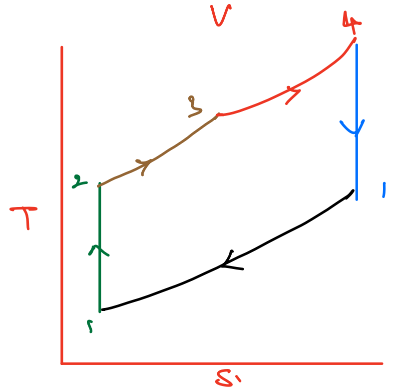
$$= \frac{941}{0.689 - 0.024}$$

$$P_m = 1437 \ \text{Kpa}.$$

10. A dual cycle with $r_c = 18$, $\rho = 2$, and $r_p = 1.2$ operates on 0.5 kg/s of air at 100 kPa and 20^0C at the beginning of the compression process. Calculate (a) the thermal efficiency, (b)the energy input, and (c) the power output.







For a dual cycle.

$$\Rightarrow \eta_{th} = \frac{W}{Q_H} = 1 - \frac{1}{r_c^{\gamma-1}}\left[\frac{\rho^{\gamma} r_p - 1}{(r_p-1)+\gamma r_p(\rho-1)}\right]$$

$$= 1 - \frac{1}{18^{1.4-1}}\left[\frac{2^{1.4}\times 1.2 - 1}{(1.2-1)+1.4\times 1.2(2-1)}\right]$$

$$\eta_{th} = 63.7\%.$$

Heat input $Q_H = m C_p(T_3 - T_2) + m C_v(T_4 - T_1)$

For the process 1-2.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} \rightarrow \frac{T_2}{T_1} = r_c^{\gamma-1}$$

$$T_2 = T_1 r_c^{\gamma-1}$$

$$= 293 \times 18^{1.4-1}$$

$$T_2 = 931\ K.$$

$$v_1 = \frac{RT_1}{P_1} = \frac{287 \times 293}{100\times 10^3}.$$

$$v_1 = 0.84\ m^3/kg.$$

$$r_c = \frac{v_1}{v_2}. \rightarrow 18 = \frac{0.84}{v_2}$$

$$v_2 = 0.046\ m^3/kg.$$

$$P_2 v_2 = R T_2$$

$$P_2 = \frac{RT_2}{v_2}.$$

$$P_2 = \frac{287\times 931}{0.046.} =$$

$$P_2 = 5.8\ Mpa.$$

For the process 2-3: $v_2 = v_3$. $P_2 v_2 = RT_2$ & $P_3 v_3 = RT_3$.

$$\frac{T_3}{T_2} = \frac{P_3}{P_2} \rightarrow r_p = \frac{P_3}{P_2}.$$

$$\therefore T_3 = \frac{P_3}{P_2} \times T_2 = r_p T_2$$

$$T_3 = 1.2 \times 931$$

$$T_3 = 1117 K.$$

For the process 3-4: $P_3 = P_4$.   ; $P_3 v_3 = RT_3$ & $P_4 v_4 = RT_4$.

$$\frac{v_4}{v_3} = \frac{T_4}{T_3} \rightarrow$$ $$\because \rho = \frac{V_4}{V_3} = 2.$$

$$T_4 = \frac{V_4}{V_3} \times T_3 = \rho . T_3.$$

$$T_4 = 2 \times 1117 = 2234 K.$$

ii) Heat supplied $\dot{Q}_H = \dot{m}\left[C_v(T_3 - T_2) + C_p(T_4 - T_3)\right]$

$$= 0.5\left[0.72 \times 10^3 (1117 - 931) + 1.005 \times 10^3 (2234 - 1117)\right]$$

$$\dot{Q}_H = 630 \text{ KW}.$$

iii) Power output/Workdone

Power $P = \frac{\text{Workdone}}{\text{sec.}} = \frac{W}{\text{sec}} = \dot{W}$

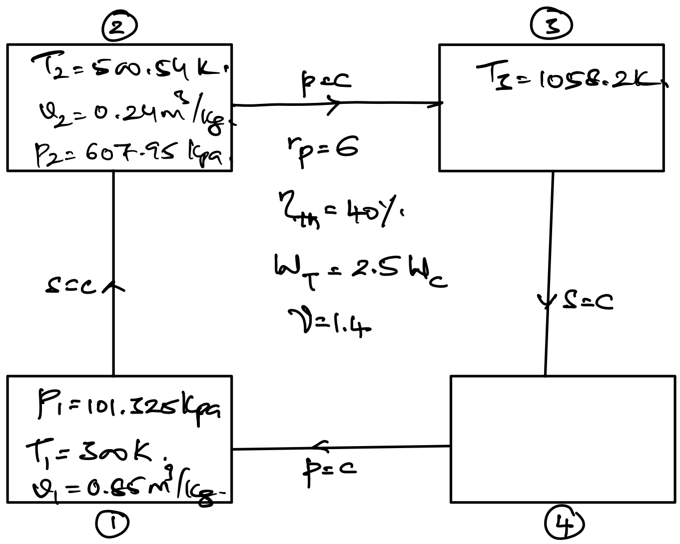
$$\eta_{th} = \frac{W}{Q_H} = \frac{P}{\dot{Q}_H}$$

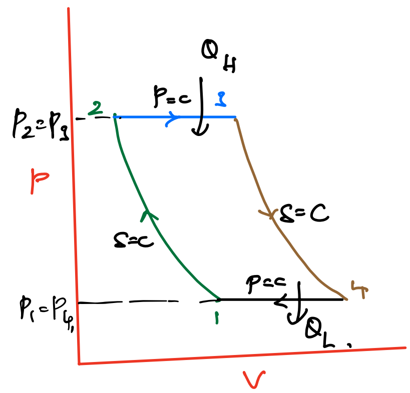
$$P = \eta_{th} * Q_H$$

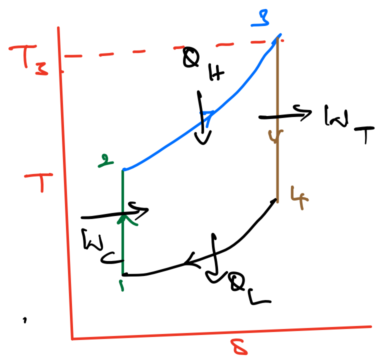
$$= 0.637 \times 630$$

$$P = 401 \text{ KW}.$$

18. Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kpa, 27^oC. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5W_C$, where W_T and W_C are the turbine and the compressor work respectively. Take $\gamma = 1.4$







For the process 1-2:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}. \qquad \left[\because\ r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}\right].$$

$$\frac{T_2}{T_1} = r_p^{\frac{\gamma-1}{\gamma}} \rightarrow T_2 = T_1\, r_p^{\frac{\gamma-1}{\gamma}}.$$

$$= 300 \times 6^{\frac{1.4-1}{1.4}}$$

$$T_2 = 500.54\,K.$$

From the characteristic gas equation

$$P_2 v_2 = RT_2. \rightarrow v_2 = \frac{RT_2}{P_2}. = \frac{287 \times 500.54}{607.95 \times 10^3}.$$

$$r_p = 6 = \frac{P_2}{P_1}$$

$$v_2 = 0.24\,m^3/kg.$$

$$P_2 = 6 \times 101.325$$

$$P_2 = 607.95\,kpa.$$

$$P_1 v_1 = RT_1 \longrightarrow v_1 = \frac{RT_1}{P_1}$$

$$= \frac{287 \times 300}{101.325 \times 10^3}.$$

$$v_1 = 0.85\,m^3/kg.$$

Thermal efficiency $\eta_{th} = 1 - \frac{1}{r_p^{\frac{\gamma-1}{\gamma}}} = \frac{W_{net}}{Q_H}$.

$$= 1 - \frac{1}{6^{\frac{1.4-1}{1.4}}}$$

$$\eta_{th} = 40\%.$$

Net workdone $W = W_T - W_C$. $\quad \because W_T = 2.5 W_C$

$$= 2.5 W_C - W_C$$

$$W = 1.5 W_C$$

As the compression process is isentropic;

$$W_C = \frac{P_2 v_2 - P_1 v_1}{\gamma - 1} \qquad \left\{\because W = \frac{P_1 v_1 - P_2 v_2}{\gamma - 1} \text{ Negative}\right\}$$

$$= \frac{607.95 \times 0.24 - 101.325 \times 0.85}{1.4 - 1}$$

$$W_C = 149.45 \text{ KJ}$$

$$\therefore W_{net} = W = 1.5 W_C = 1.5 \times 149.45$$

$$W_{net} = 224.18 \text{ KJ/kg}.$$

$$\therefore \eta_{th} = \frac{W}{Q_H} \rightarrow Q_H = \frac{W}{\eta_{th}} = \frac{224.18}{0.4}$$

$$Q_H = 560.45 \text{ KJ/kg}.$$

As heat supplied is constant pressure process

$$Q_H = m C_p (T_3 - T_2) \qquad \because m = 1\text{kg}.$$

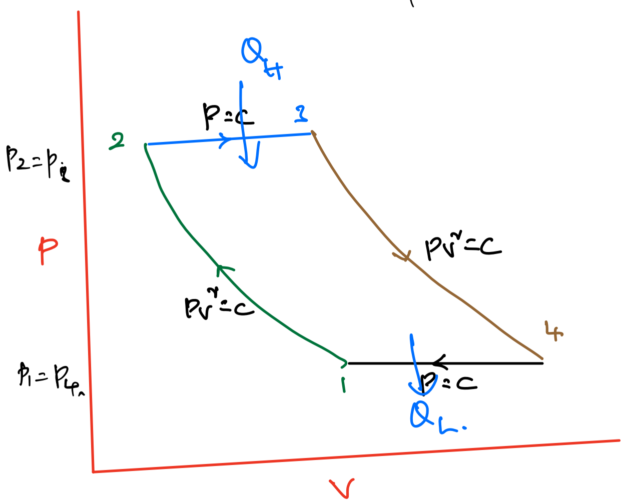
$$560.45 = 1 \times 1.005 (T_3 - 500.54)$$

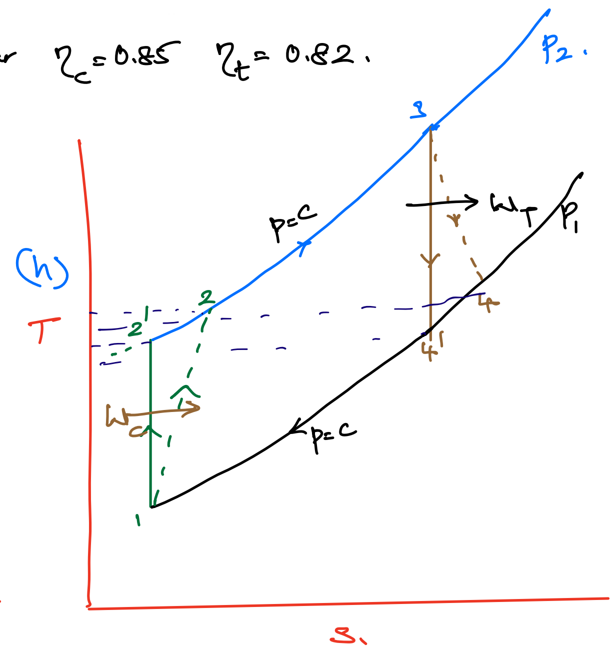
$T_3 = 1058.2\text{ K}.$ → Highest temperature in the Brayton cycle.

19. In a Gas Turbine installation, the air is taken in at 1 bar and $15^0 C$ and compressed to 4 bar. The isentropic η of turbine and the compressor are 82% and 85% respectively. Determine (i) compression work, (ii) Turbine work, (iii) work ratio, (iv) η.

Sol: $p_1 = 1$ bar; $T_1 = 288\,K$; $P_2 = 4$ bar $\eta_c = 0.85$ $\eta_t = 0.82$.

$T_3 = 825\,K$. $r_p = \frac{P_2}{P_1} = 4$





Consider the process 1-2',

$$\frac{T_2'}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \longrightarrow T_2' = T_1 (r_p)^{\frac{\gamma-1}{\gamma}} = 288 * 4^{\frac{1.4-1}{1.4}}$$

$$T_2' = 428\,K < T_2.$$

Compressor Efficiency $\eta_c = \frac{W_{isentropic}}{W_{actual}} = \frac{h_2' - h_1}{h_2 - h_1}$ $\quad$ $h \propto f(T)$ $\quad$ $h = C_p T$.

$$\eta_c = \frac{\cancel{C_p}(T_2' - T_1)}{\cancel{C_p}(T_2 - T_1)}$$

$$0.82 = \frac{428 - 288}{T_2 - 288}$$

$$T_2 = 458.7\,K.$$

Consider the process 3-4'

$$\frac{T_3}{T_4'} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}} \longrightarrow \frac{T_3}{T_4'} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \longrightarrow T_4' = \frac{T_3}{r_p^{\frac{\gamma-1}{\gamma}}} = \frac{825}{4^{\frac{1.4-1}{1.4}}}$$

$$T_4' = 555.2\,K.$$

Turbine efficiency: $\eta_t = \dfrac{W_{actual}}{W_{isentropic}} = \dfrac{(h_3 - h_4)}{(h_3 - h_4')} = \dfrac{\cancel{C_p}(T_3 - T_4)}{\cancel{C_p}(T_3 - T_4')}$

$$0.85 = \frac{825 - T_4}{825 - 555.2}.$$

$$T_4 = 595.6\ K.$$

i) Compressor Work → $W_c = C_p(T_2 - T_1)$

$$= 1.005\,(458.7 - 288)$$

$$W_c = 171.58\ kJ/kg.$$

ii) Turbine work → $W_T = C_p(T_3 - T_4)$

$$= 1.005\,(825 - 595.6)$$

$$W_T = 230.5\ kJ/kg.$$

iii) Work ratio → $W_r = \dfrac{W_{net}}{W_T} = \dfrac{W_T - W_C}{W_C}$

$$= \frac{230.5 - 171.55}{230.5}$$

$$W_r = 0.25$$

iv) Efficiency $\eta = \dfrac{W_{net}}{Q_H} = \dfrac{W_T - W_C}{C_p(T_3 - T_2)}$

$$\eta = \frac{230.5 - 171.55}{1.005\,(825 - 458.7)}$$

$$\eta = 16\%.$$